The Equation of State of Neutron Stars: Neutron-star masses, radii and internal composition.

Mariano Méndez

*Kapteyn Astronomical Institute,*
*University of Groningen,*
*The Netherlands*

Didier Barret (CESR), Frits Paerels (Columbia), Tod Strohmayer (NASA/GSFC), Phil Uttley (Southampton), Jon Miller (Michigan), Joern Wilms (Bamberg) ...

... and many others ...
Neutron-star structure

\[ \rho \sim 10^{14} \text{ gr cm}^{-3} \]

\[ \rho \sim 10^{15} - 5 \times 10^{15} \text{ gr cm}^{-3} \]
1. Use hydrostatic equilibrium and mass conservation in GR:

\[
\frac{dP}{dr} = -\frac{G\rho m}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi Pr^3}{mc^2}\right) \left(1 - \frac{2Gm}{c^2r}\right)^{-1}
\]

\[
\frac{dm}{dr} = 4\pi r^2 \rho
\]

2. Add a prescription for the relation between pressure and density, \( P = P(\rho) \).

3. Integrate from \( P(r=0) = P_c \) to \( P = 0 \), which defines \( M \) and \( R \).

For each prescription \( P = P(\rho) \), this yields a family of solutions as a function of the initial condition, \( P = P_c \).
The relation between $P = P(\rho)$, the so-called equation of state (EOS), is set by the interactions between the particles that constitute the star, and can therefore be mapped into a mass-radius relation, $M = M(R)$. 
Equation of State of nuclear matter

EOS reasonably well-known for the outer parts, but unconstrained for the high-density core.

Uncertainty due to inability to extrapolate our knowledge of normal nuclei (with 50% proton fraction) to the high-density regime of nearly 0% proton fraction.
Neutron-star EOS: Why?

- QCD (e.g., existence of Bose-Einstein condensates or free quarks at low temperatures); relevant to high-energy and particle physics.

- Dynamics of supernovae explosions.

- NS–NS mergers, which are likely progenitors of short GRBs and sources of strong gravitational waves.

- Stability of neutron stars.

- NS cooling which, compared to observed NS temperatures, provides NS ages.
Equation of State of nuclear matter.

From pulsars in binaries
\[ \langle M \rangle = 1.35 \pm 0.04 \, M_{\odot} \]

Some masses accurate down to 0.1% (!)

Mass alone not enough to exclude any EOS.

A combination of mass and radius required.

… or a massive NS.
Neutron star EOS measurements and constraints

*Time-resolved spectroscopy and photometry:*

- Redshifted photospheric lines $\rightarrow M/R$ (potentially $M/R^2$)
- Profile of photospheric lines $\rightarrow M/R$
- Pulsations waveform $\rightarrow M/R$
- Quasi-periodic oscillations $\rightarrow R(M)$ (from disc)
- Fe emission (disc) lines $\rightarrow R(M)$ (from disc)
- Frequency-resolved time-delay spectrum $\rightarrow R$ (from disc)
Photospheric absorption during X-ray bursts

EXO 0748–676

335 ks with RGS cameras

Spectra of 28 X-ray bursts co-added

Fe $^{26} n = 2 \rightarrow 3$

Fe $^{25} n = 2 \rightarrow 3$

redshift $z = 0.35 \pm 0.01$

$z = 0.35$

Cottam, Paerels & Méndez
Spectral line profile

Line profile set by:

- longitudinal and transverse Doppler shifts
- special relativistic beaming
- gravitational redshift,
- light-bending
- frame-dragging

Bhattacharyya et al.
Simulated spectral line profile

Line profile calculations courtesy of S. Bhattacharyya
XMS for bright sources

Figure from J. Wilms et al.
Simulations

Spin = 45 Hz

Spin = 400 Hz

Fe XXVI

$z = 0.35$

$n = 2 - 3$

Balmer $\alpha$

$n = 1 - 2$

Lyman $\alpha$
Pulsations during X-ray bursts

Strohmayer et al.; Spitkovsky et al.
Pulsations during X-ray bursts

Simulated pulse profile for the rising phase of an X-ray burst (T. Strohmayer).

Pulse profiles for a 1.8 solar mass NS with a spin frequency of 364 Hz. (C. Miller).
Mass and radius constraints from pulse-profile fitting. The red ellipse shows the 95% confidence regions from 5 typical bursts (C. Miller).
Quasi-periodic oscillations

Sco X–1

Leahy power

FREQUENCY (Hz)
Mass and radius constraints from timing

\[ \nu = \frac{1}{2\pi} \sqrt[3]{\frac{GM_{NS}}{r^3}} \]

\[ R_{NS} \leq r \]

\[ M_{NS} \leq 2.2(\nu/1000\text{Hz})^{-1}M_{\odot} \]

\[ R_{NS} \leq 14.6(M_{NS}/M_{\odot})^{1/3}(\nu/1000\text{Hz})^{-2/3} \text{ km} \]
Emission lines from the inner disc

Suzaku

XMM-Newton

Cackett et al.; Bhattacharyya & Strohmayer
Inner disc radius

4U 1608–52
Tracking the inner disc radius

IXO/HTRS simulations by D. Barret

![Graph showing data and continuum](Image)

- Energy (keV)
- Frequency (Hz)

1 ksec
Iron line

4U 1636-53

Pandel et al.; Altamirano et al.
Frequency-resolved time-delay spectrum
Frequency-resolved time-delay spectrum

IXO/HTRS simulations by P. Uttley

1 Crab
100 ksec

\[ R_{in} = 30 \text{ km} \]
\[ R_{in} = 18 \text{ km} \]
\[ R_{in} = 12 \text{ km} \]
Neutron-star EOS: Summary

• Multiple complementary (redundant) constraints of $M$ and $R$

• Requirements:
  - High time resolution (time scales \( \sim 1\) ms)
  - High count-rate capability (count rates \( \sim 1\) Crab or more)
  - Moderate spectral resolution (line broadening)

• HTRS is the primary instrument for this science

• Complementary information from XMS / WFI / XGS