Simulation of Junction Loss in Phononic Crystals

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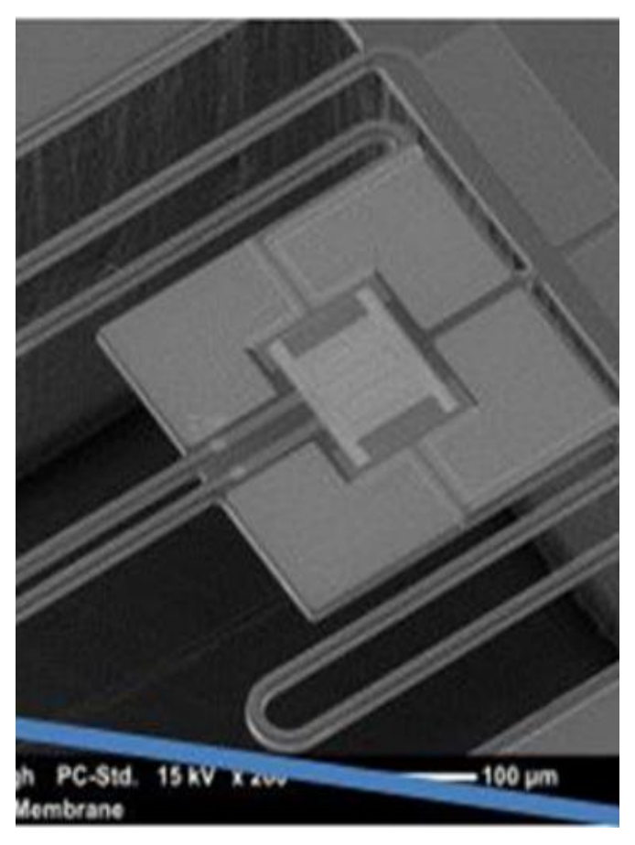
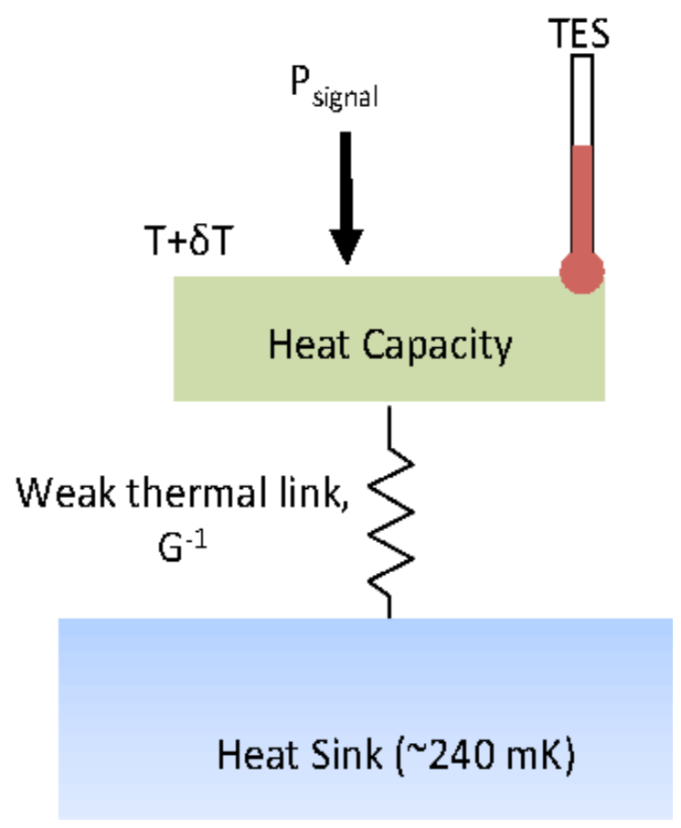
Abstract

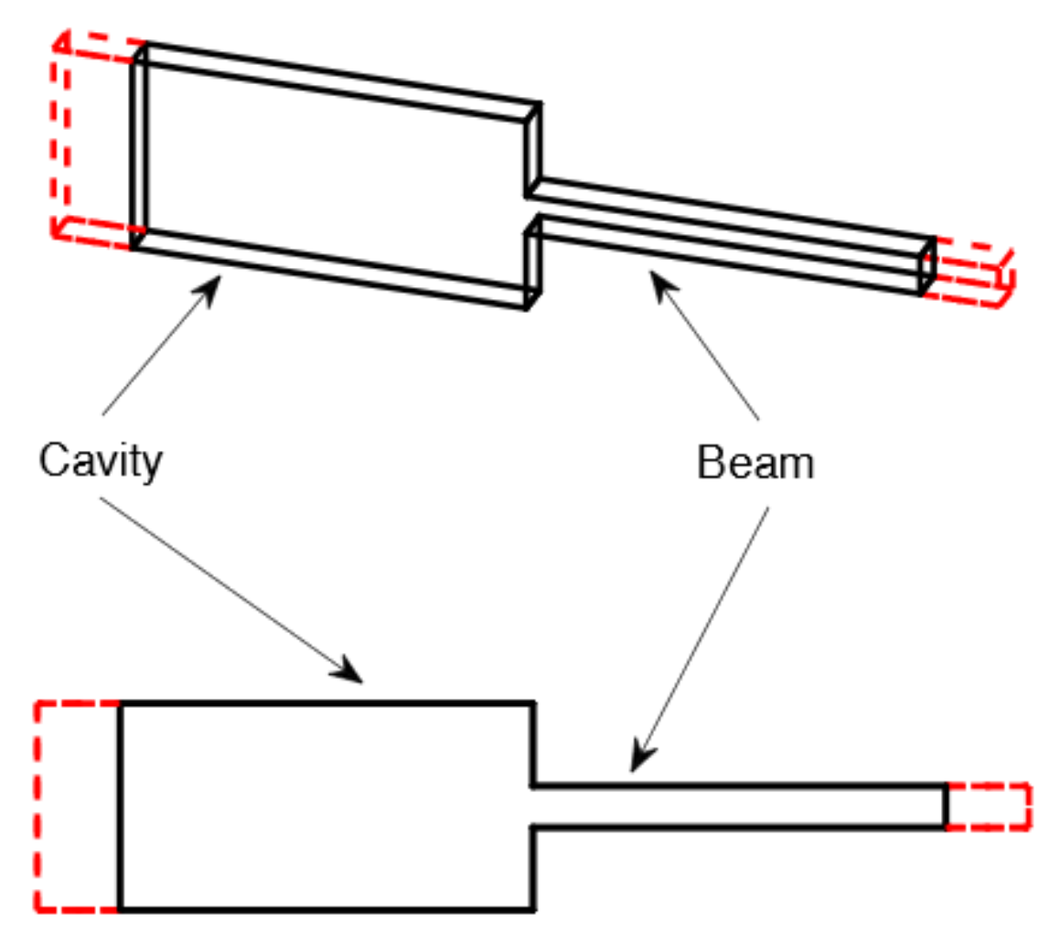
Finite element (FE) computational techniques are used to explore the transmission of vibrational waves across an abrupt geometric junction in thin single crystal silicon nitride structures. Methods of verifying accuracy of the FE models are established by determining the coupling of vibrational modes across the geometric junction in the model. Transmission coefficients of the four acoustic modes of excitement through the junction are determined with numerical simulation as a function of cavity width in units of beam thickness. Where theoretical analysis is available, simulation results are in good agreement with those predicted by thin-plate elastic theory for in-plane shear, torsional, and compressional excitation, but differ more widely for out-of-plane shear excitement. These transmission coefficients are used to characterize the quantized (phononic) thermal conductance across the junction. The results are discussed as they relate to phonon transport in far infrared wavelength cryogenic detectors.

Introduction





- Transition-edge sensors (TES) are the most advanced technology available for sensitive radiation detection
- Our application, the detection of astrophysical far infrared radiation, requires ultra-sensitive TESs
- Detector sensitivity is strongly dependent on the thermal conductance of Silicon Nitride beams linking the sense element to a thermal bath[1]
- We wish to characterize the thermal conductance across the geometric junction between the beams and cavity
- Theoretical analysis of such a junction beyond the 2-dimensional case is not tractable



- We seek to obtain transmission probabilities T_α across the junction with FE, which can then be related to thermal conductance[2]:

$$G(T) = \frac{k_B{}^2 T}{2\pi\hbar} \sum_\alpha \int_{x^c{}_\alpha}^{\infty} \mathrm{T}_\alpha(x) \frac{x^2 e^x}{(e^x - 1)^2} dx$$

where $x = \hbar\omega / k_B T$

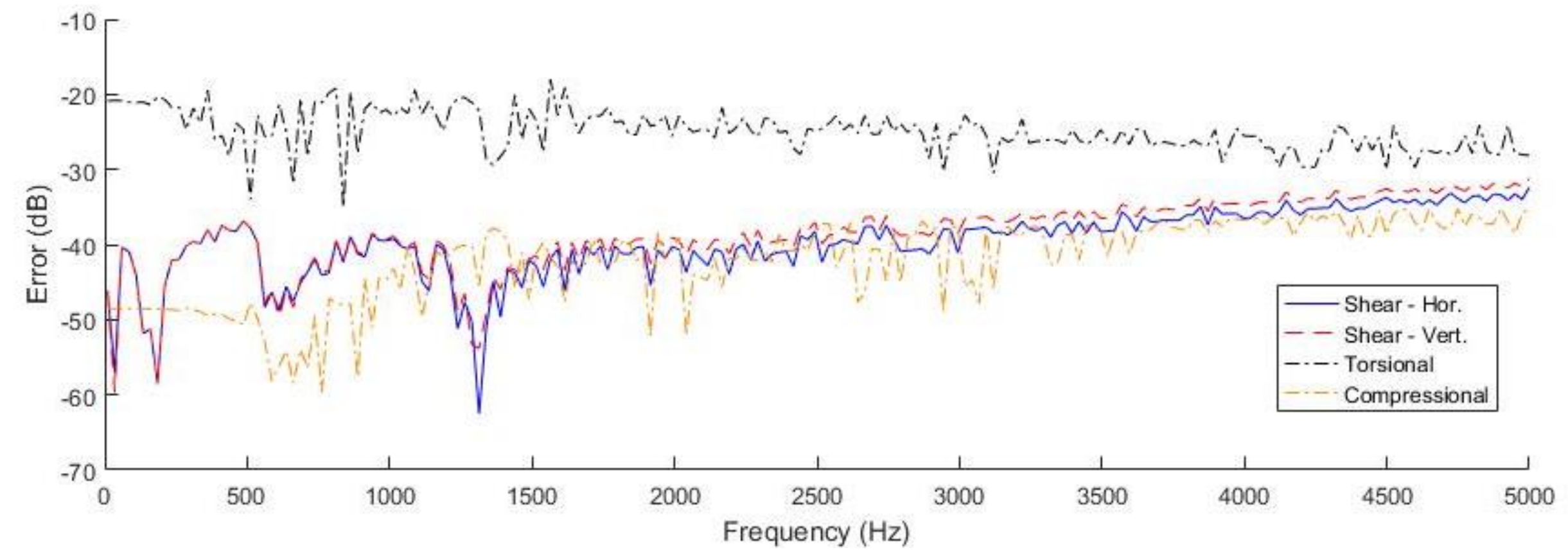
Methodology

- Transmission is determined by placing a vibrational source and probe in the beam. The complex response U is related to the response U_t for a infinite beam by the relationship
$$U = U_t + \rho U_t e^{i\phi}$$
- $\rho = 1$ for a fixed end rather than a connection to a cavity
- Therefore ρ is defined
$$|\rho|^2 = |(u_s - 1)/(u_r - 1)|^2$$
where u_r and u_s are the response with a stress-free end and an attachment to a cavity respectively, normalized by U_t

Boundary Conditions:

- Perfectly matched layers (PMLs) are used to simulate infinite or semi-infinite media
- COMSOL PMLs require a characteristic wavelength to target for simulation of perfect absorption
- Dispersion curves of the beam are generated with COMSOL Eigenmode Solver for the selection of such a characteristic wavelength for each excitation mode and frequency.

Methodology (continued)

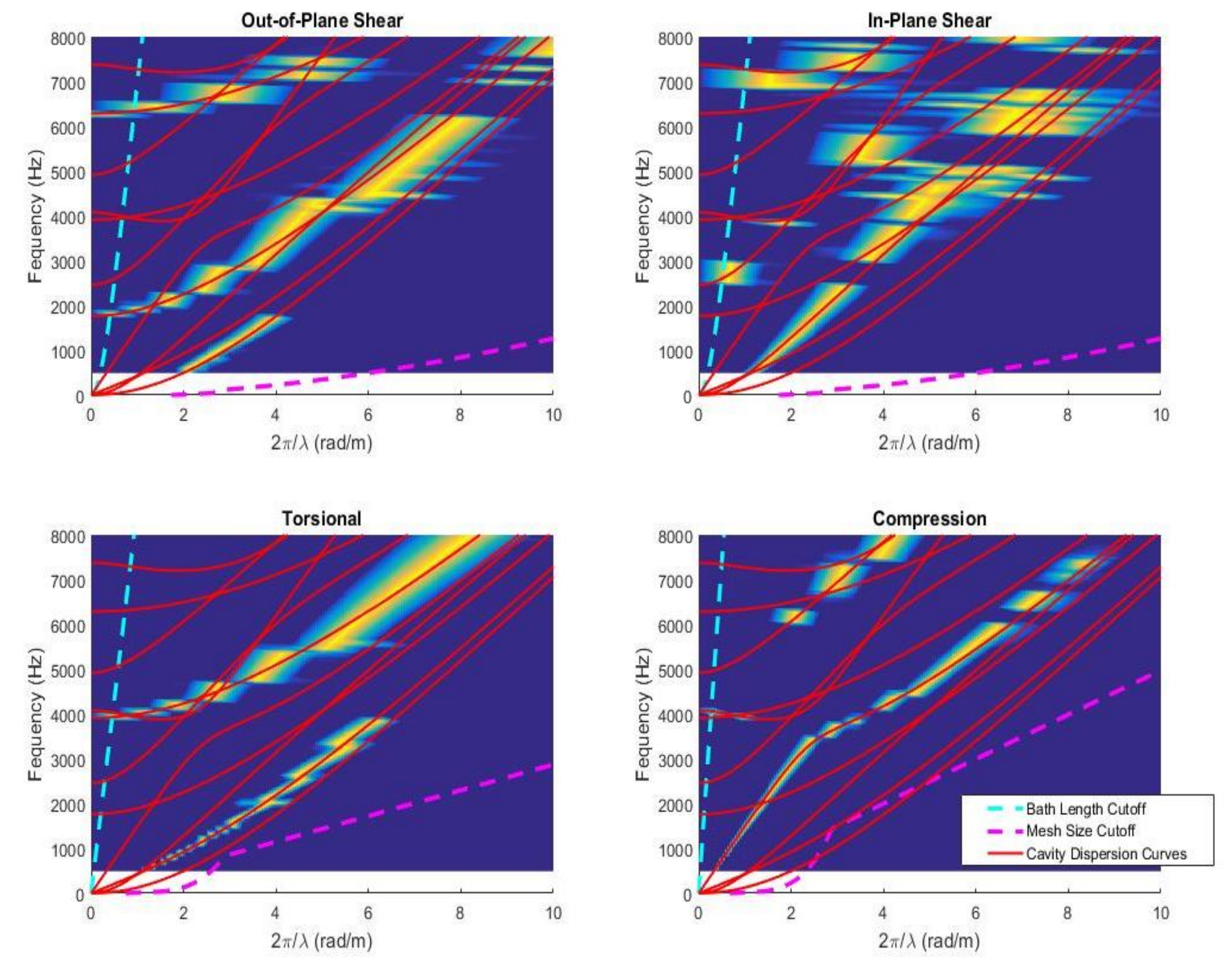


Calibrating the Beam Model:

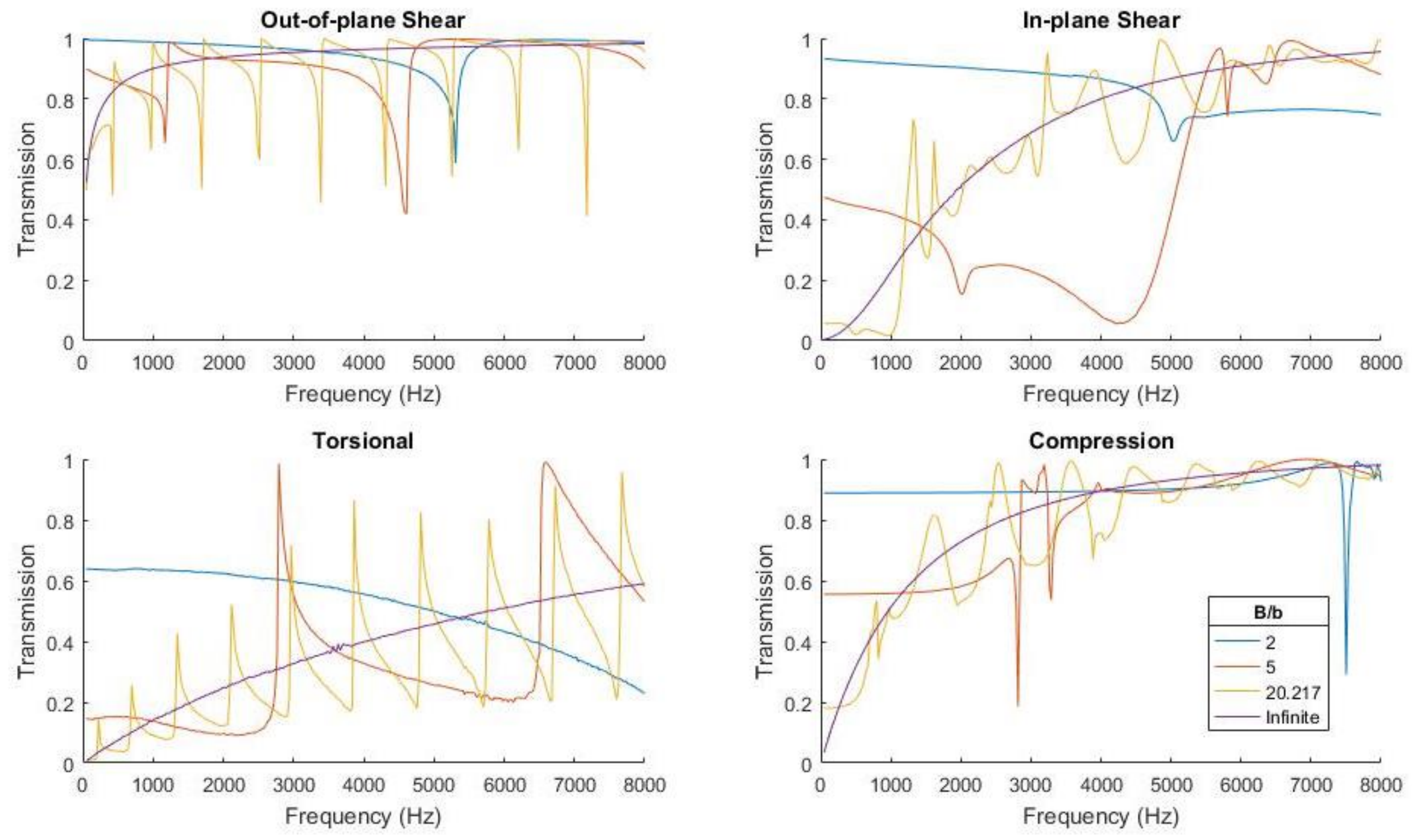
- Beam model is scaled to the characteristic wavelength of the mode and frequency being excited
- 'Thru-Reflect' calibration is performed to verify beam model accuracy[3]; accurate to -20 to -30 dB for torsional excitement, < -40dB for 3 other acoustic modes

Calibrating the Cavity Model:

- Cavity model is more difficult because determining which modes are excited for a certain beam excitation is nontrivial
- A Fourier transform is taken of the response along an edge of the cavity, excited wavelengths are plotted as a function of excitation frequency to determine mode couplings
- Model is established based on this coupling such that the cavity is large enough and mesh is fine enough to capture all excited modes of interest

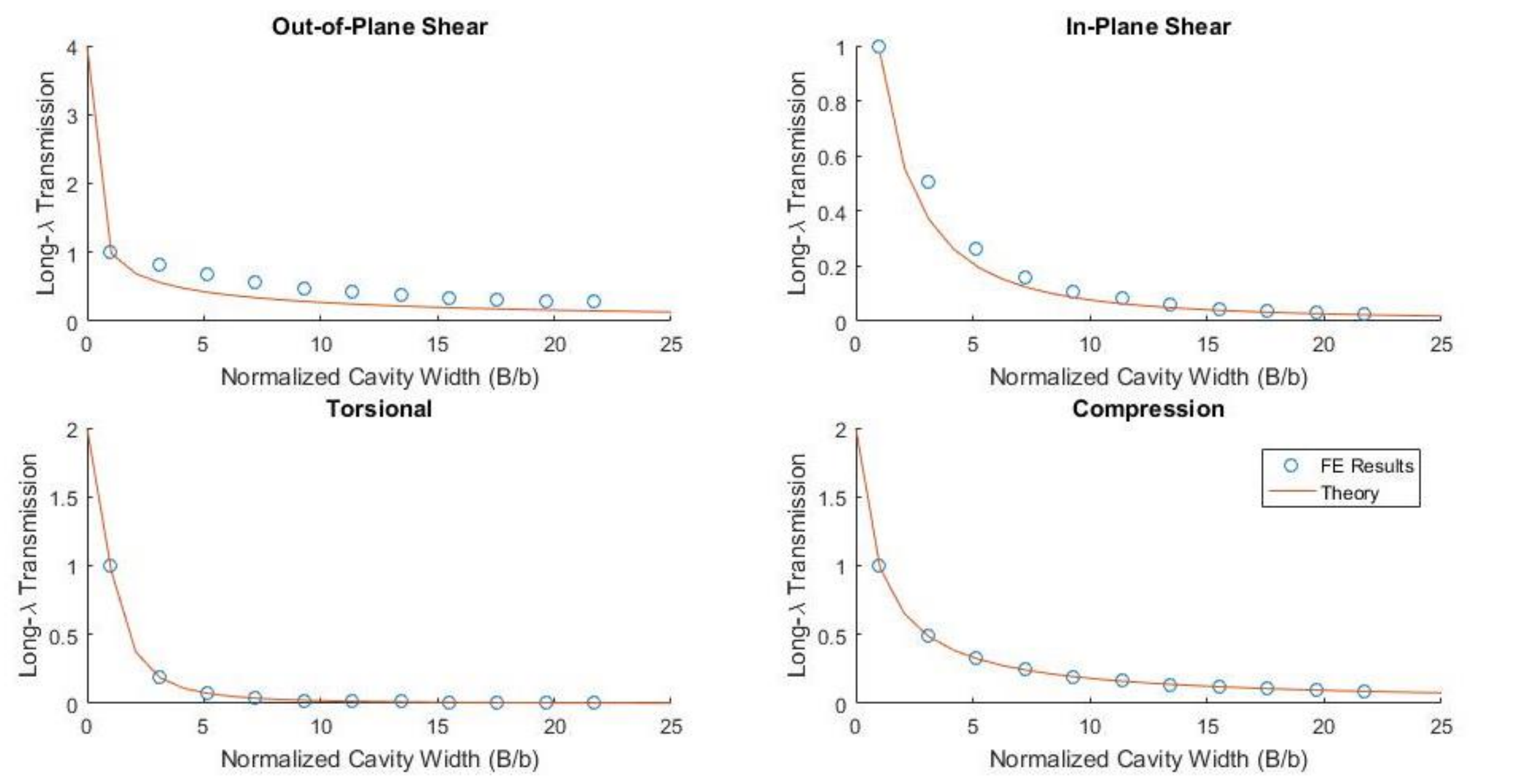


Results



- Having established reliable methods of modeling junction, we evaluate transmission for frequencies up to 8000 Hz, excited in each of the four acoustic modes
- Cavities of geometry parameter B/b = 2, 5, and 20.2 are evaluated, where B is cavity width and b is beam width
- An infinite cavity is also simulated by using a curved PML forming a 180 degree are around the junction as the boundary of the cavity

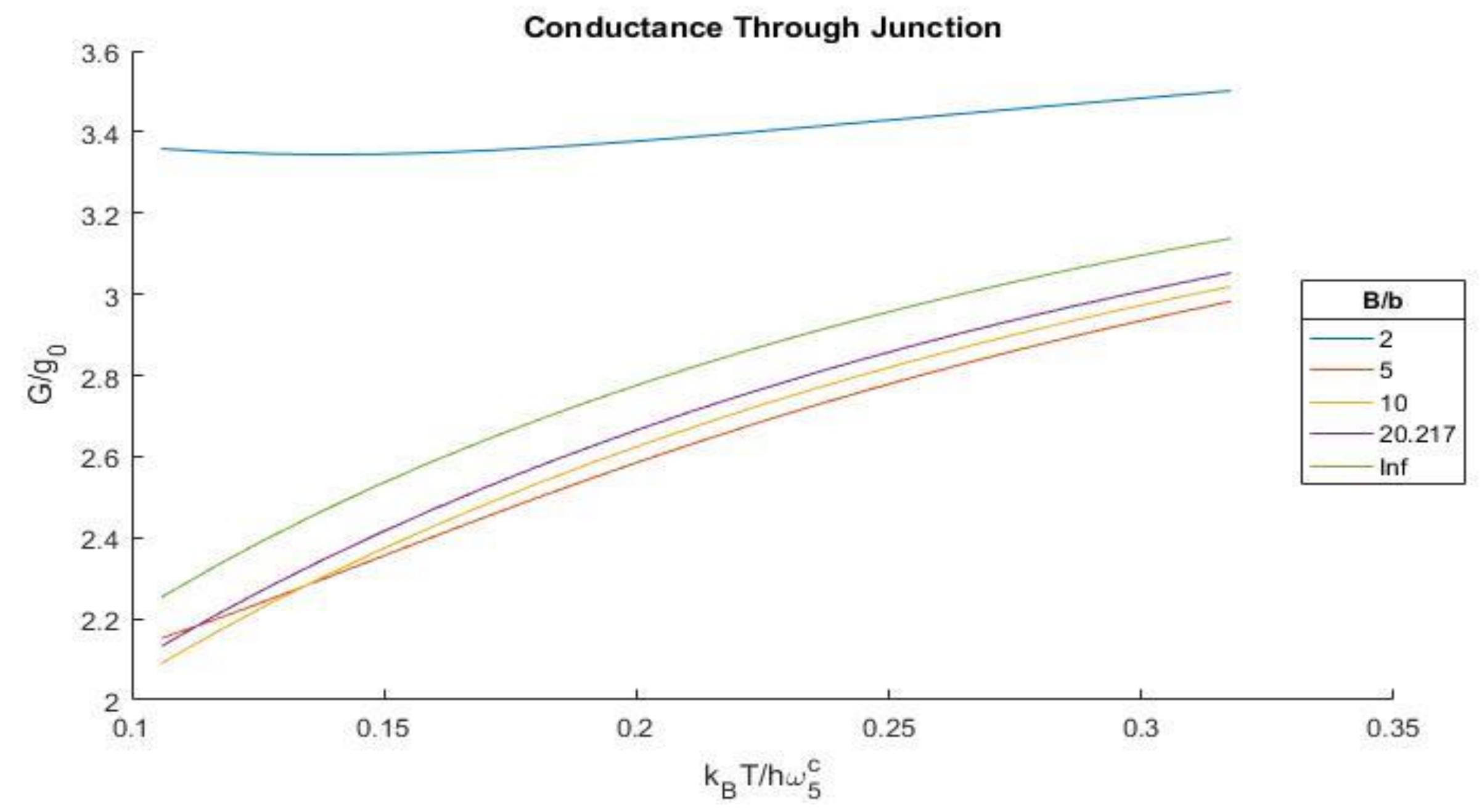
Results (continued)



- Cross and Lifshitz[4] provide analytical results for transmission probability in the long-wavelength limit
- In-plane, torsional, and compression excitation modes are in close agreement with available theoretical predictions
- Out-of-plane excitation differs from theory for narrow cavities but converges with theoretical predictions as cavity width increases

Conclusions & Next Steps

- Characterization of relationship between conductance and temperature based on transmission through the junction is shown below. Conductance and temperature are normalized by $g_0 = k_B{}^2 \pi T / 6\hbar$ and $\hbar\omega_5{}^c / k_B$ respectively, where $\omega_5{}^c$ is the cutoff frequency of the first optical mode
- These simulations have given a reliable picture of the conductance through the junction; however, the beam and cavity were assumed to be infinitely long, so it is only an approximation of a TES junction
- Next step is to model phonon transport with the true geometry of the detector.



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References

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