Microscopic model of a transition edge sensor as a weak link

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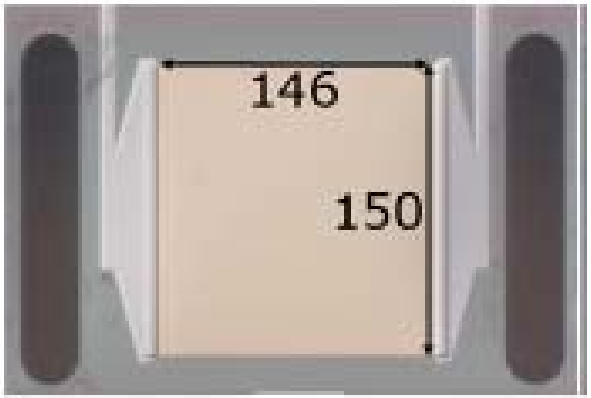
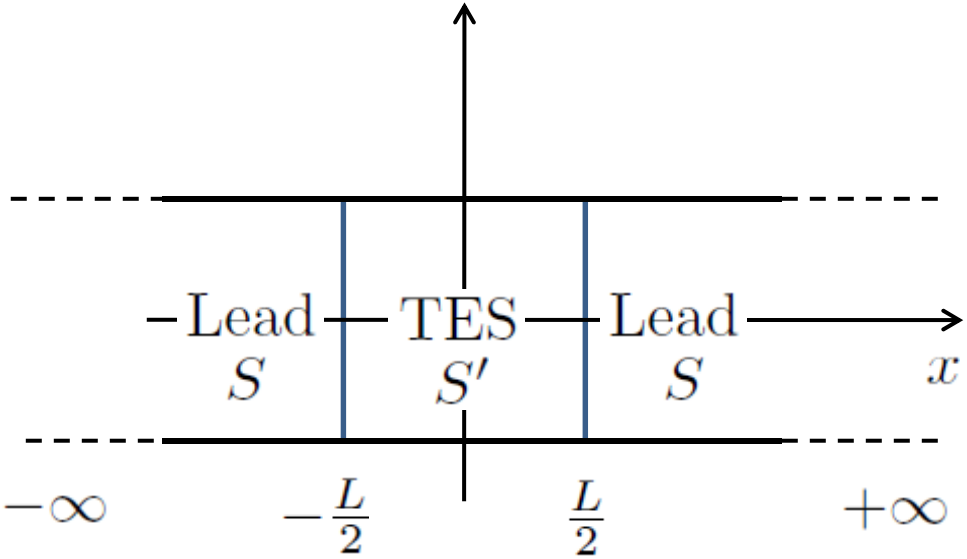
- TES AS AN INHOMOGENEOUS SUPERCONDUCTING SYSTEM.
- THE USADEL EQUATIONS. BOUNDARY CONDITIONS.
- TES AS A LATERAL SS´*S* SYSTEM. ANALYTICAL MODEL.
  THE ORDER PARAMETER PROFILE. ANALYTICAL SOLUTIONS FOR *1D SS´S* SYSTEM WITH WEAK CURVATURE PROFILES.
- NUMERICAL MODELLING

The Usadel equations. 1D geometry



a)

FIGURE 1. a) Bare TES, Ti/Au/Ti (20/50/5 nm thick)



The Usadel equations. Matsubara representation

$$\hat{g} = \begin{pmatrix} g_{\omega_n}(\mathbf{r}) & f_{\omega_n}(\mathbf{r}) \\ f^{\dagger}_{\omega_n}(\mathbf{r}) & -g_{\omega_n}(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \cos(\Theta_n(\mathbf{r})) & \sin(\Theta_n(\mathbf{r}))\exp(i\chi_n(\mathbf{r})) \\ \sin(\Theta_n(\mathbf{r}))\exp(-i\chi_n(\mathbf{r})) & -\cos(\Theta_n(\mathbf{r})) \end{pmatrix}$$

the Matsubara frequency is $\omega_n = \pi T(2n+1)$, $n = 0, \pm 1, \pm 2...$

$$\frac{d^2\Theta_n}{dx^2} - \left[\omega_n + \left(\frac{d\chi_n}{dx}\right)^2 \cos(\Theta_n)\right] + |\Delta|\cos(\Theta_n) = 0$$

$$\frac{d}{dx}\left(\sin^2(\Theta_n)\frac{d\chi_n}{dx}\right) = 0$$

x is in units of $\xi_n = \sqrt{\frac{\hbar D}{2\pi T_c}}$

ω_n and Δ are in units πT_c

Self-consistency equations:

$$\Delta(\mathbf{r})\ln\left(\frac{T}{T_c}\right) + 2\frac{T}{T_c}\sum_{\omega>0}\left[\frac{\Delta(\mathbf{r})}{\omega_n} - \sin(\Theta_n)\exp(i\chi_n)\right] = 0$$

$$\mathbf{j}_s(\mathbf{r}) = 2ieN(0)\pi TD\sum_{\omega>0}\left[f^*(\omega,\mathbf{r})\hat{\partial}f(\omega,\mathbf{r}) - f(\omega,\mathbf{r})\hat{\partial}f^*(\omega,\mathbf{r})\right]$$

$$\hat{\partial} \equiv \nabla + 2ie\mathbf{A}(\mathbf{r})$$

The Usadel equation. Boundary conditions.

$$\frac{d^2\theta}{dx^2} - \left[\omega + C^2(\omega)\frac{\cos\theta}{\sin^4\theta}\right]\sin\theta + |\Delta|\cos\theta = 0$$

$$C(\omega) = \frac{\xi_N j_s}{\sigma_N \pi T_c \Delta_L}\frac{\Delta_L^2}{\omega^2 + \Delta_L^2}$$

Boundary conditions.

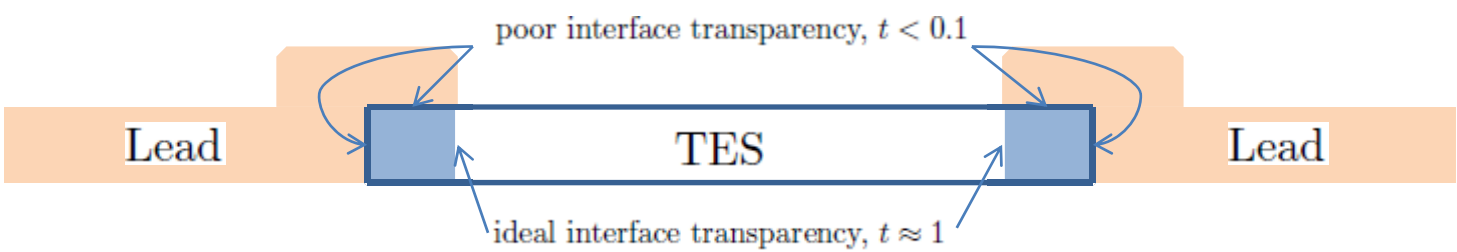
$$\left.\frac{d\theta_L}{dx}\right|_{\pm\frac{L}{2}} = \pm\frac{g_B\xi_L}{\sigma_L}\sin(\theta_L-\theta)$$

$$\left.\frac{d\theta}{dx}\right|_{\pm\frac{L}{2}} = \pm\frac{g_B\xi_L}{\sigma_N}\sin(\theta_L-\theta)$$

ideal interface, $t \approx 1$

rigid boundary conditions $\theta = \theta_L$

$\frac{g_B\xi_L}{\sigma_L} << 1$, no bending of θ_L in the leads $\Longrightarrow$ $\theta_L = \arctan\frac{\Delta_L}{\omega}$



vertically proximised volume

TES as a lateral SS´S system. Analytical model

$$\frac{L}{2\xi_N} = \int_{\theta(0)}^{\theta(L/2)} \frac{\mathrm{d}\theta}{\sqrt{2\omega\left[\cos\theta(0) - \cos\theta\right] - C^2(\omega)\left[\sin^{-2}\theta - \sin^{-2}\theta(0)\right] - 2\int_{\theta(0)}^{\theta}\mathrm{d}\theta'\cos\theta'\Delta(\theta')}}$$

$$\frac{\chi}{2C(\omega)} = \int_{\theta(0)}^{\theta(L/2)} \frac{\mathrm{d}\theta}{\sin^2\theta\sqrt{2\omega\left[\cos\theta(0) - \cos\theta\right] - C^2(\omega)\left[\sin^{-2}\theta - \sin^{-2}\theta(0)\right] - 2\int_{\theta(0)}^{\theta}\mathrm{d}\theta'\cos\theta'\Delta(\theta')}}$$

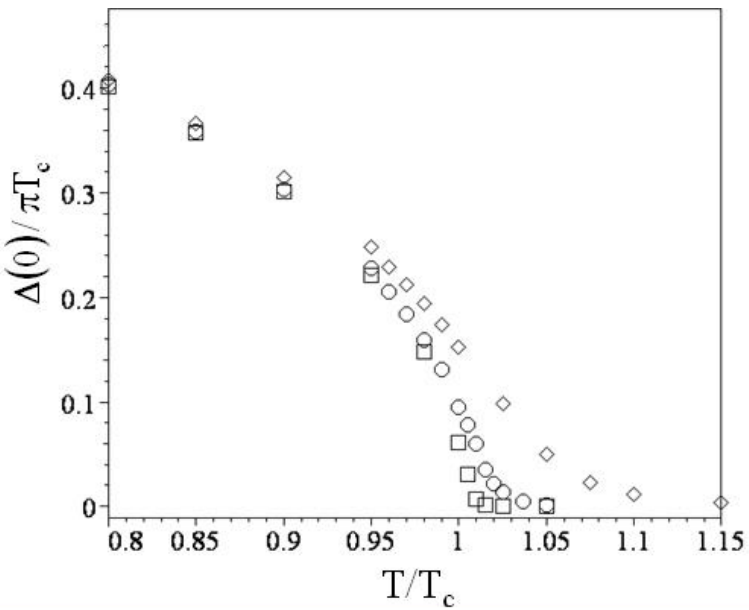
Small profile curvature: quadratic expansion

$$\theta(0) = \frac{\Delta(0)}{\omega - \frac{4\xi_N^2}{L^2}\ln^2\left[2\frac{\theta(L/2)}{\theta(0)}\frac{1}{\sqrt{1+\tan^2\chi/2}}\right]}$$

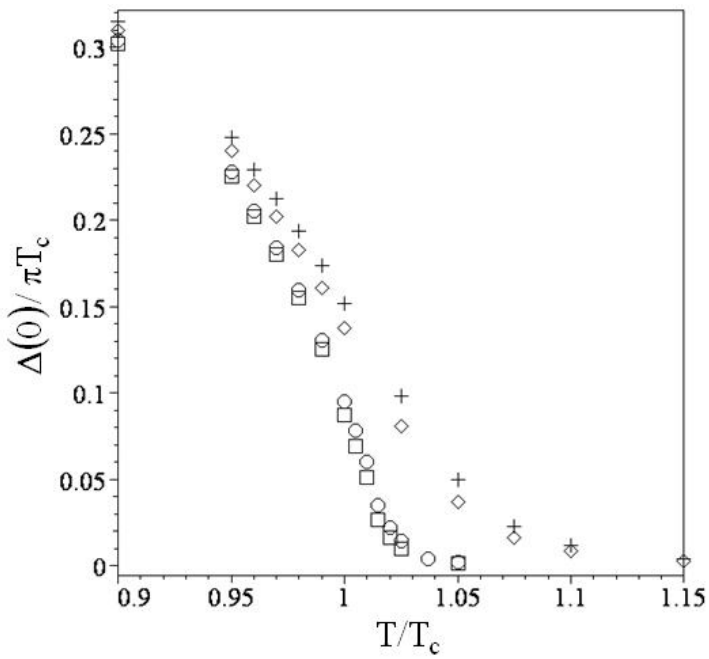
$$\sum_{\omega>0}\left\{\omega - \frac{4\xi_N^2}{L^2}\ln^2\left[2\frac{\omega\theta(L/2)}{\Delta(0)}\right]\right\}^{-1} = \frac{2T}{T_c}\left(0.5772 + \ln\frac{2T_D}{\pi T_c}\right) \qquad \chi = 0$$

$$\sum_{\omega>0}\left\{\omega - \frac{4\xi_N^2}{L^2}\ln^2\left[\sqrt{2}\frac{\omega\theta(L/2)}{\Delta(0)}\right]\right\}^{-1} = \frac{2T}{T_c}\left(0.5772 + \ln\frac{2T_D}{\pi T_c}\right) \qquad \chi = \frac{\pi}{2}$$

TES as a lateral SS´*S* system. Analytical model. Zero current

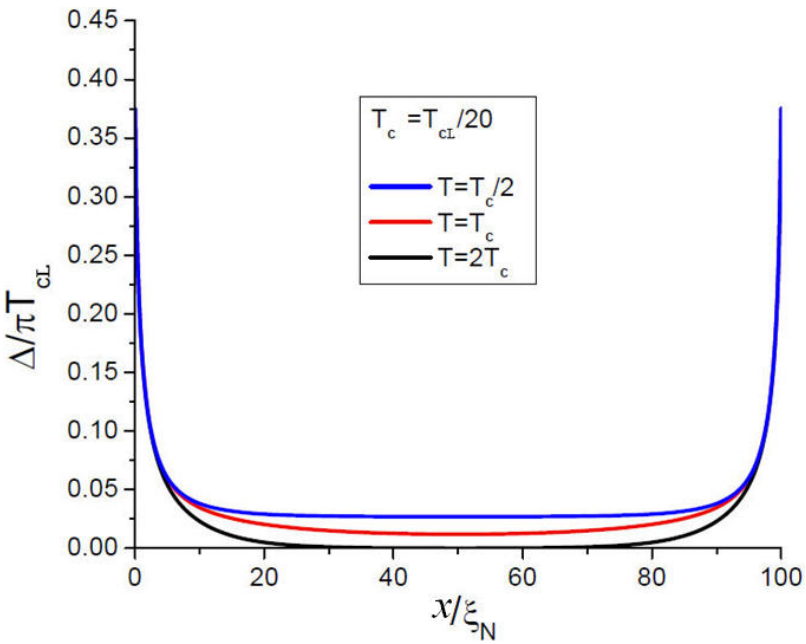


The order parameter in the center of the TES as a function of temperature close to the superconducting transition of the TES material: $T_{cL} = 20T_c$, $\theta(\pm L/2) = 0.1\theta_L$, boxes - $L/\xi_N = 200$,circles - $L/\xi_N = 100$, diamonds - $L/\xi_N = 50$

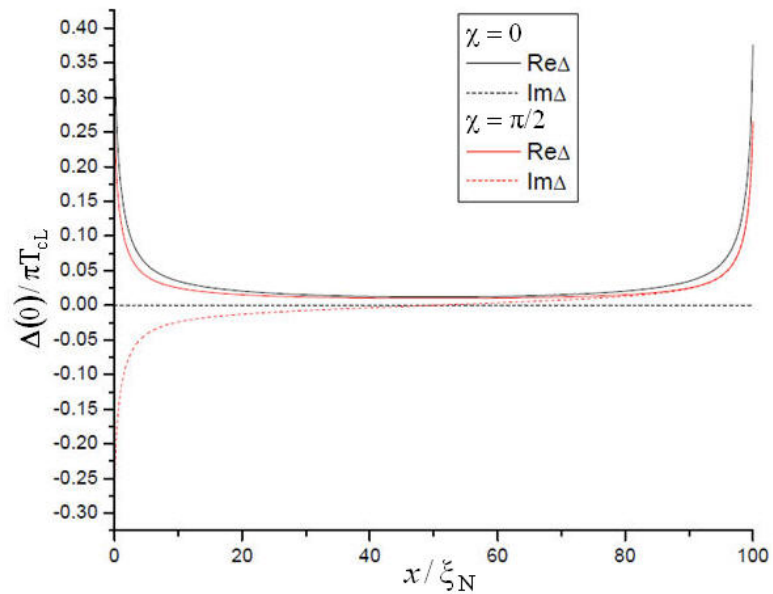


Suppression of the order parameter at the center of the TES by the critical Josephson current: boxes - $L/\xi_N = 100$, $\chi = \pi/2$, circles - $L/\xi_N = 100$, $\chi = 0$, diamonds - $L/\xi_N = 50$, $\chi = \pi/2$, crosses - $L/\xi_N = 50$, $\chi = 0$

Transparent interface: Numerical modelling

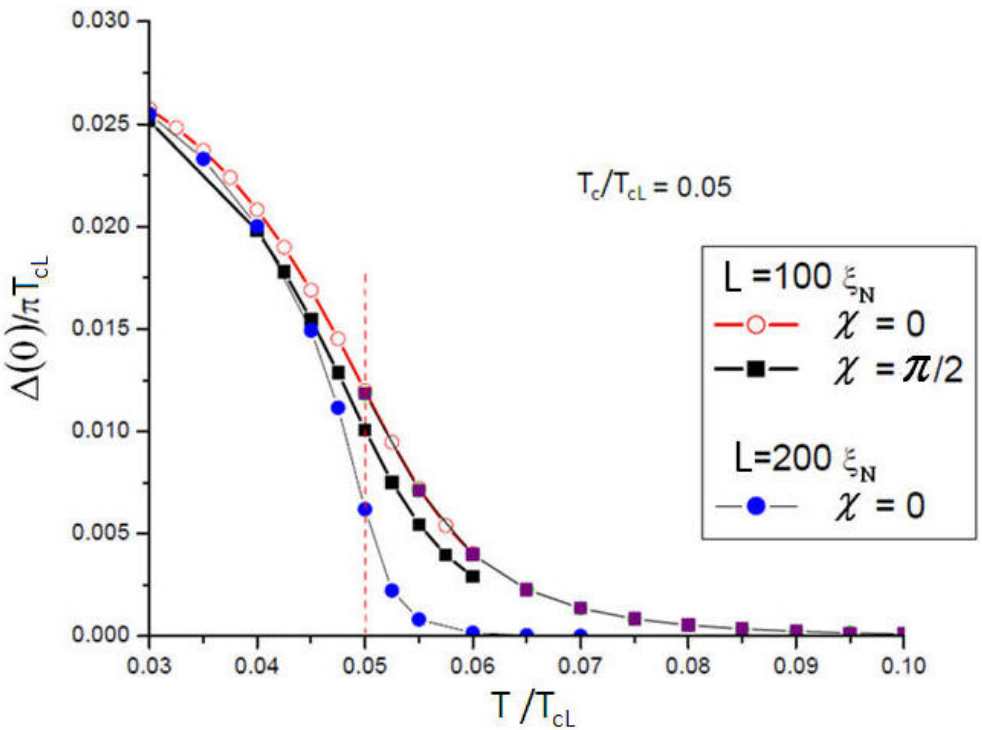


Profile of the order parameter through the TES with a transparent lead interface. The order parameter is normalized to the critical temperature in the lead



Profile of the order parameter through the TES by the critical Josephson current

Transparent interface: Numerical modelling



The order parameter at the center of the TES with the transparent lead interface as a function of temperature. The dashed line corresponds to the critical temperature of the TES material

Summary

•Both the simple analytical model and an accurate numerical analysis confirm the importance of long range lateral proximity effects in TES.

• Profile of the order parameter strongly depends on the boundary conditions at the TES/lead interfaces

• $\Delta(0)$ even for $L >> \xi_N$ decreases with temperature above the critical temperature of the TES metal very slowly

• $\Delta(0)$ is not fully suppressed by the critical Josephson current at any $T < T_{cL}$

•At the onset of the resistive transition the Usadel approach still describes most of the TES, apart from the small central region

• the Usadel approach can be valuable in modeling the rate of expansion of the central area of the TES with temperature and current