Swallow tails and Butterflies in Triple Lens Systems

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Outline

Butterfly and Swallow-tail metamorphoses:

- Metamorphoses of caustics
- Metamorphoses of amplification pattern
- Metamorphoses under finite source effect

Circumbinary planet system:

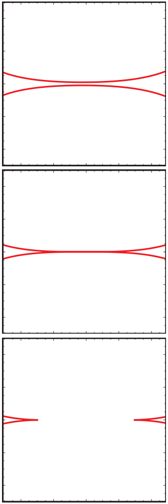
- Cusp-curve structure
- Swallowtail metamorphosis - amplification patterns

Two-planets and star system:

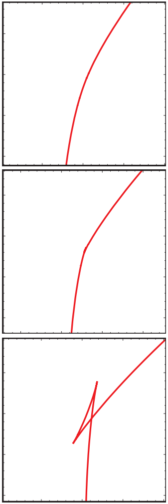
- Cusp-curve structure
- Swallowtail metamorphosis - amplification patterns

The first three elementary caustic metamorphoses

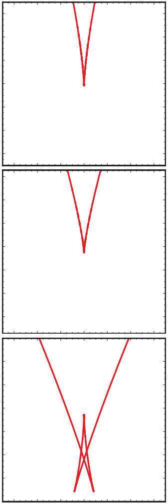
Beak-to-beak



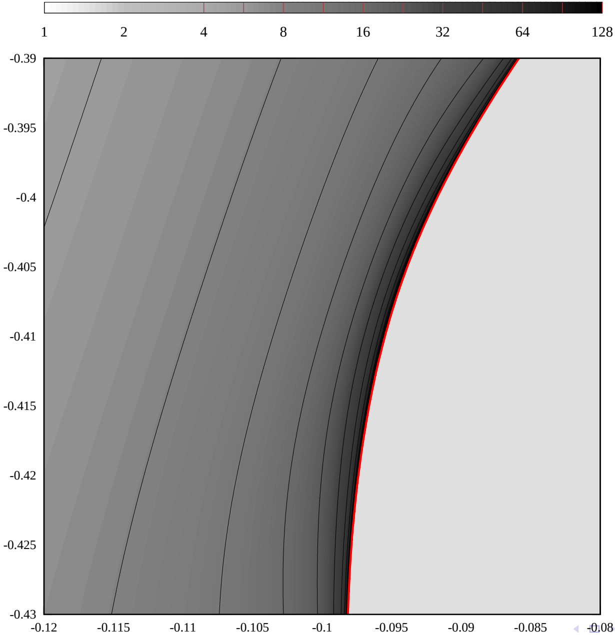
Swallow tail



Butterfly



Swallowtail caustic metamorphosis: fold



$\mu_A = \mu_B = \mu_C$

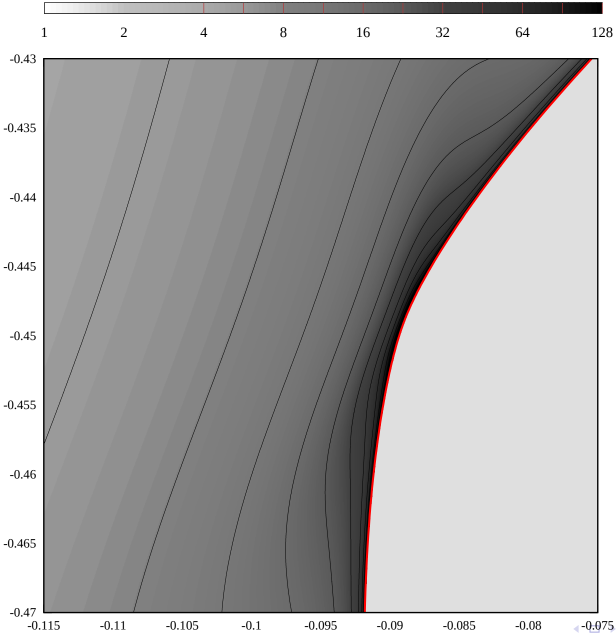
$z_A = -29/60\alpha$

$z_B = -1/30\alpha$

$z_C = 31/60\alpha$

$\alpha = 2.0$

Swallowtail caustic metamorphosis: fold



$\mu_A = \mu_B = \mu_C$

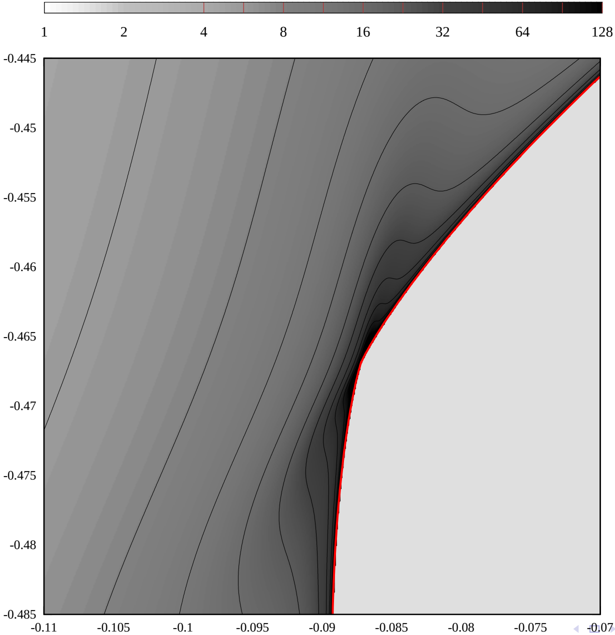
$z_A = -29/60\alpha$

$z_B = -1/30\alpha$

$z_C = 31/60\alpha$

$\alpha = 1.85$

Swallowtail caustic metamorphosis: swallowtail



$\mu_A = \mu_B = \mu_C$

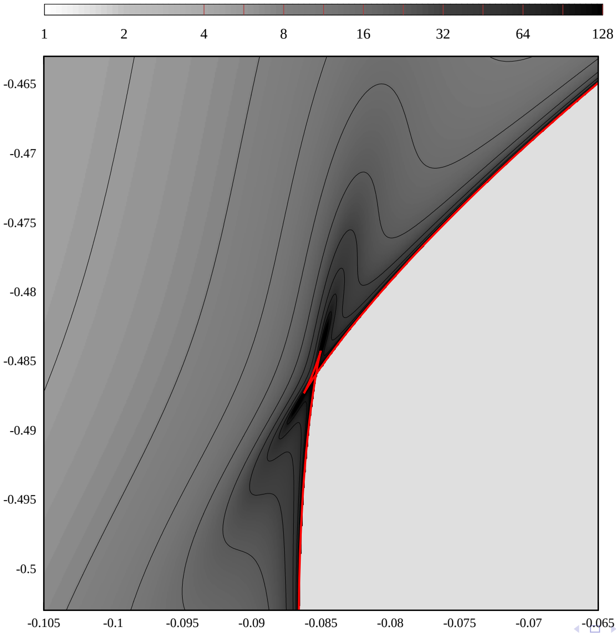
$z_A = -29/60\alpha$

$z_B = -1/30\alpha$

$z_C = 31/60\alpha$

$\alpha = 1.79465$

Swallowtail caustic metamorphosis: two cusps



$\mu_A = \mu_B = \mu_C$

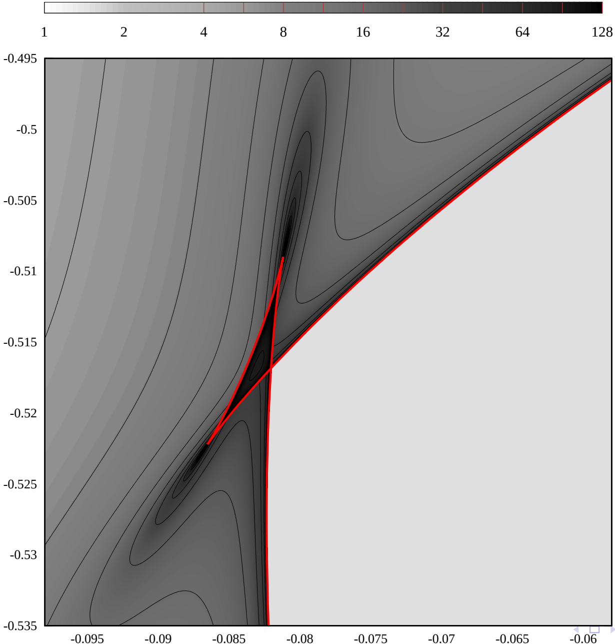
$z_A = -29/60\alpha$

$z_B = -1/30\alpha$

$z_C = 31/60\alpha$

$\alpha = 1.74$

Swallowtail caustic metamorphosis: two cusps



$\mu_A = \mu_B = \mu_C$

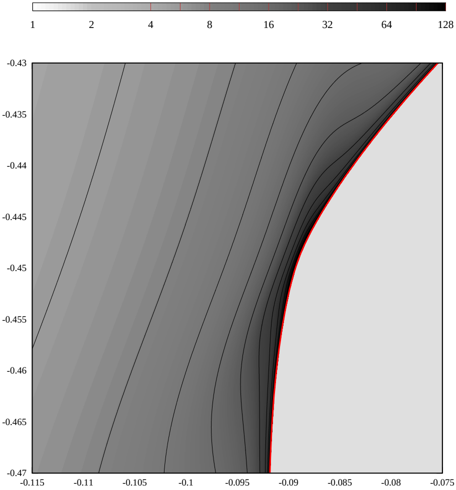
$z_A = -29/60\alpha$

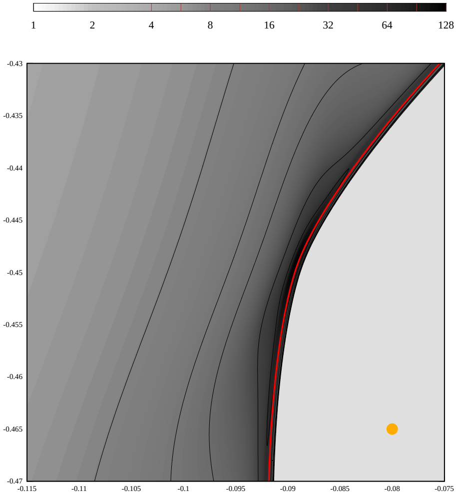
$z_B = -1/30\alpha$

$z_C = 31/60\alpha$

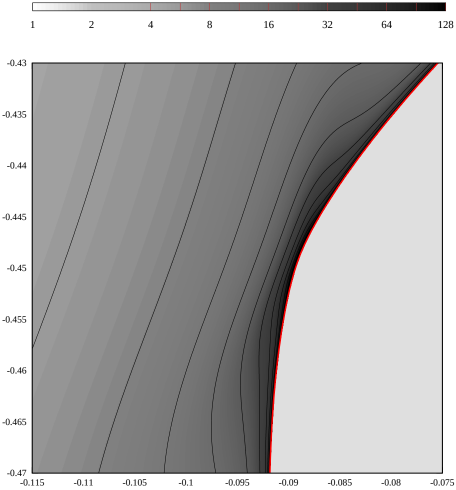
$\alpha = 1.66$

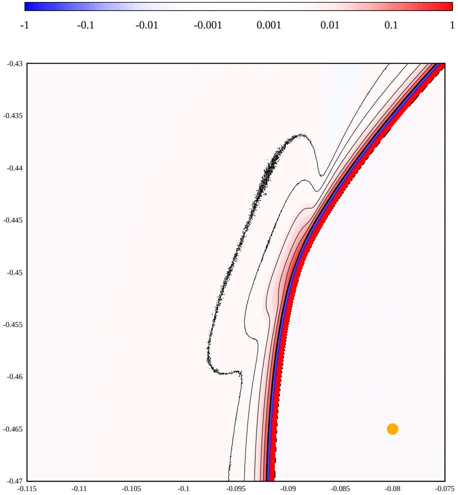
Swallowtail caustic metamorphosis: extended source



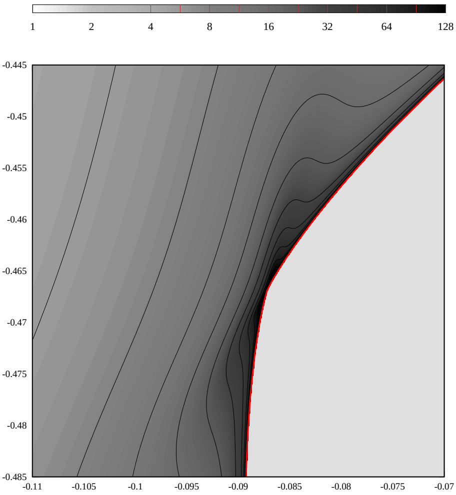


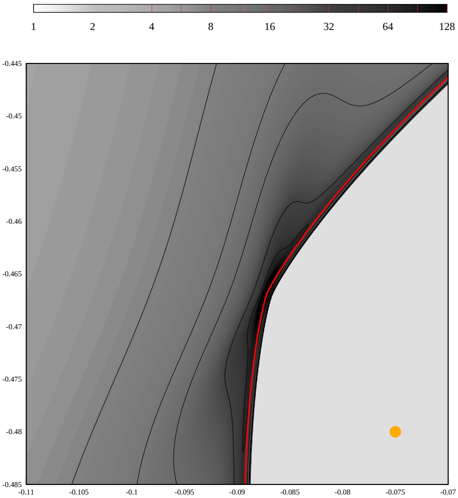
Swallowtail caustic metamorphosis: extended source



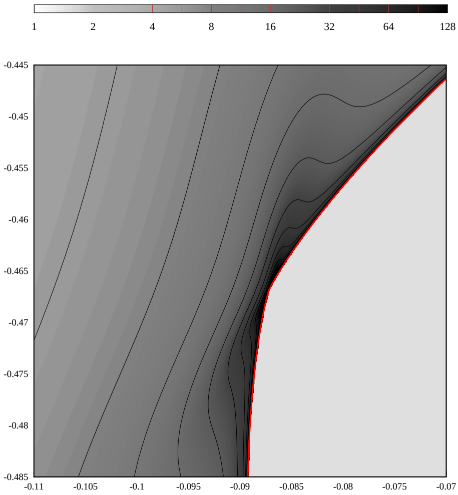


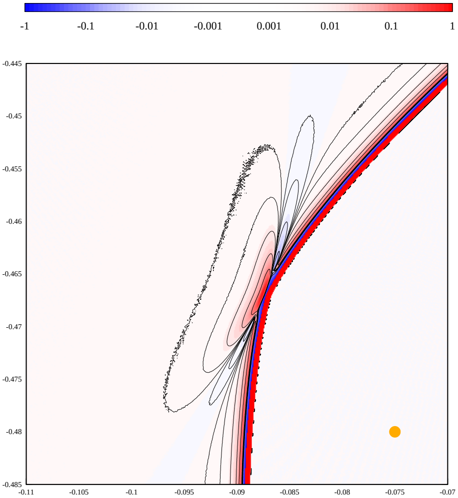
Swallowtail caustic metamorphosis: extended source



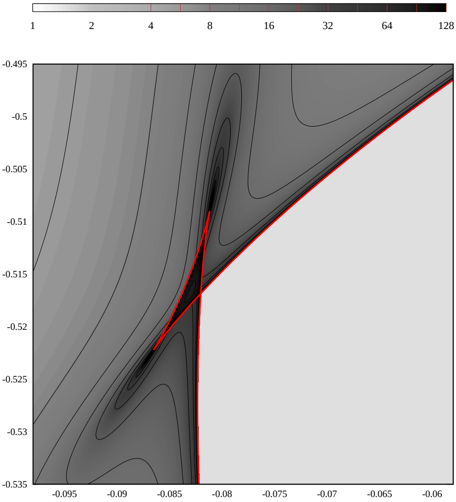


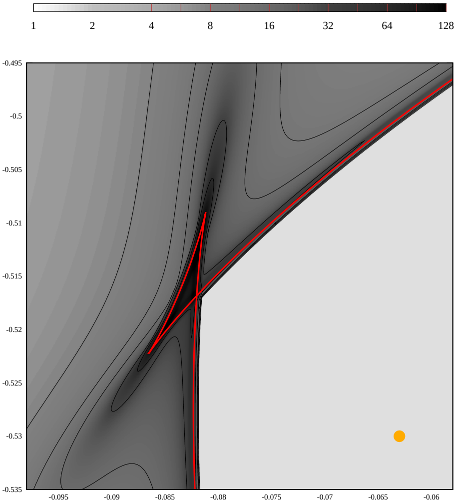
Swallowtail caustic metamorphosis: extended source



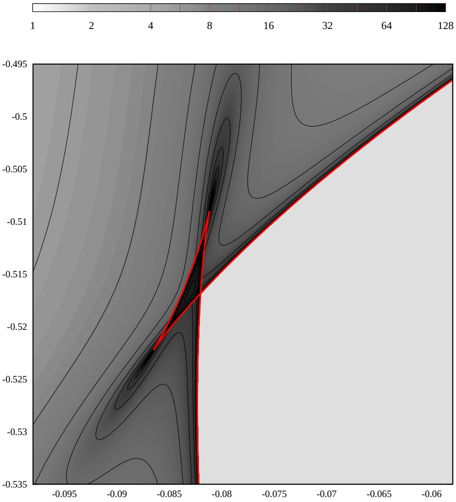


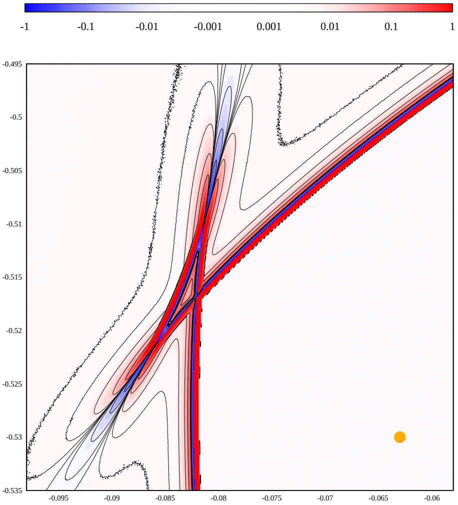
Swallowtail caustic metamorphosis: extended source



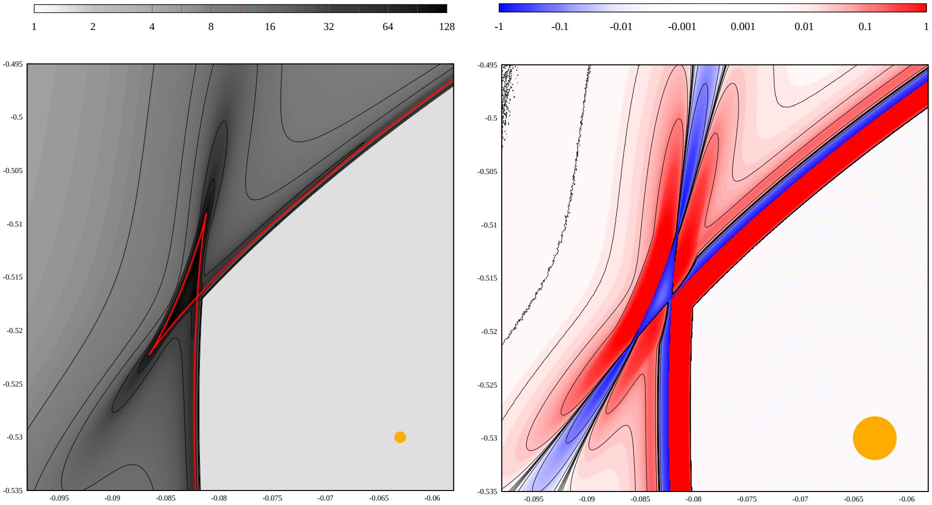


Swallowtail caustic metamorphosis: extended source

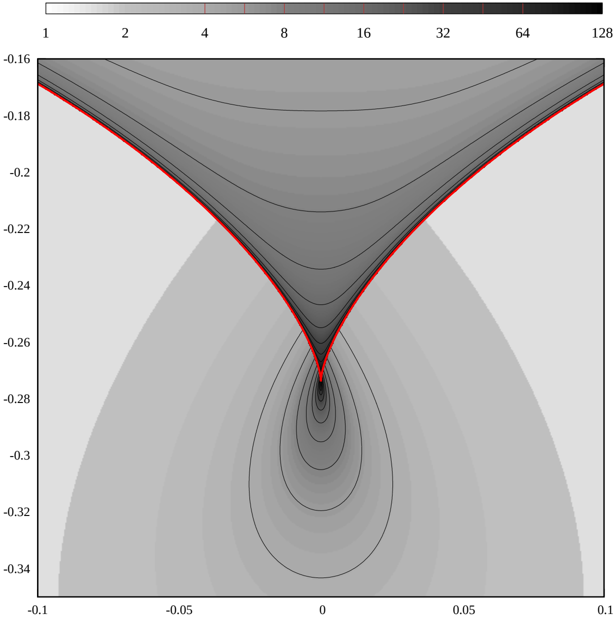




Swallowtail caustic metamorphosis: extended source



Butterfly caustic metamorphosis: cusp



$\mu_A = \mu_B = \mu_C$

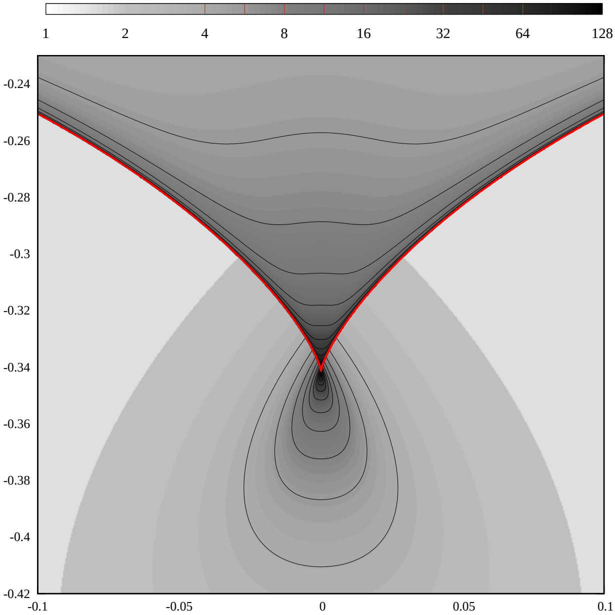
$z_A = -1/2\alpha$

$z_B = 0$

$z_C = 1/2\alpha$

$\alpha = 1.4$

Butterfly caustic metamorphosis: butterfly



$\mu_A = \mu_B = \mu_C$

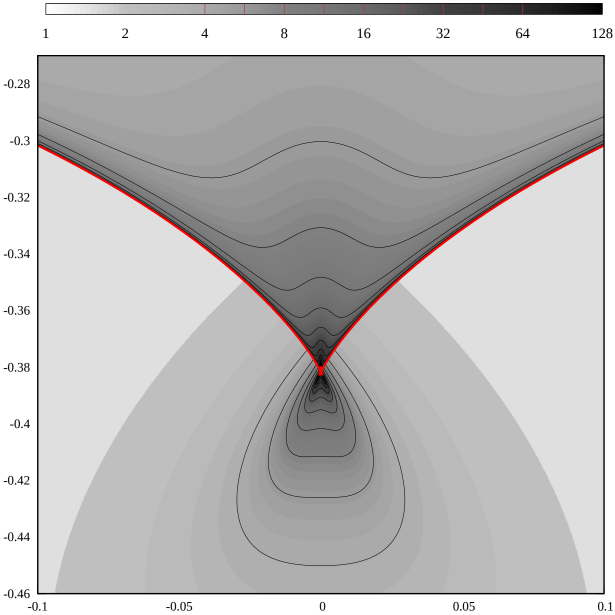
$z_A = -1/2\alpha$

$z_B = 0$

$z_C = 1/2\alpha$

$\alpha = 1.19492$

Butterfly caustic metamorphosis: three cusps



$\mu_A = \mu_B = \mu_C$

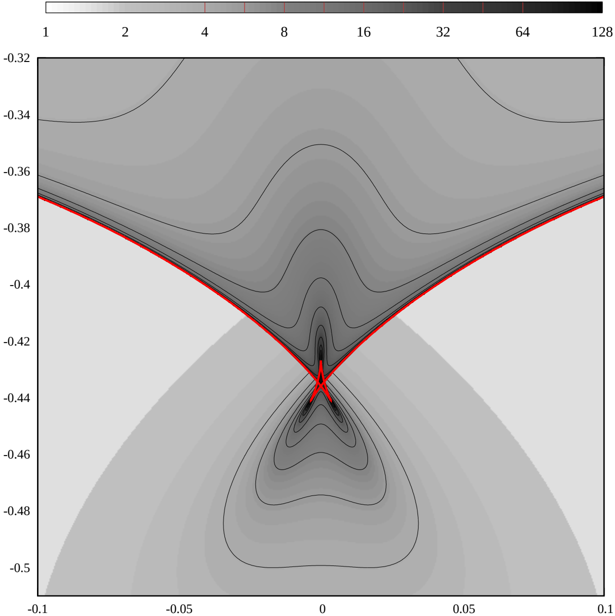
$z_A = -1/2\alpha$

$z_B = 0$

$z_C = 1/2\alpha$

$\alpha = 1.1$

Butterfly caustic metamorphosis: three cusps



$\mu_A = \mu_B = \mu_C$

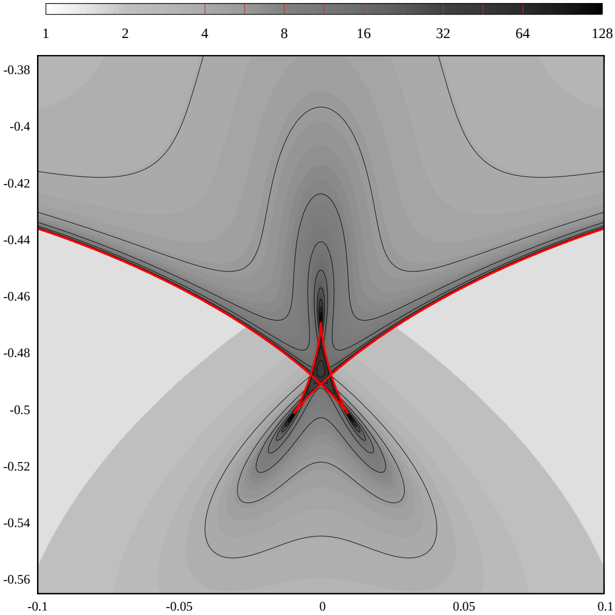
$z_A = -1/2\alpha$

$z_B = 0$

$z_C = 1/2\alpha$

$\alpha = 1.0$

Butterfly caustic metamorphosis: three cusps



$\mu_A = \mu_B = \mu_C$

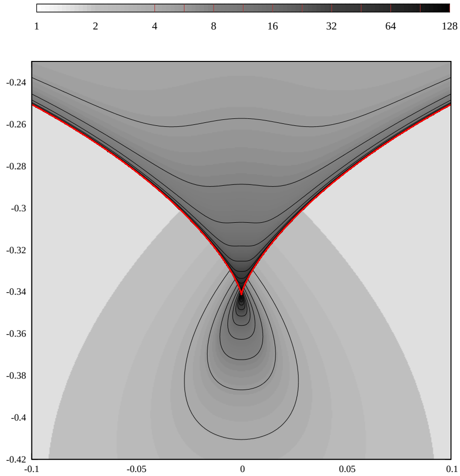
$z_A = -1/2\alpha$

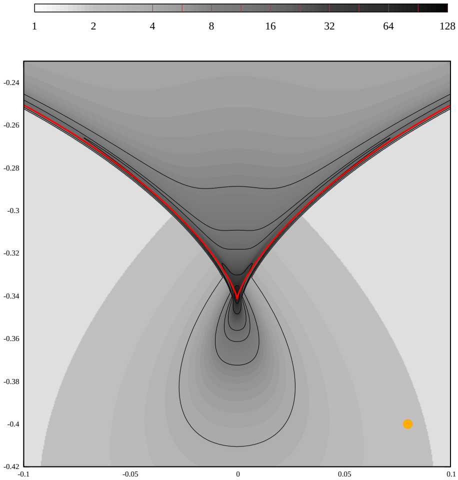
$z_B = 0$

$z_C = 1/2\alpha$

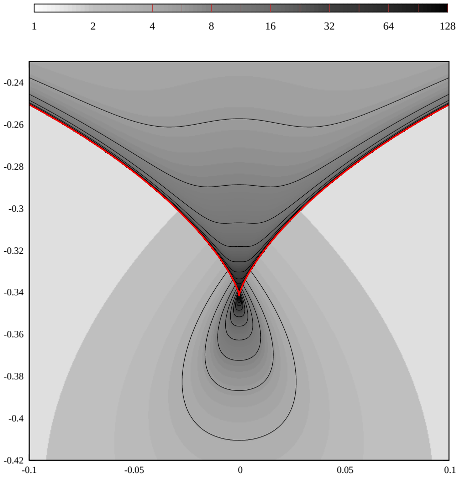
$\alpha = 0.92$

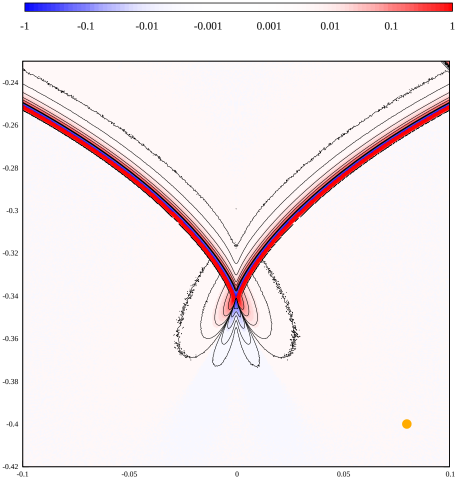
Butterfly caustic metamorphosis: extended source



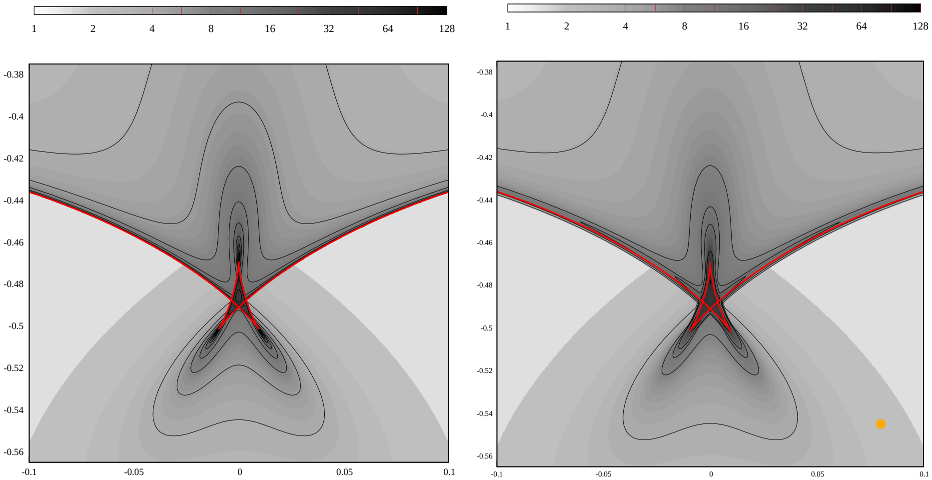


Butterfly caustic metamorphosis: extended source

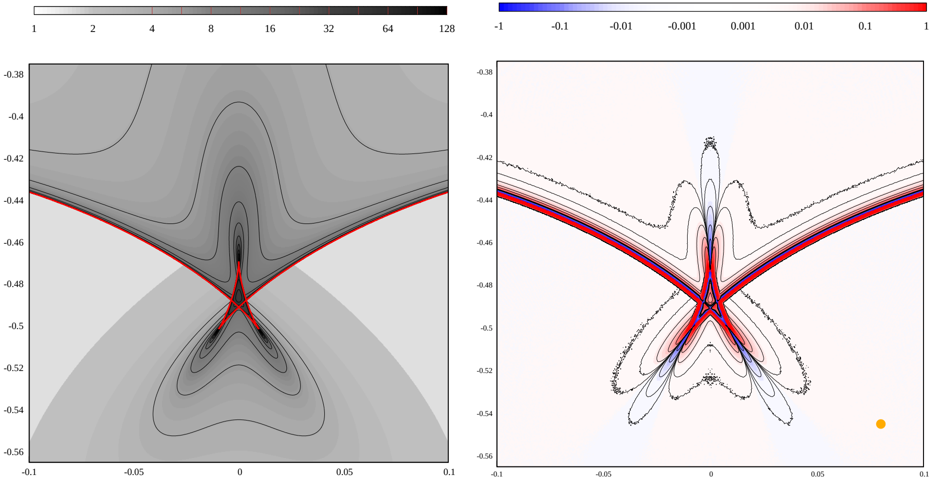




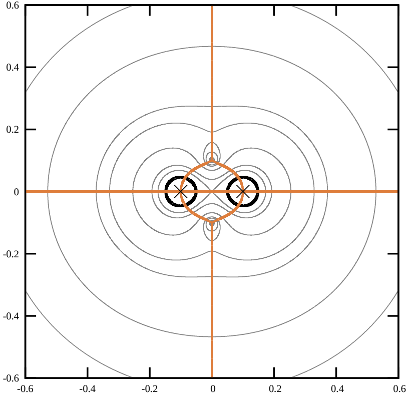
Butterfly caustic metamorphosis: extended source



Butterfly caustic metamorphosis: extended source

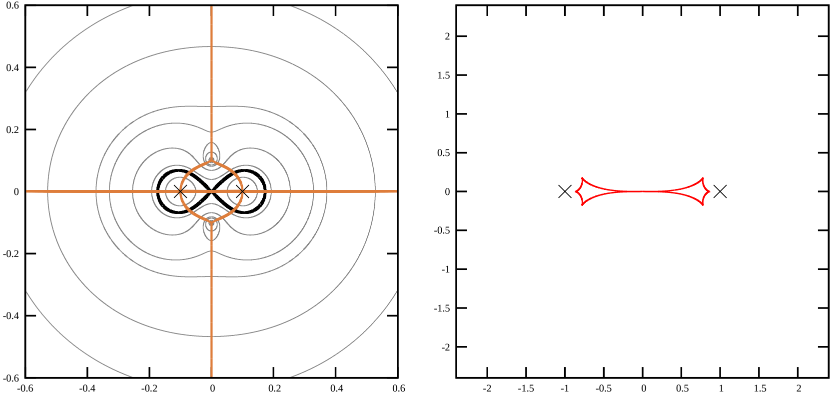


Binary star: cusp-curve structure



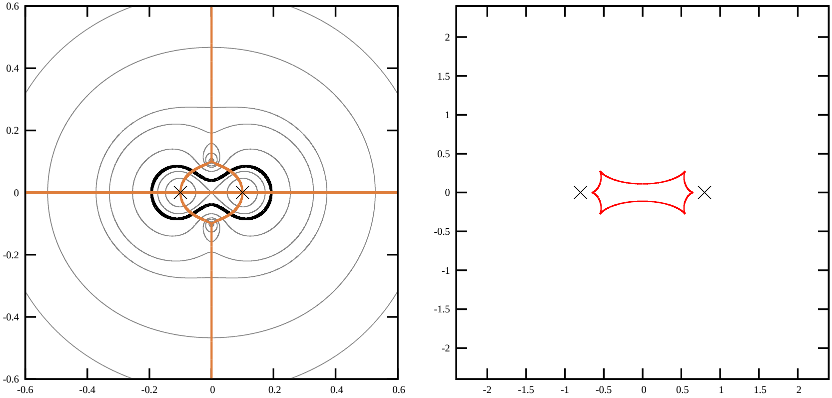
Binary star lens: μ=1/2, Jacobian contours for d=0.2 separation, bold contour correspond to critical curve of d=3.

Binary star: cusp-curve structure



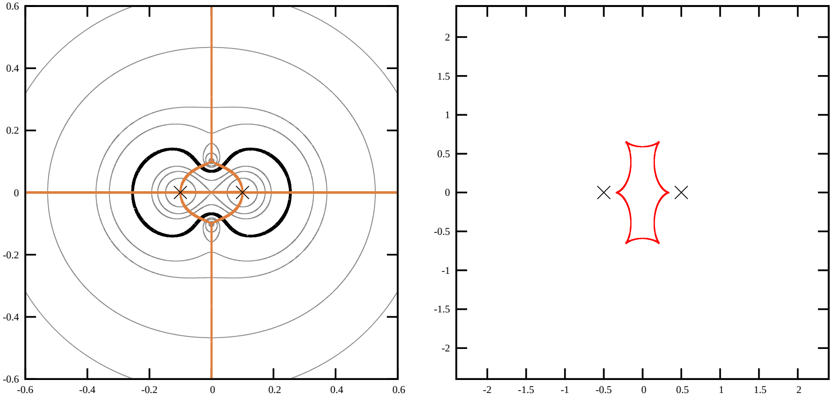
Binary star lens: μ=1/2, Jacobian contours for d=0.2 separation, bold contour correspond to critical curve of d=2.

Binary star: cusp-curve structure



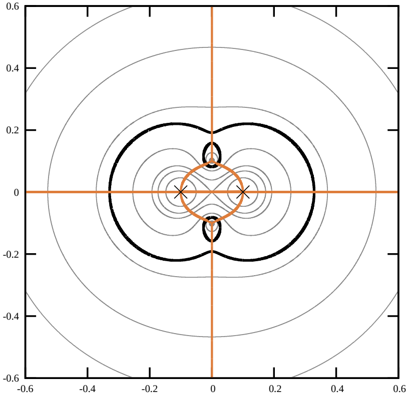
Binary star lens: μ=1/2, Jacobian contours for d=0.2 separation, bold contour correspond to critical curve of d=1.6.

Binary star: cusp-curve structure



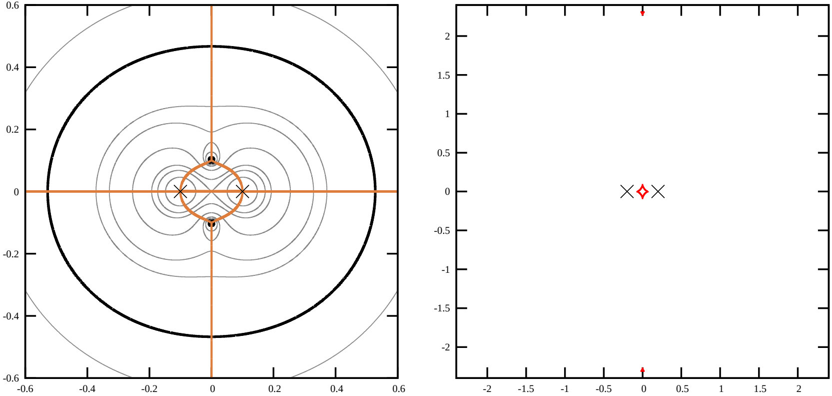
Binary star lens: μ=1/2, Jacobian contours for d=0.2 separation, bold contour correspond to critical curve of d=1.0.

Binary star: cusp-curve structure



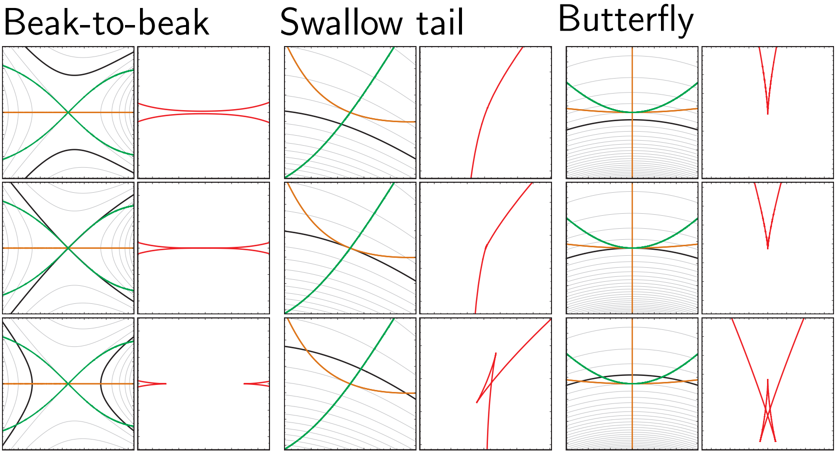
Binary star lens: μ=1/2, Jacobian contours for d=0.2 separation, bold contour correspond to critical curve of d=0.6.

Binary star: cusp-curve structure



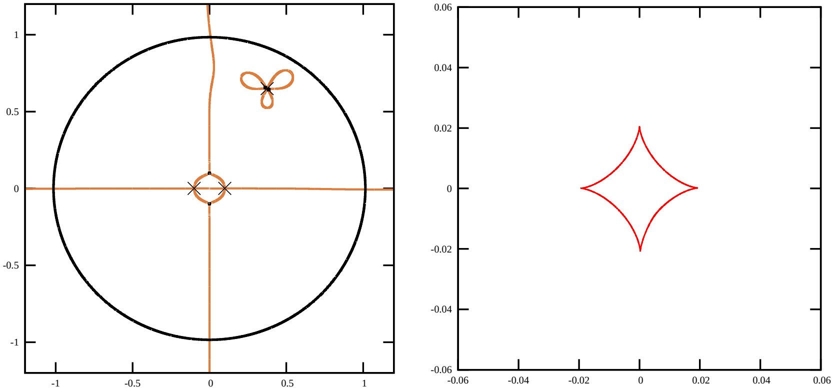
Binary star lens: μ=1/2, Jacobian contours for d=0.2 separation, bold contour correspond to critical curve of d=0.2.

Scaling method and higher-order catastrophes



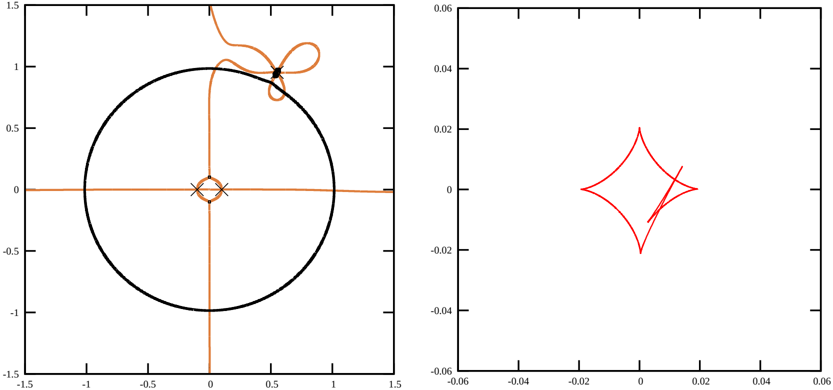
Danek K., Heyrovsky D. arXiv:1501.02722 [astro-ph.EP]

Binary star with a planet: cusp-curve structure



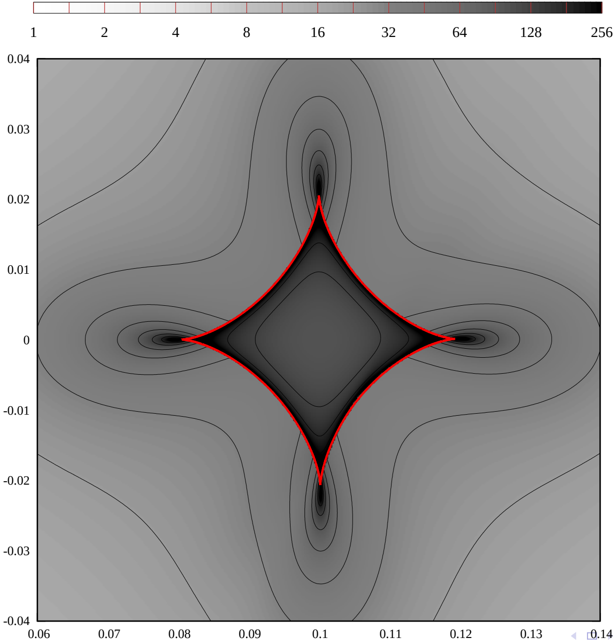
Binary star lens: $\mu_1 = \mu_2 = 0.4999$, star separation d=0.2; planet mass $\mu_3 = 0.0001$ and separation d=0.75.

Binary star with a planet: cusp-curve structure



Binary star lens: μ_1=μ_2=0.4999, star separation d=0.2; planet mass $\mu_3 = 0.0002$ and separation d=1.1.

Circumbinary planet: fold



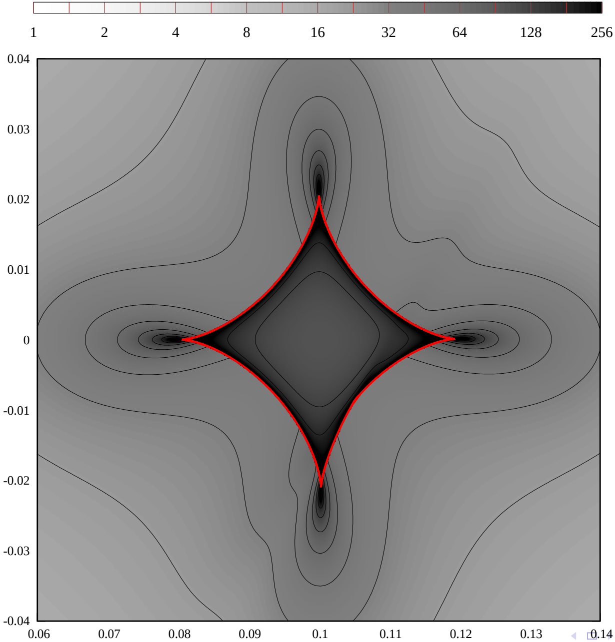
$\mu_A = \mu_B = 0.4999,$

$\mu_C = 2 \times 10^{-4}$

$z_A = 0.0, \; z_B = 0.2$

$z_C = 0.1 + 1.50 \times e^{\mathrm{i}\frac{2}{3}\pi}$

Circumbinary planet: fold



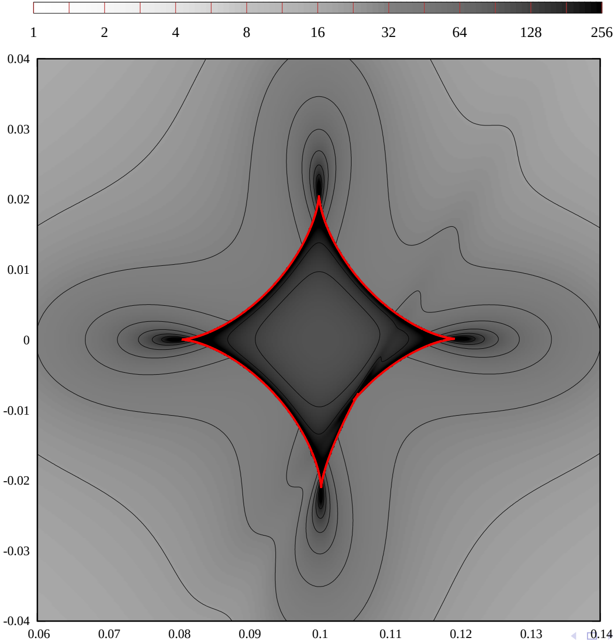
$\mu_A = \mu_B = 0.4999,$

$\mu_C = 2 \times 10^{-4}$

$z_A = 0.0, \ z_B = 0.2$

$z_C = 0.1 + 1.25 \times e^{i\frac{2}{3}\pi}$

Circumbinary planet: two cusps



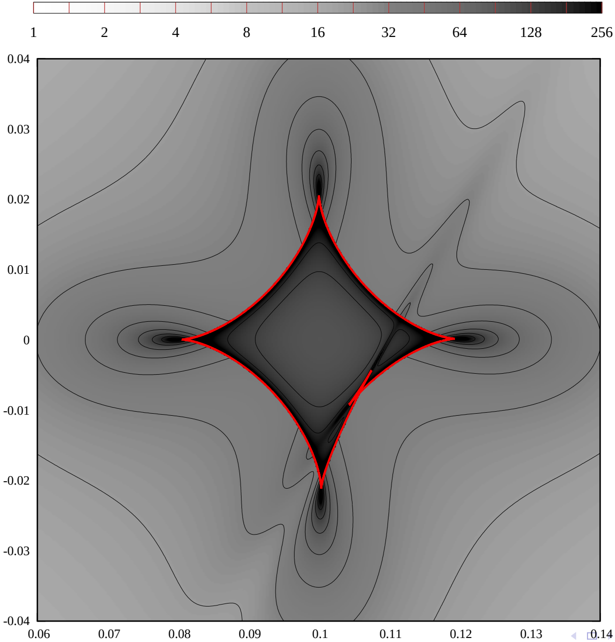
$\mu_A = \mu_B = 0.4999,$

$\mu_C = 2 \times 10^{-4}$

$z_A = 0.0, \; z_B = 0.2$

$z_C = 0.1 + 1.20 \times e^{i\frac{2}{3}\pi}$

Circumbinary planet: two cusps



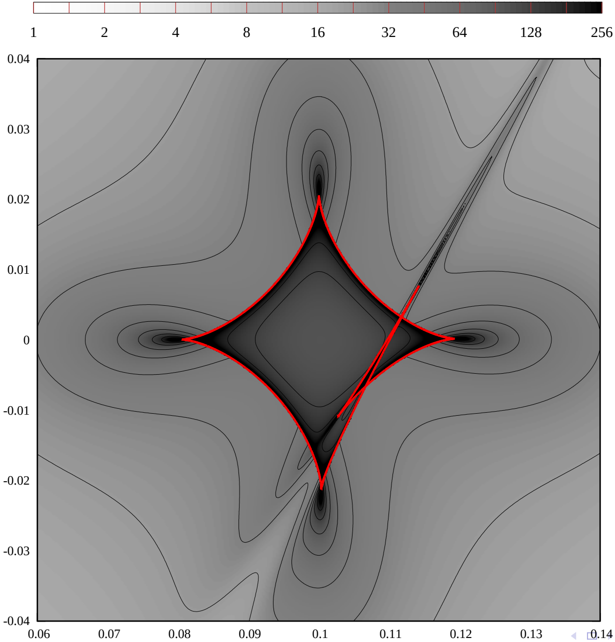
$\mu_A = \mu_B = 0.4999,$

$\mu_C = 2 \times 10^{-4}$

$z_A = 0.0, \; z_B = 0.2$

$z_C = 0.1 + 1.15 \times e^{i\frac{2}{3}\pi}$

Circumbinary planet: two cusps



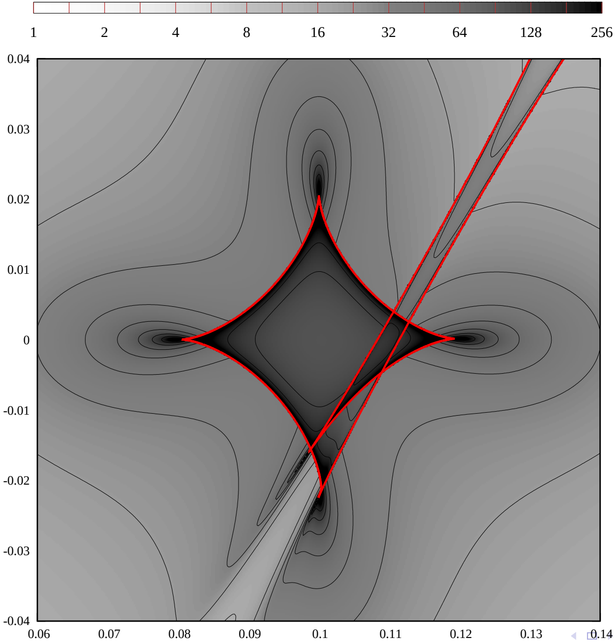
$\mu_A = \mu_B = 0.4999,$

$\mu_C = 2 \times 10^{-4}$

$z_A = 0.0,\ z_B = 0.2$

$z_C = 0.1 + 1.10 \times e^{i\frac{2}{3}\pi}$

Circumbinary planet: resonant caustic



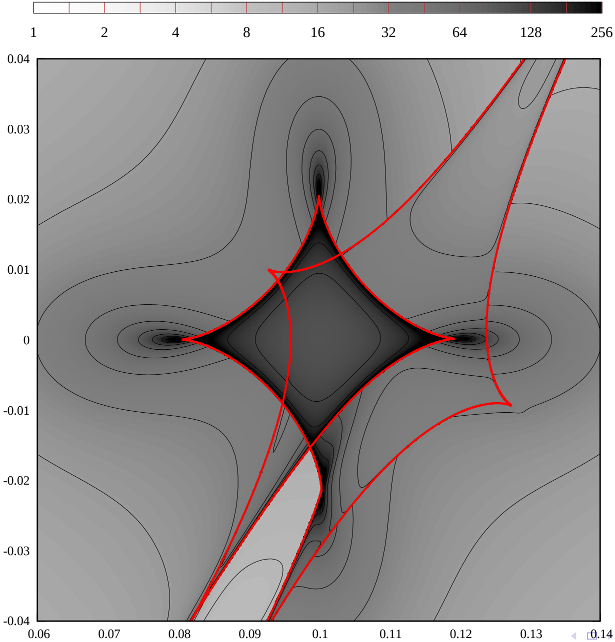
$\mu_A = \mu_B = 0.4999,$

$\mu_C = 2 \times 10^{-4}$

$z_A = 0.0, \ z_B = 0.2$

$z_C = 0.1 + 1.05 \times e^{i\frac{2}{3}\pi}$

Circumbinary planet: resonant caustic



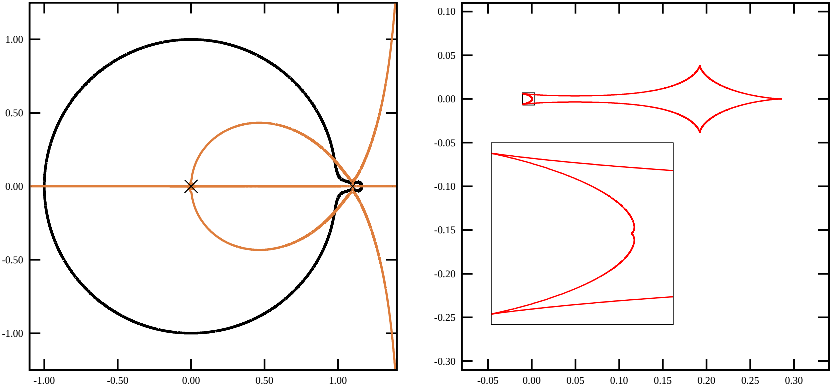
$\mu_A = \mu_B = 0.4999,$

$\mu_C = 2 \times 10^{-4}$

$z_A = 0.0, \; z_B = 0.2$

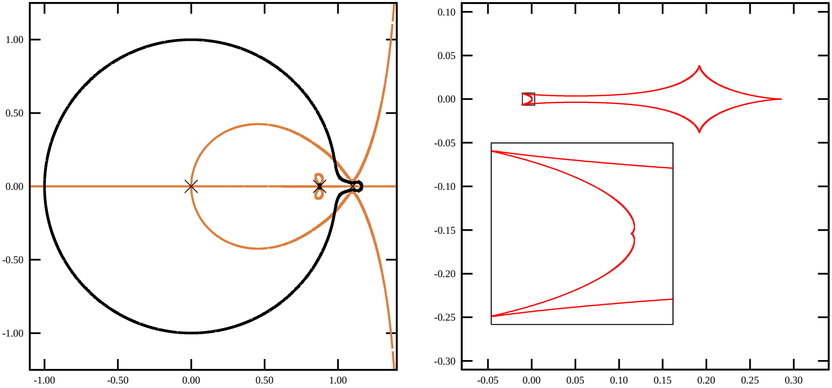
$z_C = 0.1 + 1.00 \times e^{i\frac{2}{3}\pi}$

Star with one planet: cusp-curve structure



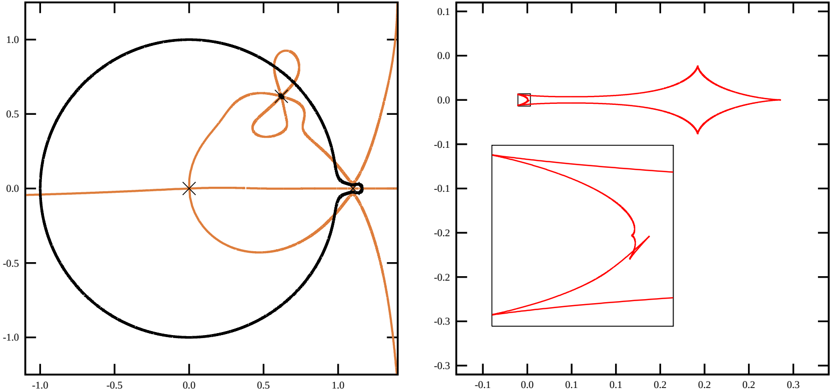
Star with one planet: mass ratio $\mu = 10^{-3}$, separation d=1.1.

Star with two planets: cusp-curve structure



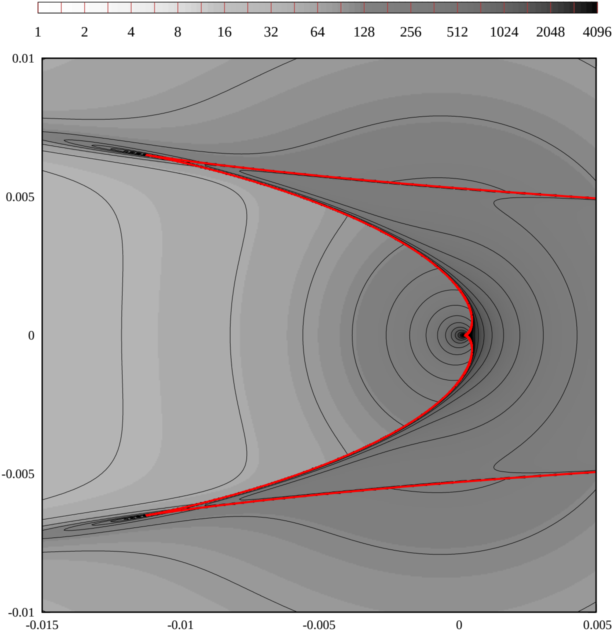
Star with two planet: mass ratio $\mu_2 = 10^{-3}$, $\mu_3 = 5 \times 10^{-5}$, separation $s_{12} = 1.1$, $s_{23} = 0.875$, angle $\theta_{23} = 0\pi$.

Star with two planets: cusp-curve structure



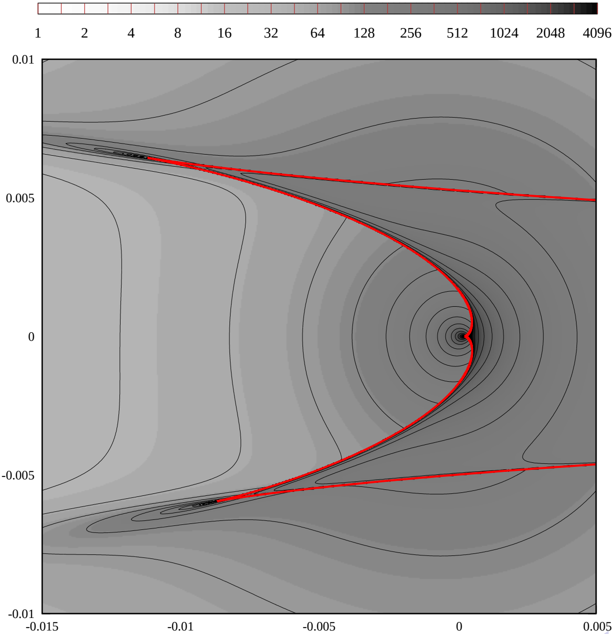
Star with two planet: mass ratio $\mu_2 = 10^{-3}$, $\mu_3 = 5 \times 10^{-5}$, separation $s_{12} = 1.1$, $s_{23} = 0.875$, angle $\theta_{23} = \pi/4$.

Two planets: fold



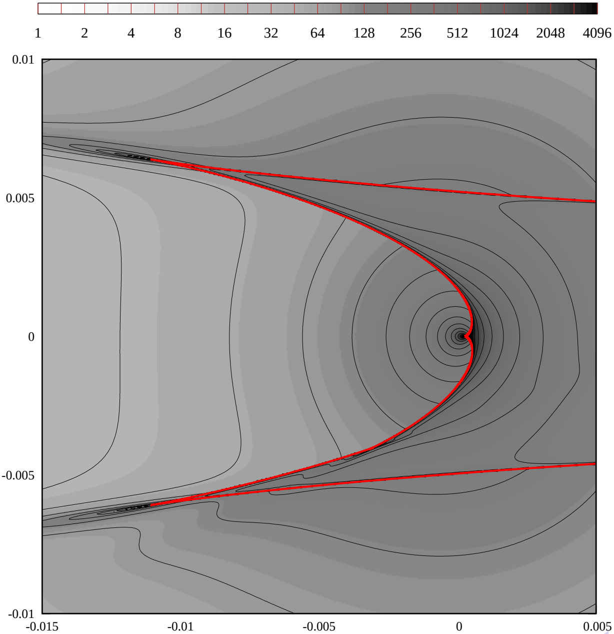
$\mu_A = 0.99895, \mu_B = 10^{-3}$,
$\mu_C = 5 \times 10^{-5}$,
$z_A = 0.0,\ z_B = 1.1$
$z_C = 0.875 \times e^{0.0 i\pi}$

Two planets: fold



$\mu_A = 0.99895, \mu_B = 10^{-3},$
$\mu_C = 5 \times 10^{-5},$
$z_A = 0.0, \ z_B = 1.1$
$z_C = 0.875 \times e^{0.05 i \pi}$

Two planets: swallow tail vicinity



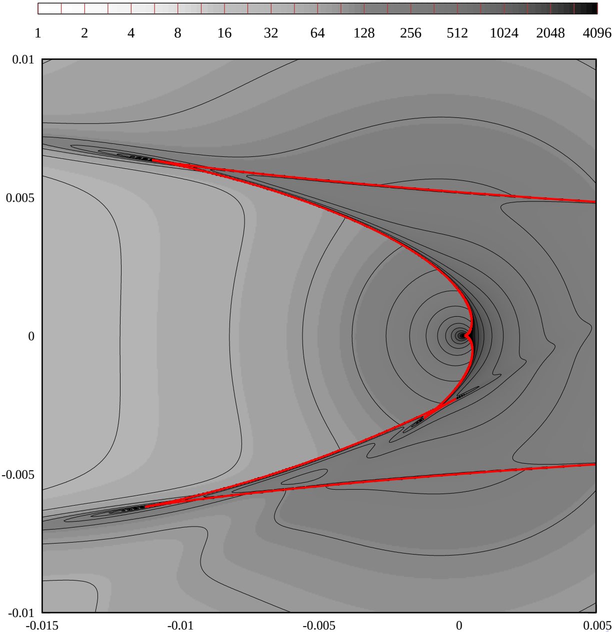
$\mu_A = 0.99895, \mu_B = 10^{-3},$

$\mu_C = 5 \times 10^{-5},$

$z_A = 0.0,\ z_B = 1.1$

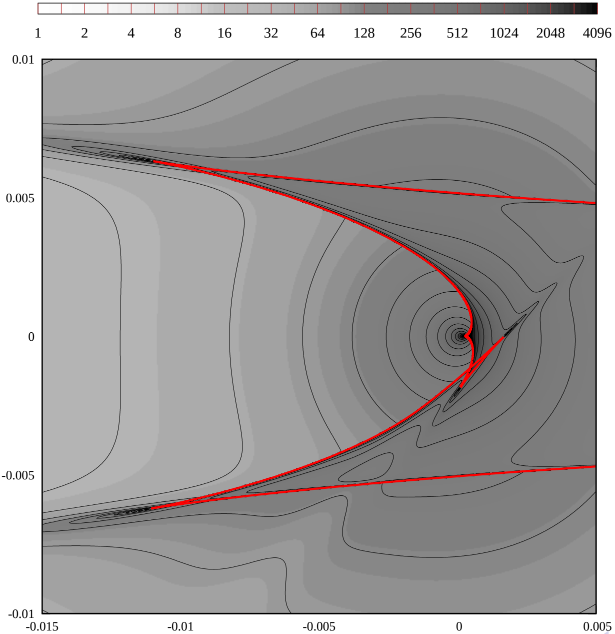
$z_C = 0.875 \times e^{0.10 i\pi}$

Two planets: two-cusps



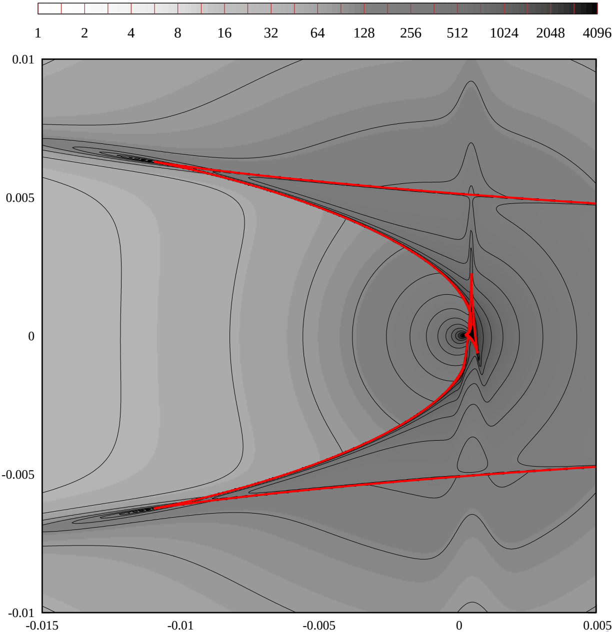
$\mu_A = 0.99895, \mu_B = 10^{-3},$
$\mu_C = 5 \times 10^{-5},$
$z_A = 0.0,\ z_B = 1.1$
$z_C = 0.875 \times e^{0.15 i\pi}$

Two planets: two cusps



$\mu_A = 0.99895, \mu_B = 10^{-3},$
$\mu_C = 5 \times 10^{-5},$
$z_A = 0.0, \; z_B = 1.1$
$z_C = 0.875 \times e^{0.25 i \pi}$

Two planets: entangled caustic(s)



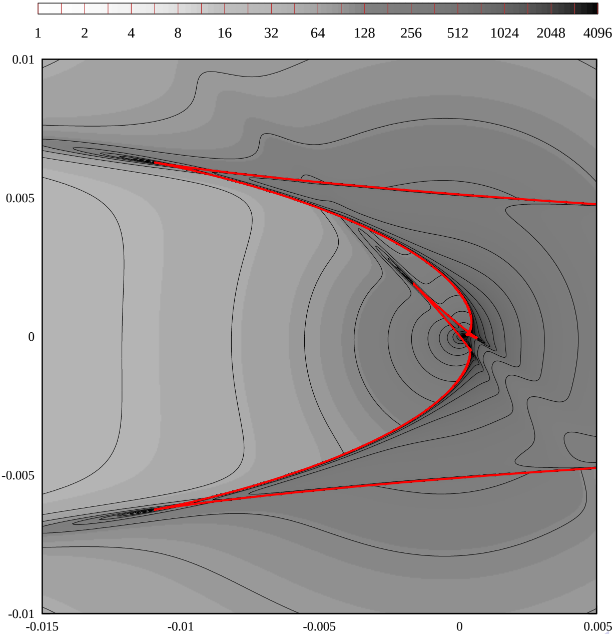
$\mu_A = 0.99895, \mu_B = 10^{-3},$

$\mu_C = 5 \times 10^{-5},$

$z_A = 0.0,\ z_B = 1.1$

$z_C = 0.875 \times e^{0.5i\pi}$

Two planets: entangled caustic(s)



$\mu_A = 0.99895, \mu_B = 10^{-3},$

$\mu_C = 5 \times 10^{-5},$

$z_A = 0.0, \ z_B = 1.1$

$z_C = 0.875 \times e^{0.75 i\pi}$

Conclusion

- We developed tools of localizing butterfly and swallow-tail catastrophes in parameter space of triple lens.
- We described amplification patterns in vicinity of metamorphosis points.
- Characteristic patterns are distinguishable even before the metamorphoses.
- In planetary system, the metamorphoses occur as first significant change in primary caustic geometry due to planet.

Thank you!

Acknowledgements

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