The MLDC effort at APC (Paris)
29th of December 2006

I. The method

The APC in Paris has started to test different methods to extract the physical parameters of Galactic Binary sources and has applied them to the LMDC Training_111a and Challenge_111a data sets.

The method is based on 3 steps:

1) A study of the FFT of 1 year data and the extraction of the “mean” frequency of the Galactic Binary.
2) The extraction of approximate values of the source direction (+ polarisation) by the study of the amplitude modulation, for each TDI variable, with an analytical formula which relies on 3 assumptions:
    - Low frequencies $2\pi fL \ll 1$
    - The variations of the envelopes are $\ll f$
    - $h_x(t) = \rho\, h_+(t-\tau)$ or $\rho_x h_x(t) = \rho_+ h_+(t-\tau)$

    The following formulae give the “envelope” of the amplitude modulation, with parameters β, λ (ecliptic angles), τ (phase between h_+ and h_x) and $\rho+$ and ρ_x (amplitude of h_+ and h_x) :

$$\left.\frac{\tilde{\delta\nu}}{\nu_{opt}}\right|_{AB}(f) = i e^{2i\pi f\left(\widehat{w}.\vec{r}_{AB} - \frac{L}{2}\right)} \frac{\pi f L}{1+\widehat{w}.\widehat{n}} \tilde{h}(f) \left(\xi_{AB,+} + \rho e^{-2i\pi f\tau} \xi_{AB,\times}\right)$$

$$\begin{cases} \xi_{+i} = \left(\widehat{\beta}.\widehat{n_i}\right)^2 - \left(\widehat{\lambda}.\widehat{n_i}\right)^2 \\ \xi_{\times i} = 2\left(\widehat{\beta}.\widehat{n_i}\right)\left(\widehat{\lambda}.\widehat{n_i}\right) \end{cases}$$

$$\tilde{X}(f) = 8(\pi f L)^2 e^{-3i\pi f L} e^{2i\pi f \widehat{w}.\vec{r}_{AB}} \tilde{h}(f) \left[\xi_{+3} - \xi_{+2} + \rho e^{2i\pi f\tau}\left(\xi_{\times 3} - \xi_{\times 2}\right)\right]$$

$$\left|\tilde{X}\right| = 64(\pi f L)^4 \left|\tilde{h}\right| \left[(\xi_{+3}-\xi_{+2})^2 + \rho^2(\xi_{\times 3}-\xi_{\times 2})^2 + 2(\xi_{+3}-\xi_{+2})(\xi_{\times 3}-\xi_{\times 2})\rho\cos(2\pi f\tau)\right]$$

$$\boxed{\left|\tilde{X}\right| = 64(\pi f L)^4 \left|\tilde{h}\right| \left[\rho_+^2(\xi_{+3}-\xi_{+2})^2 + \rho_\times^2(\xi_{\times 3}-\xi_{\times 2})^2 + 2(\xi_{+3}-\xi_{+2})(\xi_{\times 3}-\xi_{\times 2})\rho_+\rho_\times\cos(2\pi f\tau)\right]}$$

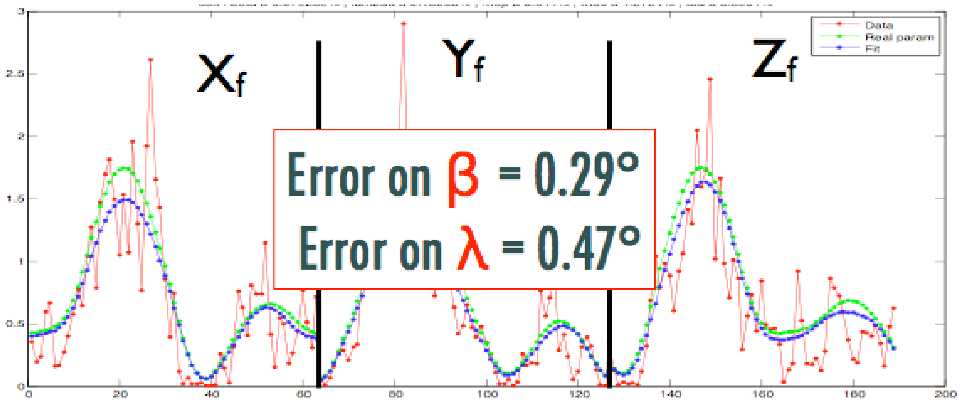
    When applying this method, the year sample is divided into N segments of m days (eventually overlapping) and the amplitude of the TDI variable is extracted automatically from the FFT. The values of N and m have been optimized on the training set.

3) Using the parameters thus determined, a X^2 minimisation using all the parameters and the Fourrier frequencies components (amplitude and phase of the FFT of the 1-year data set) as data is performed. The Fourrier frequencies, kept for this search, correspond to the “mean” frequency (f_0) ± 10 frequency bins ($\Delta f = 1/\text{year} \approx 3\ 10^{-8}$ Hz), which appears sufficient to encompass the frequency spread due to Doppler broadening.

The (amplitude) error associated to each Fourrier frequency vector is extracted from the amplitude of the noise to the left and right of f_0.

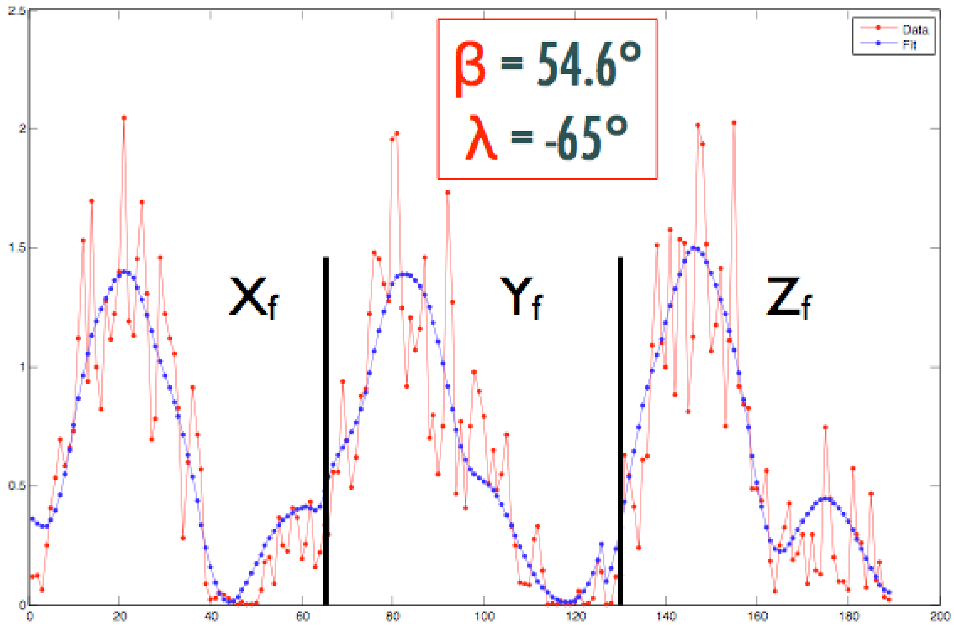
II. Results

Applied to the Training_111a set, the best results are obtained for N=64 sets of m=11 days.
The red curve represents the data, the green curve the simulation (with the exact parameters) as obtained from LISACode and the blue curve is the fit using the formula above.

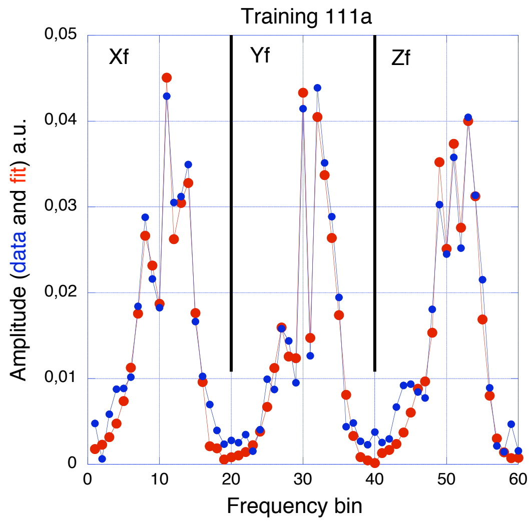


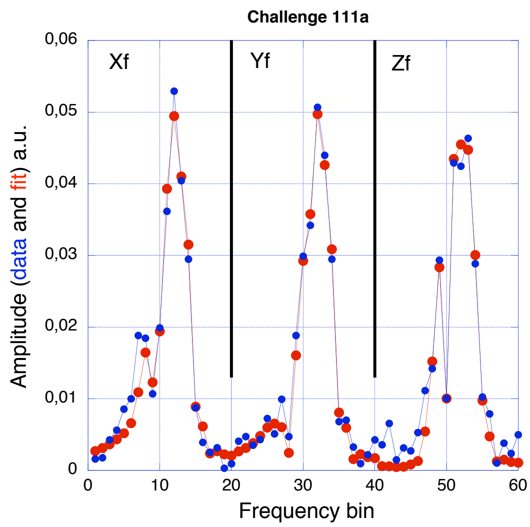
The errors (difference between the exact parametervalues and those found) are given on the figure.

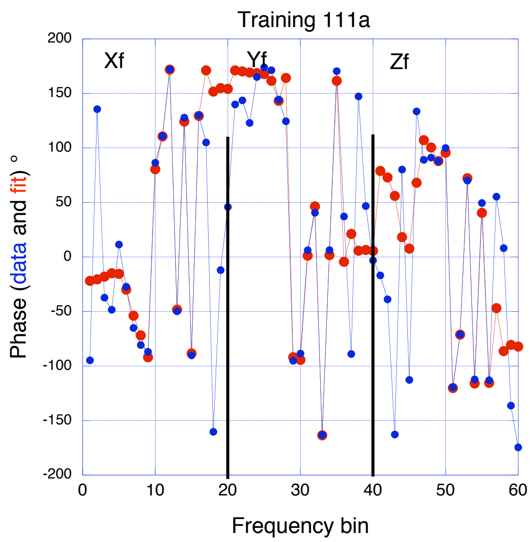
Applying the same method to Challenge_111a, yields :

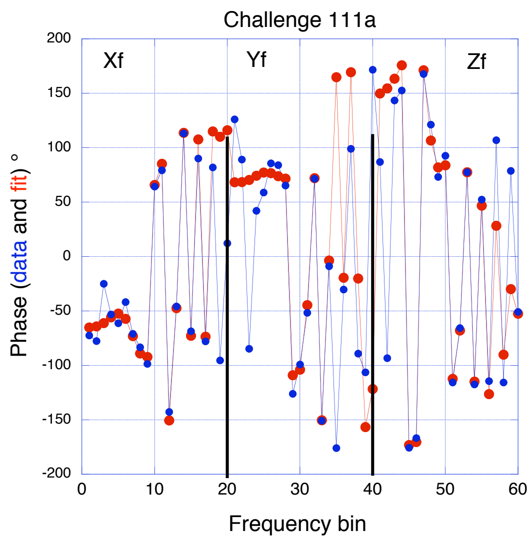


After this first step and using the above parameters as starting values, A X^2 search is performed first by a grid sampling and then by minimisation using the SIMPLEX method.
The resulting fits are given for the Training set (figures on the left column) and for the Challenge set (figures on the right column).









The resulting parameters are :

Training_111a :

The latitude and longitude obtained from the study of the modulation (step 2 of the method) are: $\beta = 27.45°$ and $\lambda = 298.3°$.

After X2 minimisation on the Fourrier components, the following parameters are obtained:

Frequency = 0.99303430 mHz, $\beta = 25.26820°$ (0.441 radians), $\lambda = 297.10652$ (5.185), Amplitude = 0.36947 10^{-22}, $\iota = 10.30163$ (0.1798), $\psi = 136.51447$ (3.901) , $\varphi_0 = -116.42669$ ($5.821-\pi/2$)

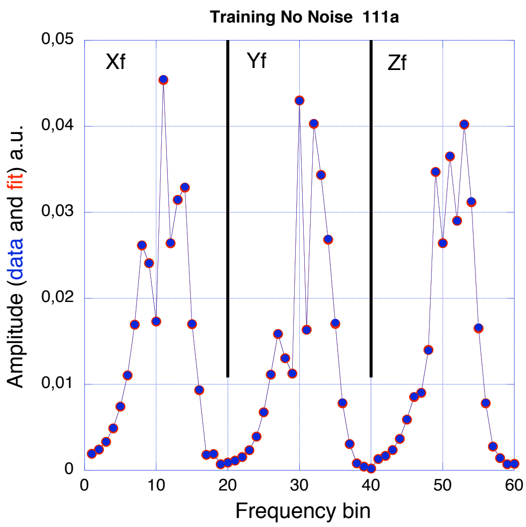
As can be seen form the "true" values of the parameters (see table below) most parameters agree well, except the latitude which, after X^2 minimisation, shows an offset of -2°. This, at this moment, is not explained and may be the result of a bias in the X^2 minimisation on the Fourrier components. The initial phase is shifted by $\pi/2$ because of our formulation.

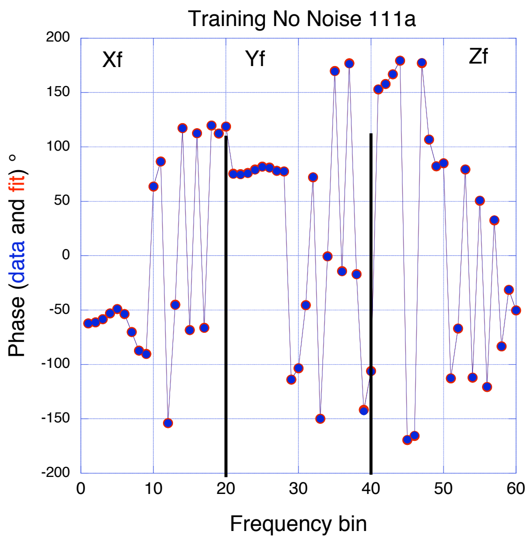
EclipticLatitude	0.4741143268 (27.1648°)	Radian
EclipticLongitude	5.19921 (297.8928°)	Radian
Polarization	3.975816 (227.797°=360-132.203)	Radian
Frequency	0.0009930348535	Hertz
InitialPhase	5.781211 (331.238°)	Radian
Inclination	0.1793956 (10.278°)	Radian
Amplitude	1.789229908e-22	1

Training_111a_noise_free:

Using the noise free data, we have tested the method. The results are the following:

Frequency = 0.99303475 mHz, β = 27.22992 ° , λ = 297.72629,
Amplitude = 0.37071 10^{-22}, ι = 10.09817, ψ = 134.15095, φ_0 = -116.71502





One observes that the value of β is much closer and that the fit is excellent.

The table below gives the error on each parameter when using the training set and the noise-free training set.

Parameter	Error (set with noise)	Error (noise free set)
Frequency	6.0 10^{-10} Hz	-1.0 10^{-10} Hz
β (Ec latitude)	-1.8966°	0,065°
λ (Ec longitude)	-0,78°	-0,163°
Amplitude	0.36947	0.37071
ι (inclination)	0.0236°	-0,18°
ψ (polarization)	4.31°	1,948°
φ_0 (initial phase)	0,335°	0.047°

A study to explain the β discrepancy is ongoing.

Challenge_111a :

The latitude and longitude obtained from the study of the modulation (step 2 of the method) are: $\beta = 54.6°$ and $\lambda = 295°$.

After X2 minimisation on the Fourrier components the following parameters are obtained:

Frequency = 1.0627301 mHz, β = 54.49987 (0.951), λ= 291.02475 (5.078),
Amplitude = $0.46090\ 10^{-22}$, ι = 34.12910 (0.596), ψ = 112.18533 (1.958),
φ_0 = -58.39495 (0.551-π/2)

Pour l'amplitude, cf l'attenuation de TDI

One observes that, in this case, β and λ do not appear to have changed very much.

Conclusions.

We report here our first attempts to tackle the LMDC and the methods presented here are not final. The determination of the "principal" observation angles (β and λ) by using the modulation "envelope" formula appears promising and can still be improved (direct determination of the minima of the modulation amplitude). It is possible that the suggested X2 minimisation on the Fourrier components introduces some bias as has been seen for the training set.

We are also looking into the estimation of the errors (systematic and statistical) that can apply to the extraction of the parameters.