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Estimating Orbital Period of Exoplanets in Microlensing Events

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Binary Lens with Orbital Motion

The parameters needed to model microlensing events by binary lens with orbital motion are

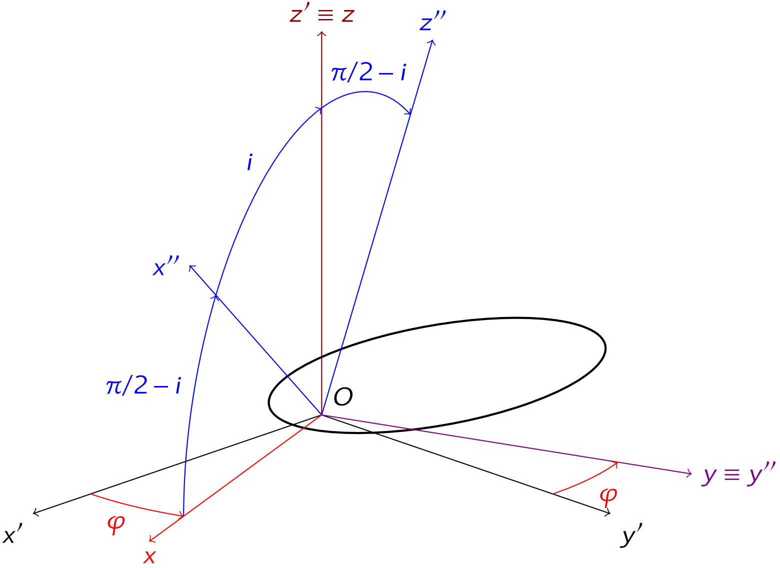
- Paczyński curve parameters: $t_0 \quad u_0 \quad t_\mathrm{E} \quad \theta$
- finite source effects: $\rho_\star$
- binary lens: $s \quad q$
- binary lens with orbital motion: $a \quad e \quad i \quad \varphi$

In addition, with small mass ratios q there is the close-wide degeneracy $s \longleftrightarrow s^{-1}$
What if we knew the orbital period of the lenses

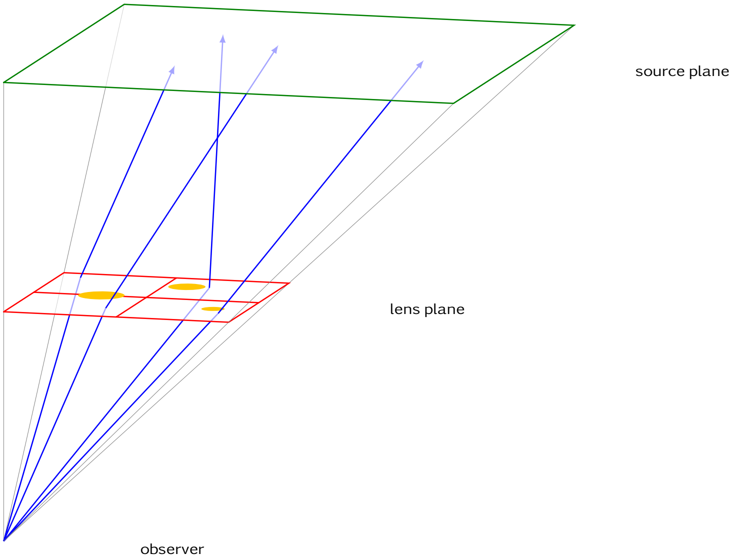
$$P = 2\pi\sqrt{\frac{a^3}{G(m_1 + m_2)}} = 2\pi\sqrt{\frac{a^3}{Gm_1(1+q)}}$$

independently from a fit?

Geometry of the System



Inverse Ray Shooting



Inverse Ray Shooting (cont.)

Solve the lens equation “backwards”

$$\zeta = z - \sum_{i=1}^{N} \frac{\varepsilon_i (z - z_i)}{\|z - z_i\|^2}$$

Conditions

- source area subdivided in at least 10^3 pixels
- each pixel on the source plane matches at least 100 pixels on the lens plane

Pros and cons

✓ precise, also on caustics

✗ very slow, high number of photons to be “shot”

✓ any lens configuration

✗ only point-like source

Witt & Mao Method

Binary-Lens Equation in complex formalism (details?)

$$\zeta = z + \frac{\varepsilon_1}{\overline{z}_1 - \overline{z}} + \frac{\varepsilon_2}{\overline{z}_2 - \overline{z}}$$

Put the lenses on points $z_1 = -z_2$ along the real axis $(z_j = \overline{z}_j)$

$$p_5(z) = \sum_{i=0}^{5} c_i z^i = 0$$

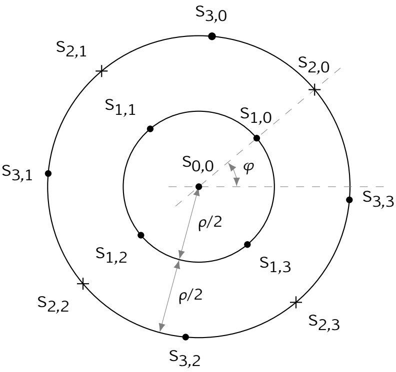
Amplification

$$\mu(\zeta) = \sum_{i=1}^{N} |\mu_i| = \sum_{i=1}^{N} \frac{\pi_i}{\det \mathcal{J}}\bigg|_{z=z_i}$$

Pros and cons

- ✓ fast
- ✗ only point-like source
- ✓ any lens configuration
- ✗ doesn't work near caustics

Hexadecapole Approximation



Approximation of the amplification function with a Taylor series up to the fourth order

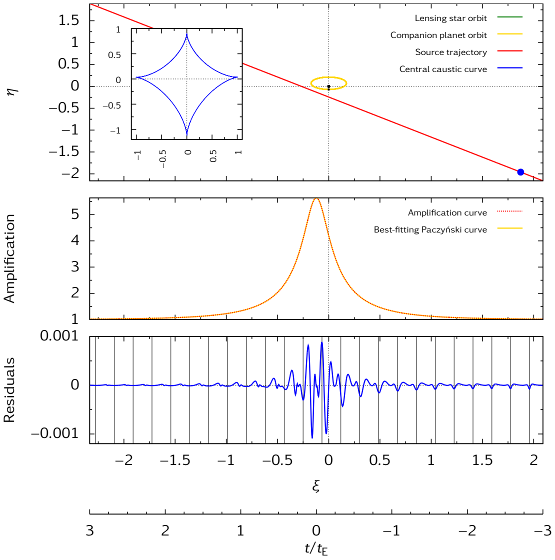
$$\begin{aligned}\mu_{\text{finite}}(\rho) &= \frac{2\pi}{F}\sum_{n=0}^{\infty}\mu_{2n}\int_0^{\rho} S(w)w^{2n+1}\,\mathrm{d}w \\ &= \mu_0 + \frac{\mu_2\rho^2}{2}\left(1-\frac{\Gamma}{5}\right) \\ &\quad + \frac{\mu_4\rho^4}{3}\left(1-\frac{11\Gamma}{35}\right)+\cdots\end{aligned}$$

Pros and cons

- ✓ fast (no amplification map required)
- ✓ extended source
- ✓ any lens configuration and any radial luminosity profile of the source
- ✗ far enough from the caustics

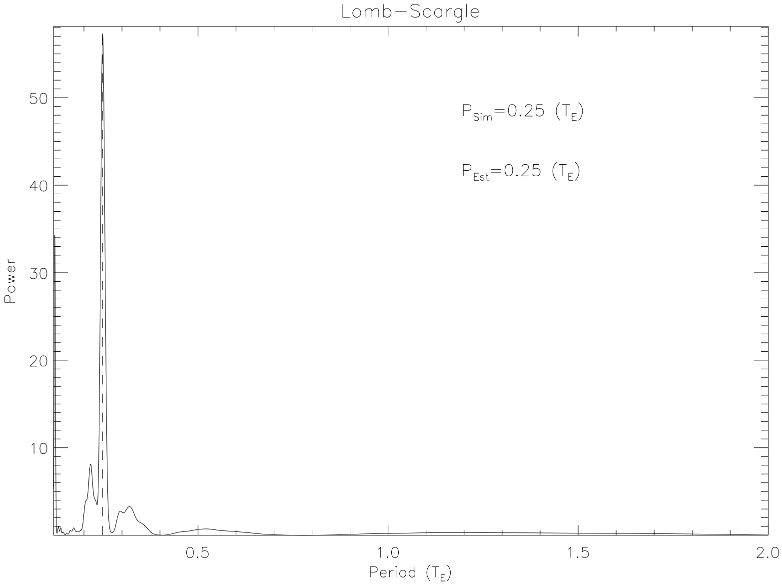
Details?

Simulation 1

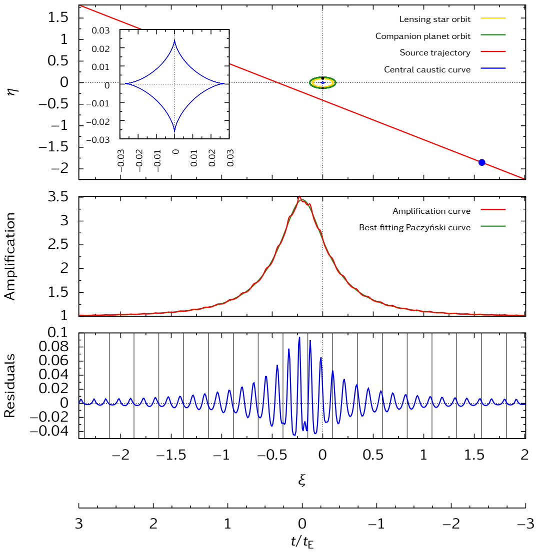


$$q = 10^{-3},\ a = 0.2,\ e = 0.5,\ i = 45^\circ,\ \varphi = 0^\circ,\ P = t_E/4$$

Simulation 1 (periodogram)

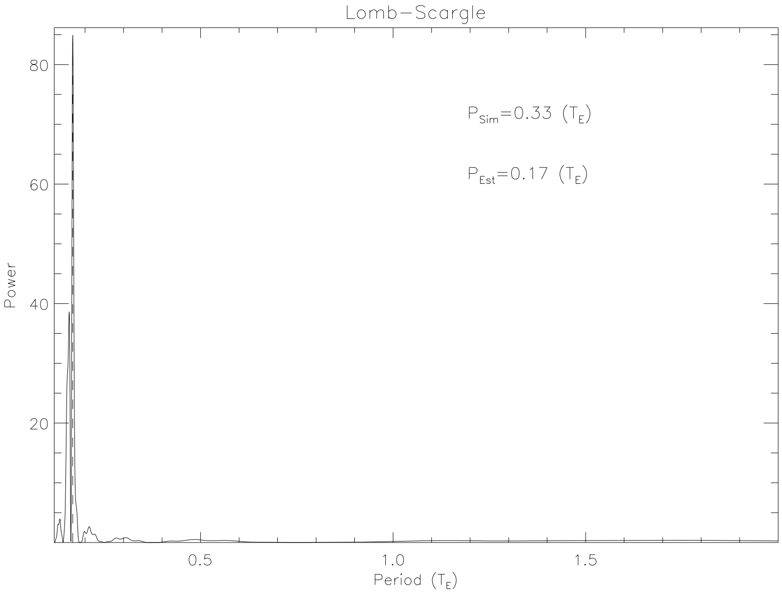


Simulation 2

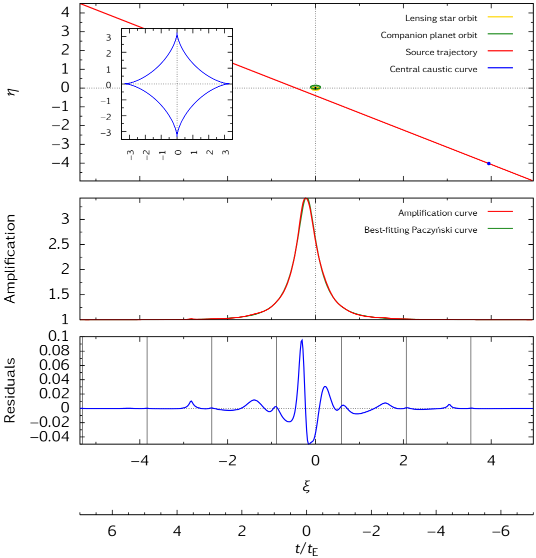


$q = 0.8$, $a = 0.23$, $e = 0$, $i = \varphi = 0°$, $P = t_E/3$

Simulation 2 (periodogram)

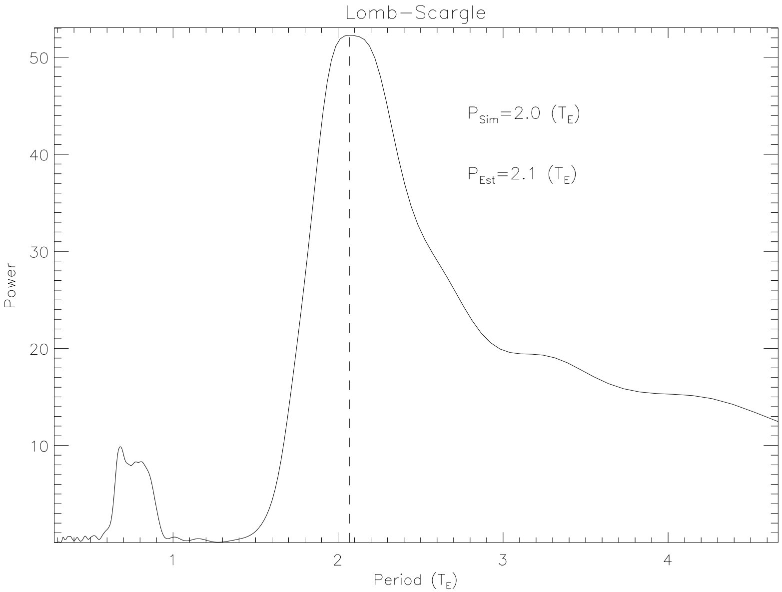


Simulation 3

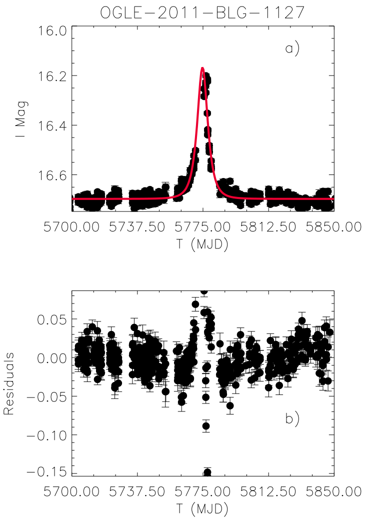


$q = 0.8$, $a = 0.23$, $e = 0.5$, $i = 45°$, $\varphi = 0°$, $P = 2t_E$

Simulation 3 (periodogram)



Fit to Real Data



Event OGLE-2011-BLG-1127/MOA-2011-BLG-322

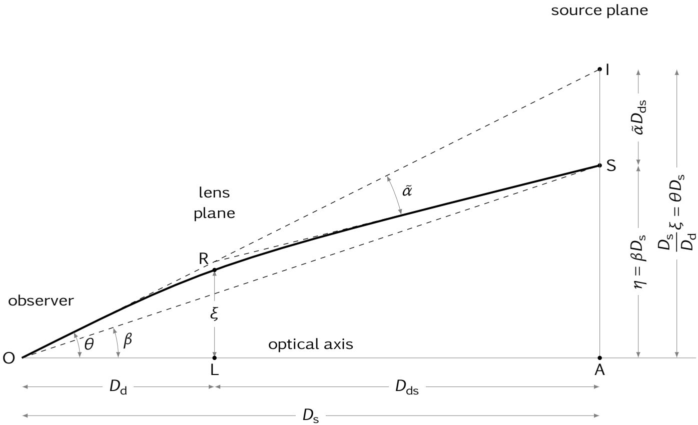
Conclusions

- Orbital period of the lenses should be shorter than the Einstein time of the event or we must have a long observational window
- We fit the observed amplification curve to a simple Paczyński curve, with four easily-guessable free parameters, and then perform a periodogram on the residuals: the period so obtained is the period of the binary system
- We need to remove a very small region around the central peak from the residuals before performing the periodogram
- Periodic feature with the same period far from the peak $\implies$ source periodicity (binary system, intrinsic variable, etc...)

Reference

A. Nucita, M. Giordano, F. De Paolis, and G. Ingrosso. "Signatures of rotating binaries in microlensing experiments". In: *Monthly Notices of the Royal Astronomical Society* 438 (Mar. 2014), pp. 2466–2473. DOI: 10.1093/mnras/stt2363. arXiv: 1401.6288.

Lens Equation



Lens Equation

$$\vec{\beta} = \vec{\theta} - \vec{\tilde{\alpha}} \frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}} \iff \vec{\eta} = \vec{\xi} \frac{D_{\mathrm{s}}}{D_{\mathrm{d}}} - \vec{\tilde{\alpha}} D_{\mathrm{ds}} \iff \vec{y} = \vec{x} - \vec{\alpha}$$

Critic and Caustic Curves

Amplification Matrix

$$\mathcal{J}_{ij} = \frac{\partial y_i}{\partial x_j}$$

Amplification

$$\mu = \frac{1}{\det \mathcal{J}}$$

Critic Curves

Locus of the points in the lens plane in which $\mu \to \infty \iff \det \mathcal{J} \to 0$

Caustic Curves

Locus of the points in the source plane in which $\mu \to \infty \iff \det \mathcal{J} \to 0$

Dimensionless Quantities

Einstein Radius

$$R_{\mathrm{E}} = \sqrt{\frac{4GM}{c^2}\frac{D_{\mathrm{ds}}D_{\mathrm{d}}}{D_{\mathrm{s}}}}$$

Einstein Angle

$$\theta_{\mathrm{E}} = \frac{R_{\mathrm{E}}}{D_{\mathrm{d}}} = \sqrt{\frac{4GM}{c^2}\frac{D_{\mathrm{ds}}}{D_{\mathrm{s}}D_{\mathrm{d}}}}$$

Critical Superficial Mass Density

$$\Sigma_{\mathrm{cr}} = \frac{c^2 D_{\mathrm{s}}}{4\pi G D_{\mathrm{d}} D_{\mathrm{ds}}}$$

Complex Formalism

Introduced by Witt (1990)
Complex Coordinates:
Source Plane: $z = x + \mathrm{i}\, y$
Lens Plane: $\zeta = \xi + \mathrm{i}\,\eta$
Mass Distribution

$$\Sigma(z) = \sum_{j=1}^{N} m_j \delta^2(z - z_j)$$

Lens Equation

$$\zeta = (1-\kappa)z + \gamma\bar{z} - \sum_{j=1}^{N} \frac{\varepsilon_j}{\bar{z} - \bar{z}_j}$$

Critic Curves Parametrization

$$\sum_{j=1}^{N} \frac{\varepsilon_j}{(\bar{z} - \bar{z}_j)^2} = (1-\kappa)\,\mathrm{e}^{\mathrm{i}\varphi} - \gamma$$

Hexadecapole Approximation: details

Far from the caustics, amplification can be expanded in Taylor series

$$\mu(\xi,\eta) = \sum_{n=0}^{\infty}\sum_{i=0}^{n}\mu_{n,i}(\xi-\xi_0)^i(\eta-\eta_0)^{n-i}$$

Amplification of an extended source

$$\begin{aligned}\mu_{\text{finite}}(\rho;\xi_0,\eta_0) &= \frac{\int_0^\rho wS(w)\,\mathrm{d}w\int_0^{2\pi}\mu(\xi_0+w\cos\theta,\eta_0+w\sin\theta)\,\mathrm{d}\theta}{\int_0^\rho wS(w)\,\mathrm{d}w\int_0^{2\pi}\mathrm{d}\theta}\\ &= \frac{2\pi}{F}\sum_{n=0}^{\infty}\mu_{2n}\int_0^\rho S(w)w^{2n+1}\,\mathrm{d}w\end{aligned}$$

With linear limb-darkening ($S(w) = (1-\Gamma(1-(3/2)\sqrt{1-w^2/\rho^2}))F/\pi\rho^2$)

$$\mu_{\text{finite}}(\rho;\xi_0,\eta_0) = \mu_0 + \frac{\mu_2\rho^2}{2}\left(1-\frac{\Gamma}{5}\right) + \frac{\mu_4\rho^4}{3}\left(1-\frac{11\Gamma}{35}\right) + \cdots$$

Hexadecapole Approximation: details (cont.)

$$
\begin{aligned}
M_{w,+} &= \frac{1}{4}\sum_{j=0}^{3} \mu(\xi_0 + w\cos(\varphi + j\pi/2), \eta_0 + w\sin(\varphi + w\sin(\varphi + j\pi/2))) - \mu_0 \\
&\approx \frac{1}{4}\sum_{j=0}^{3}\sum_{n=0}^{4}\sum_{i=0}^{n} \mu_{n,i} w^n (\cos(\varphi + j\pi/2))^i (\sin(\varphi + j\pi/2))^{n-i} - \mu_0 \\
&= \frac{(\mu_{4,0} + \mu_{4,4})(3 + \cos(4\varphi)) + (\mu_{4,3} + \mu_{4,1})\sin(4\varphi) + \mu_{4,2}(1 - \cos(4\varphi))}{8} \\
&\quad + \mu_2 w^2 \\
M_{w,\times} &= \frac{1}{4}\sum_{j=0}^{3} \mu(\xi_0 + w\cos(\varphi + (2j+1)\pi/4), \eta_0 + w\sin(\varphi + w\sin(\varphi + (2j+1)\pi/4))) \\
&\quad - \mu_0 \\
&\approx \frac{(\mu_{4,0} + \mu_{4,4})(3 - \cos(4\varphi)) - (\mu_{4,3} + \mu_{4,1})\sin(4\varphi) + \mu_{4,2}(1 + \cos(4\varphi))}{8} w^4 \\
&\quad + \mu_2 w^2
\end{aligned}
$$

Hexadecapole Approximation: details (cont.)

Recipe:

- determine amplification on the thirteen points
- use these amplifications to calculate $M_{\rho,+}$, $M_{\rho,\times}$, and $M_{\rho/2,+}$
- calculate $\mu_2\rho^2$ and $\mu_4\rho^4$ with relations

$$\mu_2\rho^2 = \frac{16M_{\rho/2,+} - M_{\rho,+}}{3}$$
$$\mu_4\rho^4 = \frac{M_{\rho,+} + M_{\rho,\times}}{2} - \mu_2\rho^2$$

- insert $\mu_2\rho^2$, $\mu_4\rho^4$, and amplification μ_0 of the central monopole inside equation

$$\mu_{\text{finite}}(\rho;\xi_0,\eta_0) = \mu_0 + \frac{\mu_2\rho^2}{2}\left(1 - \frac{\Gamma}{5}\right) + \frac{\mu_4\rho^4}{3}\left(1 - \frac{11\Gamma}{35}\right) + \cdots$$

to get the amplification of a finite source