LISA Mock Data Challenge 1.2 - MBH Binaries

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1. Overview

(a) *HHT Method*

Our analysis technique is the HHT (Hilbert-Huang Transform), an adaptive technique that first "sifts" the data by identifying extrema in a time series, and then produces a set of orthonormal basis sets (Intrinsic Mode Functions) containing the signal (see accompanying article for details - Camp, Cannizzo, & Numata 2006). Since in general the higher IMFs contain lower frequency information, a signal such as a chirp appears spread across several IMFs.

There are three relevant derived time series for each IMF one can produce from a given data set: the amplitude c, the instantaneous frequency f, and the instantaneous power p. The power p is taken to be the square root of the sum of the squares of the real and imaginary parts of the IMF.

(b) *Data Analysis*

The GW frequency can be expressed

$$f(\mathrm{Hz}) = \frac{5^{3/8}}{8\pi}\left(\frac{GM_t}{c^3}\right)^{-5/8}[\mu(t_c-t)]^{-3/8}\,, \tag{1.1}$$

where M_t is the total binary mass, and μ is the normalized reduced mass $\mu = M_1M_2/M_t^2$.

If we define $y = \log(f)$ and $x = \log[t_c(\mathrm{d}) - t(\mathrm{d})]$, then this equation can be rewritten in the simpler form

$$y = 0.32785 - (5/8)\log m_{\mathrm{chirp}} - (3/8)x. \tag{1.2}$$

Therefore if the instantaneous frequency $f(t)$ is plotted in terms of x and y, one expects the slope to be $-3/8$ and the $y-$intercept to be $0.32785 - (5/8)\log m_{\mathrm{chirp}}$, where m_{chirp} is the chirp mass in solar units, $m_{\mathrm{chirp}} = (M_t/M_\odot)\mu^{3/5}$. A least squares fit through the $f(t)$ data therefore enables a precise determination of the chirp mass to be made. In addition, if a localization on the sky can be made, then the coincidence in functional form of $f(t)$ and strain amplitude $h(t)$ on M_t and μ (Schutz 1986, Holz & Hughes 2006) can be used to derive the distance in terms of f and h.

2. Challenge 1.2.1 - Training

This data set contains a chirp signal from MBH with $D_L = 27\text{Gpc}$, corresponding to a redshift $z \simeq 3.11$ for standard cosmology. The chirp mass is given as $1.02 \times 10^6 M_\odot$, and the inspiral time $t_c = 191.5$ d into the 1 yr of data.

Figs. 1-3 cover a 20 d interval encompassed the inspiral, and show the breakdown of the IMFs for c, f, and p. The signal appears spread out over IMFs 3-9. The regions of strong elevation in power p indicate the signal. Fig. 4 shows a co-adding of the amplitudes c for IMFs 3-9, over a 2 d interval including t_c. There appears to be an inconsistency between the data and the theoretical strain amplitude as given by eqn. (3.12) in the Document for Challenge 1. This is evident in Fig. 5, where we plot the log of the amplitude over a 12 d interval covering t_c, and show on the same scale eqn. (3.12). We have also calculated the strain using the antenna pattern in combination with the calculated h_+ and $h_\times$ values (Cornish & Rubbio 2003, Krolak et al. 2004), and the result is about the same; there seems to be about a factor of $\sim 30 - 100$ discrepancy in the strain amplitude as given in the data set.

Fig. 6 shows the result of taking a data cut on regions of high power in IMFs 3-9, forming a moving average in the time series of $f(t)$ and then using eqn. [2] above in a least squares fit to determine the slope and $y-$intercept. The chirp mass so derived agrees well with the given value, and the slope is close to the expected theoretical value $-3/8$. The quoted errors are only the internal error to the fit, and do not reflect that fact that the slope of the relation is not exactly $-3/8$.

Fig. 7 shows the result of plotting the final chirp in frequency as deduced from the cut $f(t)$ values on a log-linear scale, and superposing the theoretical chirp relation deduced from adopted the fitted value for $m_{\rm chirp}$.

Due to the problem with the amplitude in the data we cannot determine a sky location nor distance. The data discussed in this section is for the X column of the data set. For the Y and Z components, the discrepancy in amplitude between theory and data is even larger.

3. Challenge 1.2.2 - Training

This data set contains a chirp signal from MBH with $D_L = 3\text{Gpc}$, corresponding to a redshift $z \simeq 0.53$ for standard cosmology. The chirp mass is given as $1.48 \times 10^6 M_\odot$, and the inspiral time $t_c = 410.$ d, after the end of the 1 yr of data.

Figs. 8-10 cover the final 45 d of the 1 yr data, and show the breakdown of the IMFs for c, f, and p. The signal appears spread out over IMFs 9-11. Fig. 11 shows the log of the signal amplitude over the final 45 d for IMFs 9-11, along with the theoretical strain amplitude from eqn. (3.12) in the Document. There also seems to be a problem with this data set; the discrepancy between theory and data, for the given parameters, is again about 100-fold.

Fig. 12 shows the result of taking a data cut on regions of high power in IMFs 9-11, forming a moving average in the time series of $f(t)$ and then using eqn. [2] above in a least squares fit to determine the slope and $y-$intercept. The chirp mass so derived agrees well with the given value, and the slope is close to the expected theoretical value $-3/8$.

Fig. 13 shows the result of plotting the final chirp in frequency as deduced from the cut $f(t)$ values on a linear-linear scale, and superposing the theoretical chirp relation deduced from adopted the fitted value for $m_{\rm chirp}$.

Due to the problem with the amplitude in the data we cannot determine a sky location nor distance. The data discussed in this section is for the X column of the data set. For the Y and Z components, the discrepancy in amplitude between theory and data is even larger.

4. Challenge 1.2.1 - Blind

The inspiral time is determined to be $t_c = 154.775$ d. Figs. 14-16 cover a 20 d interval with the chirp near the end and show the breakdown of the IMFs for c, f, and p. The signal appears spread out over IMFs 4-8.

Fig. 17 shows moving average of $f(t)$, and least squares fit. The derived chirp mass $m_{\rm chirp} = 1.26 \times 10^6 M_\odot$, and the slope is close to the expected theoretical value $-3/8$.

Fig. 18 shows the final chirp in frequency as deduced from the cut $f(t)$ values on a log-linear scale, and superposing the theoretical chirp relation deduced from adopted the fitted value for $m_{\rm chirp}$.

The signal cannot be followed for enough time to determine a sky position. This might perhaps be explained if the data signal is too weak for the given parameters by a factor $\sim 40 - 100$ as in the Training sets 1.2.1 and 1.2.2.

5. Challenge 1.2.2 - Blind

No signal is evident in the data set. This might perhaps be explained if the data signal is too weak for the given parameters by a factor $\sim 40 - 100$ as in the Training sets 1.2.1 and 1.2.2.

6. References

References

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