Relationships between complex impedance, thermal response, and noise in microcalorimeters and bolometers

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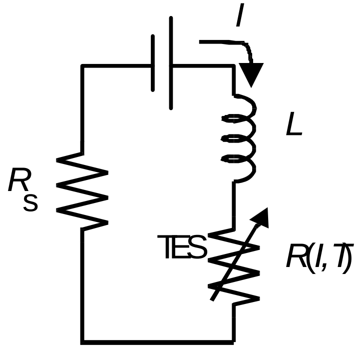
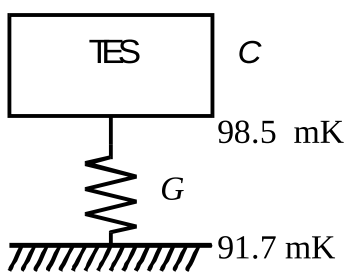
First order model

Electrical Circuit



Thermal Circuit



$$\frac{d}{dt}\begin{pmatrix}I\\T\end{pmatrix} = \begin{pmatrix} -\frac{1}{L}\left(R_s+R+I_0\frac{\partial R}{\partial I}\right) & -\frac{I_0}{L}\frac{\partial R}{\partial T}\\ \frac{I_0 R}{C}\left(2+\frac{I_0}{R}\frac{\partial R}{\partial I}\right) & \frac{I_0^{2}}{C}\frac{\partial R}{\partial T}-\frac{G}{C}\end{pmatrix}\begin{pmatrix}I\\T\end{pmatrix} + \text{NOISE}$$

$$Z_{TES}(\omega) = R\left((1+\beta)+\frac{2+\beta}{2}\frac{I_0^{2}R}{CT}\alpha\tau_{eff}\left(-1+\frac{1+i\omega\tau_{eff}}{-1+i\omega\tau_{eff}}\right)\right)$$

Free parameters

$$\alpha \equiv \frac{T_0}{R}\frac{\partial R}{\partial T} \qquad \beta \equiv \frac{I_0}{R}\frac{\partial R}{\partial I}$$

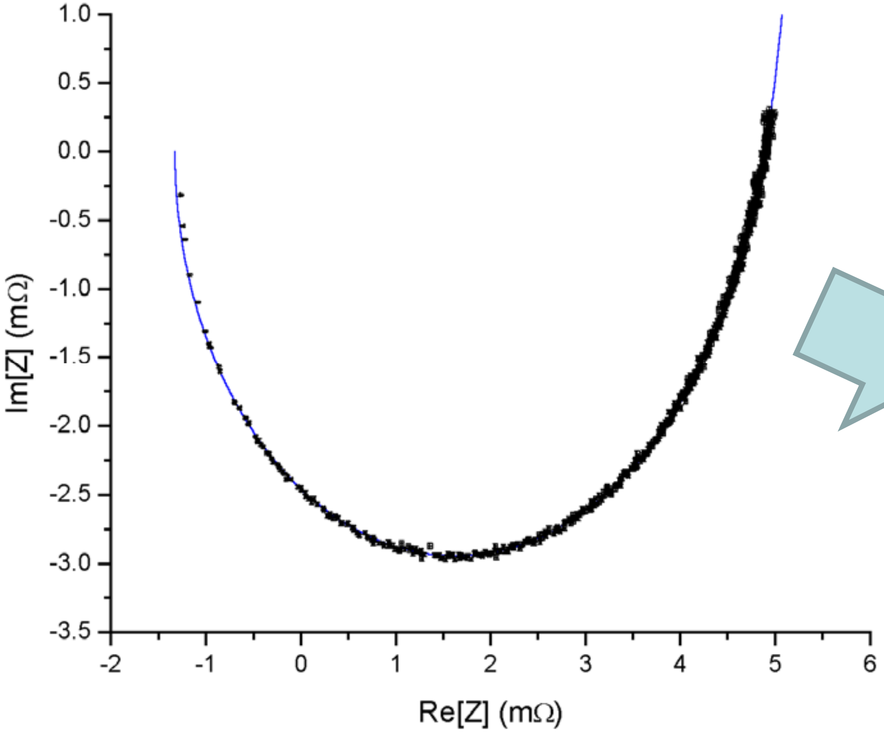
L = Inductance

C = Heat capacity

$$R(I,T) \cong R_0\left(1+\alpha\frac{\Delta T}{T_0}+\beta\frac{\Delta I}{I_0}\right)$$

Fit to impedance to find parameters

$$Z(f) = R_s + i(2\pi f)L + Z_{\mathrm{TES}}(f)$$



<u>Measured parameters</u>

G=344 pW/K

I_0=29.5 μA

R=2.05 mΩ

R_S=1.45 mΩ

T_0=98.5 mK

<u>Free parameters and statistical errors</u>

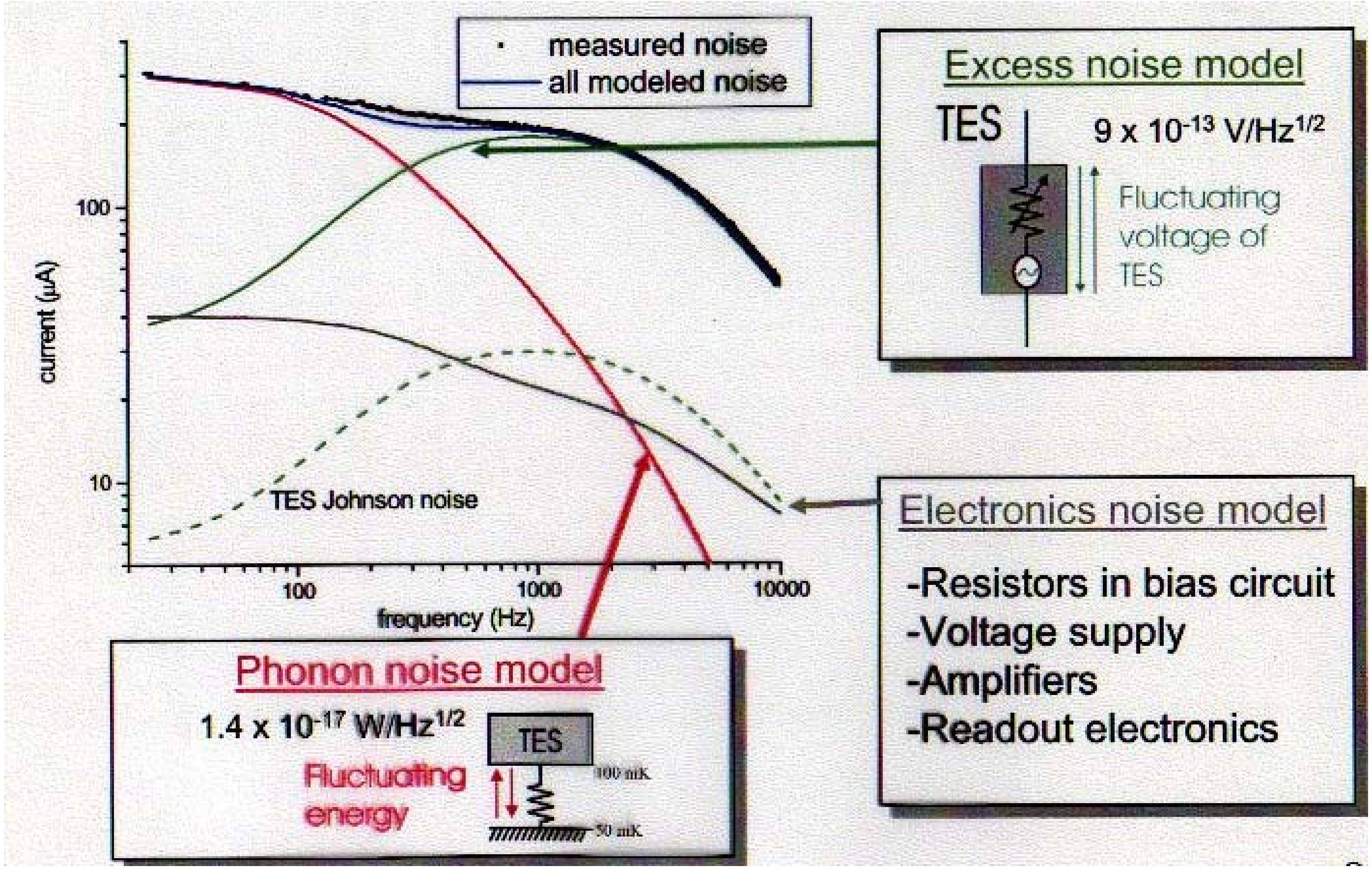
α=172.3 +/- 0.7

β= 0.8518+/- 0.0008

C=(2.034 +/- 0.008) pJ/K

L=(262.9 +/- 0.6) nH

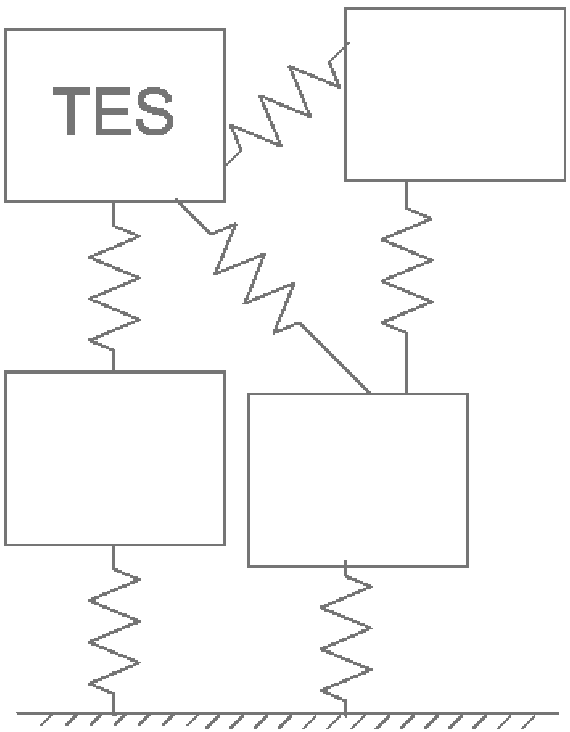
Use parameters to calculate noise



Complex calorimeters

$$M_S \begin{bmatrix} \Delta I \\ \Delta T \\ \Delta T_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} V_{JN} + V_{Stimulus} \\ -I_0 V_{jN} + P_1 \\ P_2 \\ \vdots \end{bmatrix}$$

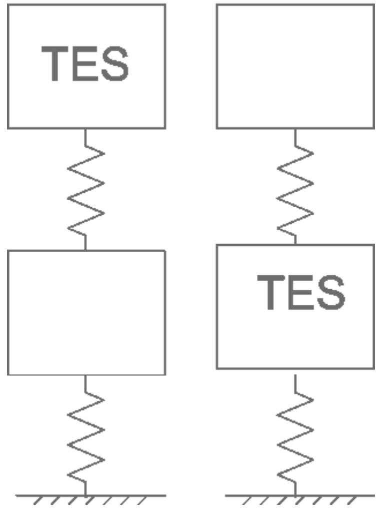
$$M_S(\omega) = \begin{bmatrix} Z_{eq} + R_0(1+\beta_I) & R_0 I_0 \frac{\alpha_I}{T} & 0 & \cdots \\ -R_0 I_0 (2+\beta_I) & i\omega C_1 - P\frac{\alpha_I}{T} + G_{11} & G_{12} & \\ 0 & G_{21} & iwC_2 + G_{22} & \\ \vdots & & & \ddots \end{bmatrix}$$

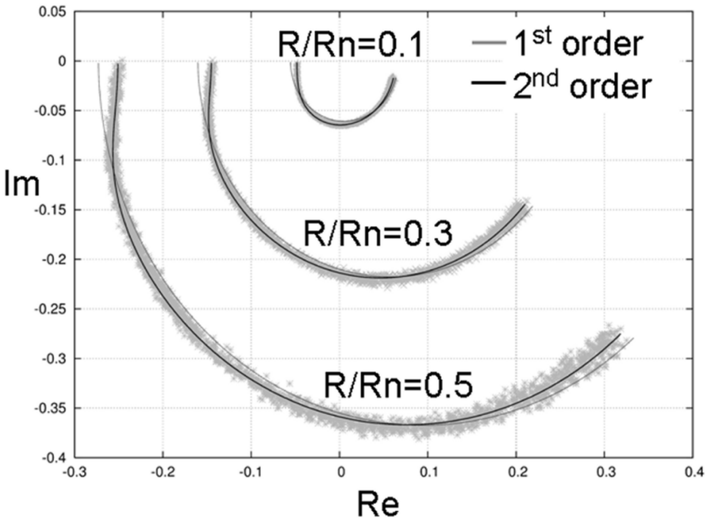


Model and fit to find
- Additional thermal couplings
- Hanging heat capacities

Similar models

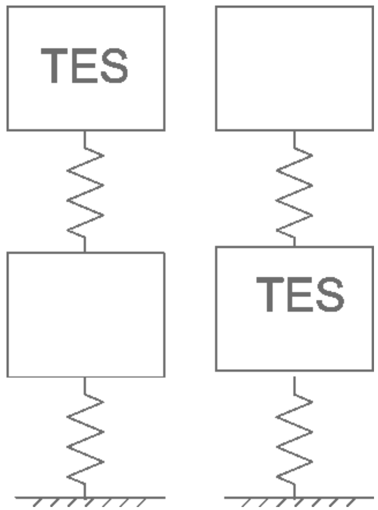
- Two different hypotheses of where extra heat capacity is
- Both described by 3x3 matrix
- Can fit impedance equally well
- Fitting gives about the same α and β
- Produce different noise terms
- Total noise appears to be about the same



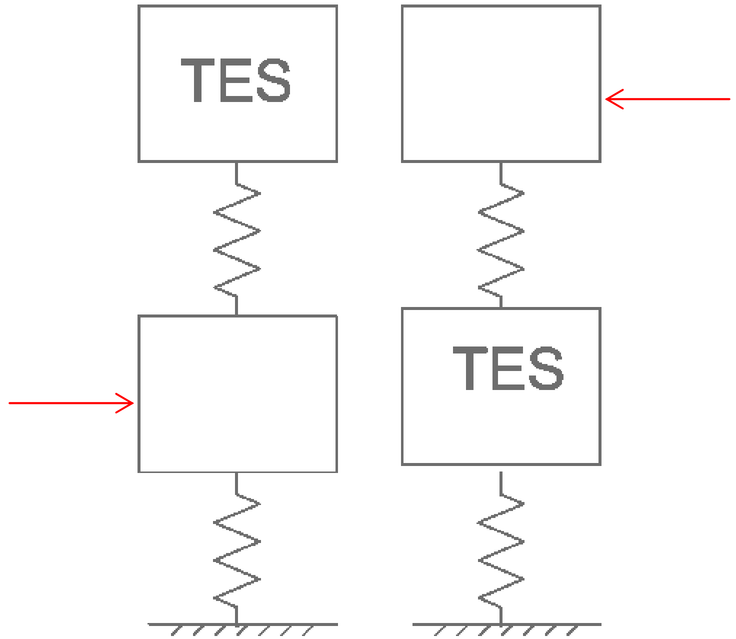


Is there a simple relationship between Z, signal, and noise?

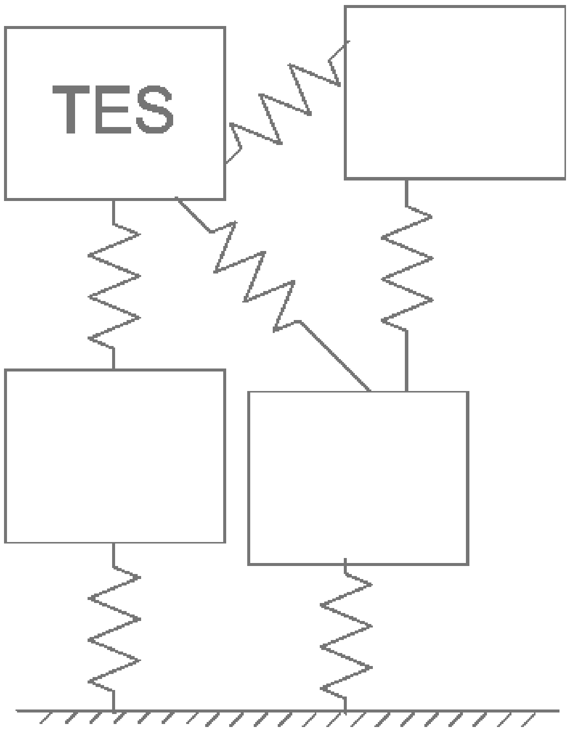
- Heat capacities and thermal conductances differ
- The individual phonon noise terms differ between the two models
- However, Z is the same
- The total calculated signal and noise appeared to be the same
- Is this true in general?
- **Is there a simple relationship between Z, signal, and noise?**

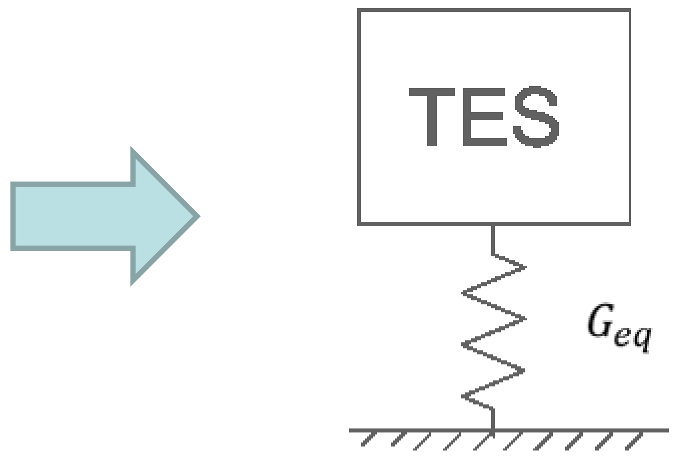


Distinguishable by injecting power into hanging heat capacities



Simplify math to find answer





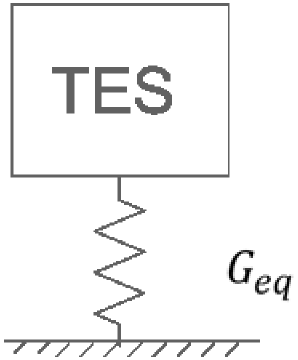
Equivalent thermal conductance

$$G_{eq}(\omega) = \frac{R_0 + Z(\omega)}{R_0 + Z_0} \frac{Z_\infty - Z_0}{Z_\infty - Z(\omega)} G_0$$

The real thermal conductance from the TES to the bath which is measured by I-V curves

Equivalent bolometer

- Equivalent in two ways
  - Response to power in the TES is the same
  - Response to noise in the Bias circuit is the same
- Models with same equivalent bolometer have same impedance
- Do they have the same total noise?

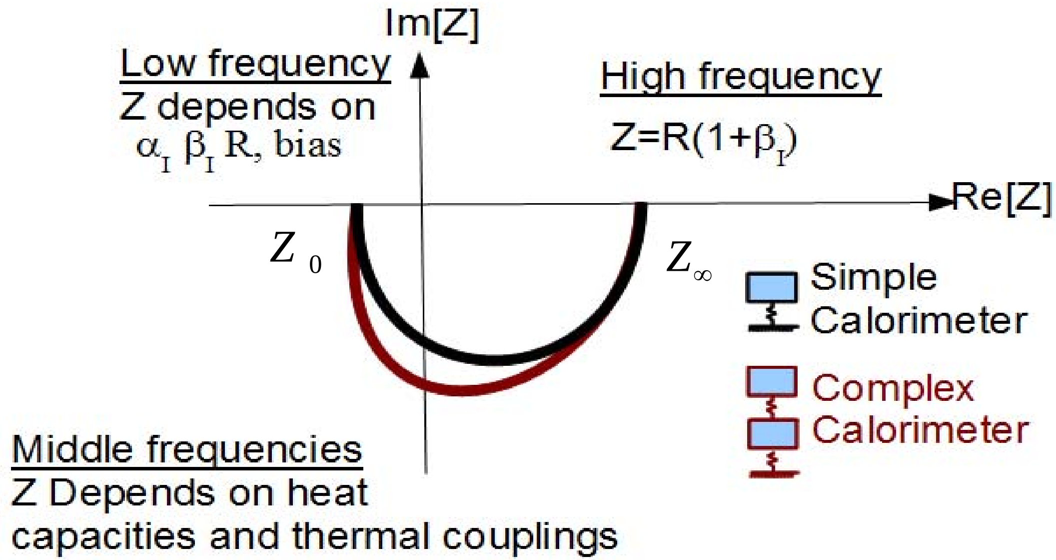


Model independent parameters

- α and β are a model independent function Z
- Should be able to get these directly from Z at two frequencies
- No modeling

$$\alpha_I = \frac{G_0 T_0}{P} \frac{Z_0 - Z_\infty}{Z_0 + R_0}$$

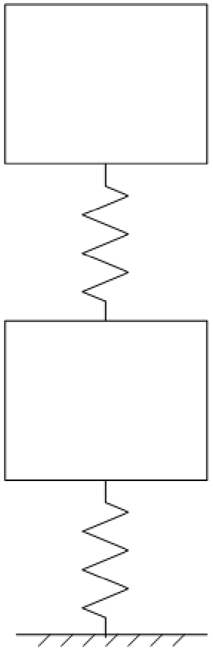
$$\beta_I = Z_\infty / R_0 - 1$$



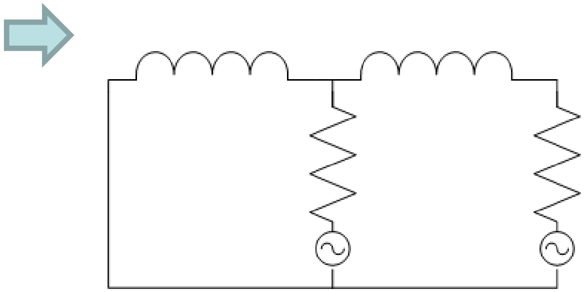
Circuit analogy to compute noise

Thermal	Electrical
Temperature	Current
Heat capacity	Inductor
Phonon noise	Fluctuating voltage noise
Thermal conductance	Resistance

Thermal Circuit

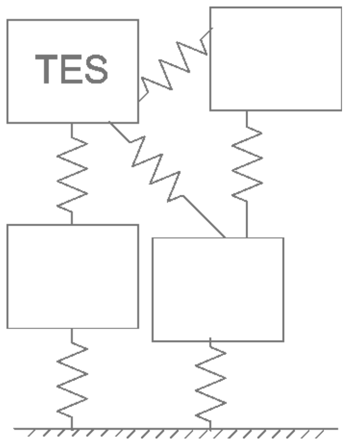


Electrical analog



Fluctuation-dissipation

- For a network of resistors, inductors, and capacitors at the same temperature noise is $V = \sqrt{4kTRe[Z]}$
- Want a similar expression for noise in a network capacitances and thermal couplings in a bolometer



Noise in equivalent bolometer

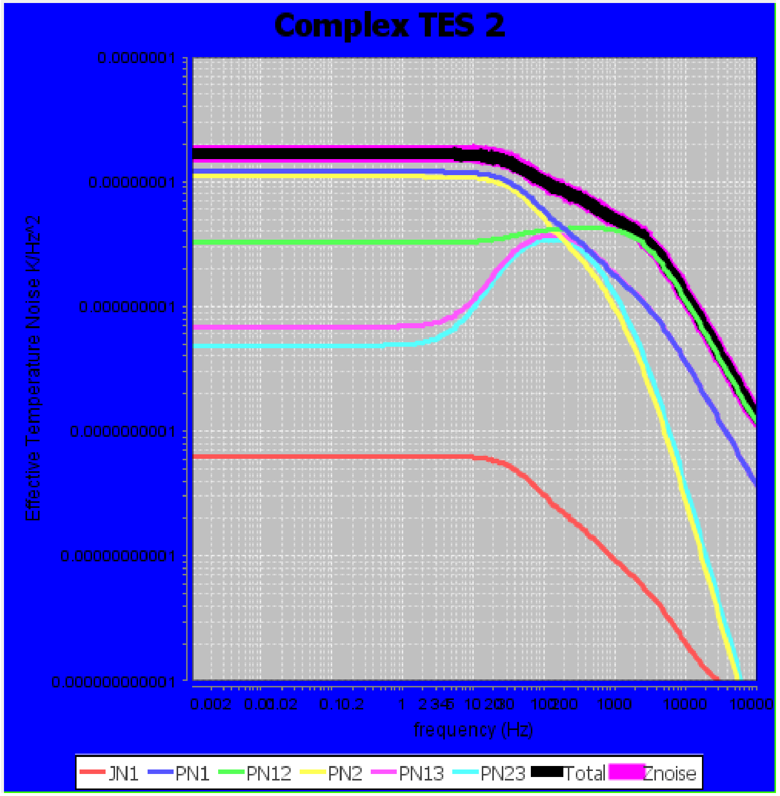
- Assuming that the bath temperature is near the temperature of the TES
- Noise $P_{TN}(\omega) = \sqrt{4 k_B T^2 \,\mathrm{Re}[\,G_T(\omega)]}$
- Noise is a function of the complex impedance because $G_T(\omega) = \frac{R_0 + Z(\omega)}{R_0 + Z_0} \frac{Z_\infty - Z_0}{Z_\infty - Z(\omega)} G_0$
- Similar to Johnson noise of resistors
- Can also calculate the Johnson noise of the bolometer from the impedance
- Don’t have to know a lot about internal structure of a bolometer to estimate thermal noise

Bath temperature complicates things

- The previous calculation assumed all the elements were at the same temperature
- If TES is at twice bath temperature, <21% error
- If TES is 10% above bath temperate,< 5% error
- Only affects frequencies where external noise dominates
- Closer approximation may be possible

Numerical verification

- Computed noise for a number of different complex models
- Computed noise directly from Z
- They agree



Conclusions

- Found a relationship between impedance, signal and noise
  - Useful for validating calculations and experiments
- Can find α and β without model fitting by measuring impedance
- Difficult to distinguish between models that reproduce the same impedance on the bases of noise measurements
- Detector response also similar between models if absorber is well coupled to TES in the models
- Modeling may not be needed to estimate noise from impedance because it is fairly model independent