Accurate Binary Black Hole Evolutions Without Excision

Yosef Zlochower (UTB)
Center For Gravitational Wave Astronomy
The University of Texas at Brownsville

Manuela Campanelli (UTB)
Carlos Lousto (UTB)
John Baker (NASA-Goddard)

Bernard Kelly (UTB)
Mark Hannam (UTB, Jena)

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NASA Goddard Space Flight Center

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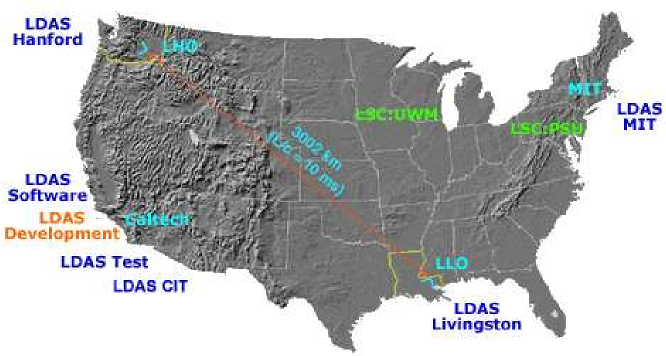
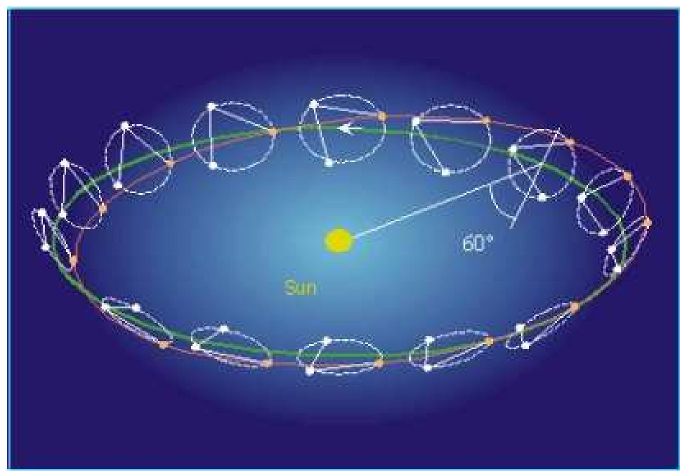
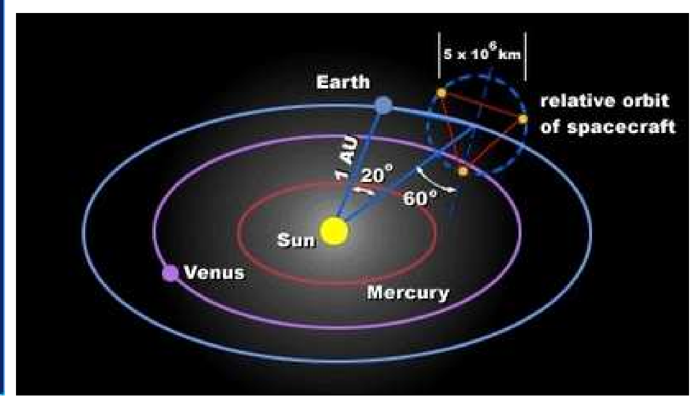
Overview

- Numerical Relativity at UTB.
- The puncture approach.
- The LazEv Framework.
- Higher order finite-differencing.
- Stability issues with punctures.
- Headon collision results.
- Moving punctures, and QC0 results.
- Conclusion

Motivation


LDAS
Hanford
LHO
LSC:UWM
MIT
LDAS
MIT
LSC:PSU
3002 km
(L/c = 10 ms)
LDAS
Software
LDAS
Development
Caltech
LDAS Test
LDAS CIT
LLO
LDAS
Livingston

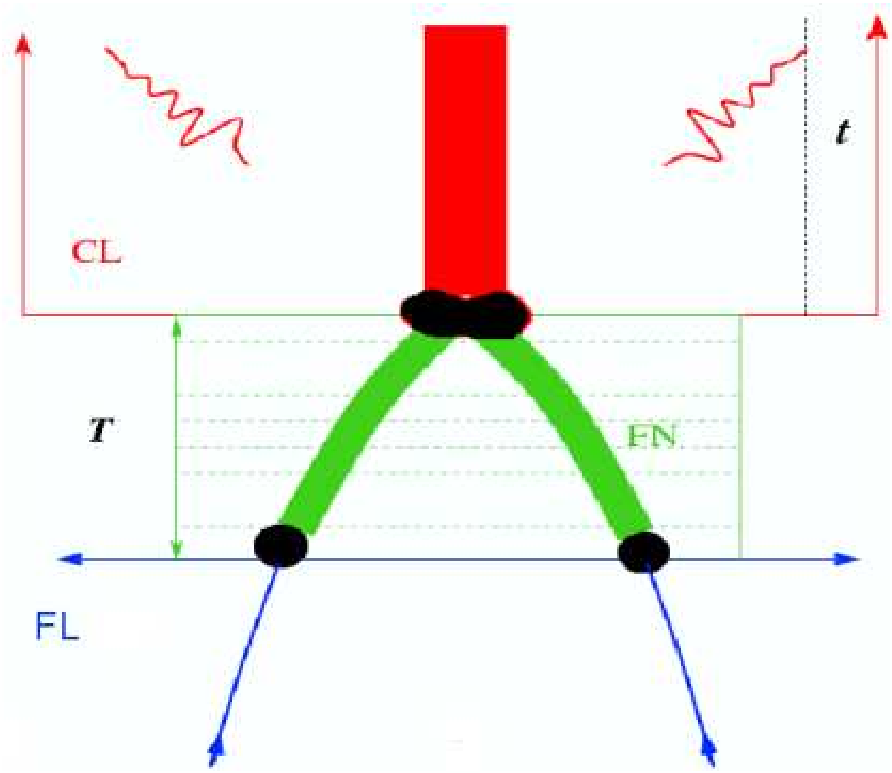

60°
Sun


5 x 10⁶ km
Earth
relative orbit
of spacecraft
1 AU
20°
60°
Sun
Venus
Mercury

Numerical Relativity at UTB

- Goal: to obtain highly accurate BBH waveforms (critical for extracting physics from LISA signals)
- Lazarus and the numerical to perturbative transition (gr-qc/0510122)
- Conformal thin-sandwich initial data (gr-qc/0505120 gr-qc/0502067)
- Post-Newtonian initial data (gr-qc/0207011)
- BSSN style numerical evolutions with punctures (gr-qc/0505055)

Lazarus

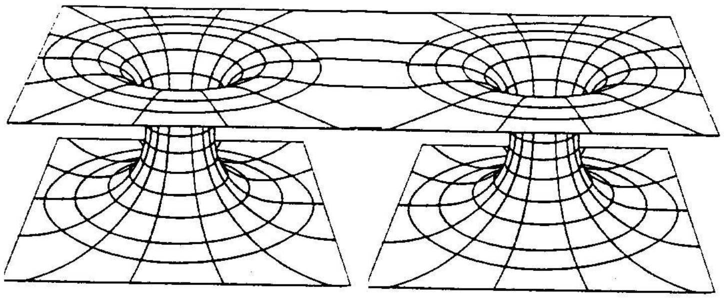


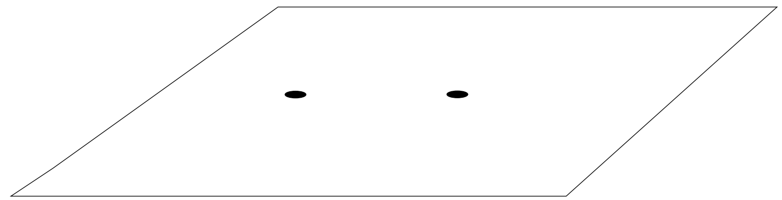
- Consider the orbiting BBH problem as 3 different problems: inspiral, merger, ringdown
- Solve each problem using tools most suited for that problem
- merge the solution together

Nonlinear evolutions

- The LazEv framework (PRD 72 (2005) 024021, qr-qc/0505055)
- We use various flavors of BSSN
- Dynamical gauges
- Higher order finite differencing
- Puncture style evolutions
- Fisheye coordinates
- Radiative BCs

The puncture approach





$\psi = 1 + \sum \frac{m_i}{2r_i}$

- No excision with Singularity avoiding slicing
- No inner boundary conditions
- Physically motivated data
- Ideally suited for BSSN (ϕ can handle singular behavior analytically)

Puncture vs. excision

- advantages
  - Simpler to implement
  - Don't need to worry inner boundaries and excision inconsistencies
  - Superior waveforms
- problems
  - Can never resolve all features near the puncture
  - May not have continuum limit
  - Puncture induces high frequency features that can kill a run
  - Fixing puncture position can lead to grid distortions that kill a run

How we handle puncture problems

- Reduce the differencing order near the puncture (higher-order $\longrightarrow$ higher error)

- upwinding – avoid differencing across the puncture 

- corotation – reduce distortions due to fixed punctures

- What about moving punctures ?

- Can't use KO dissipation near the punctures

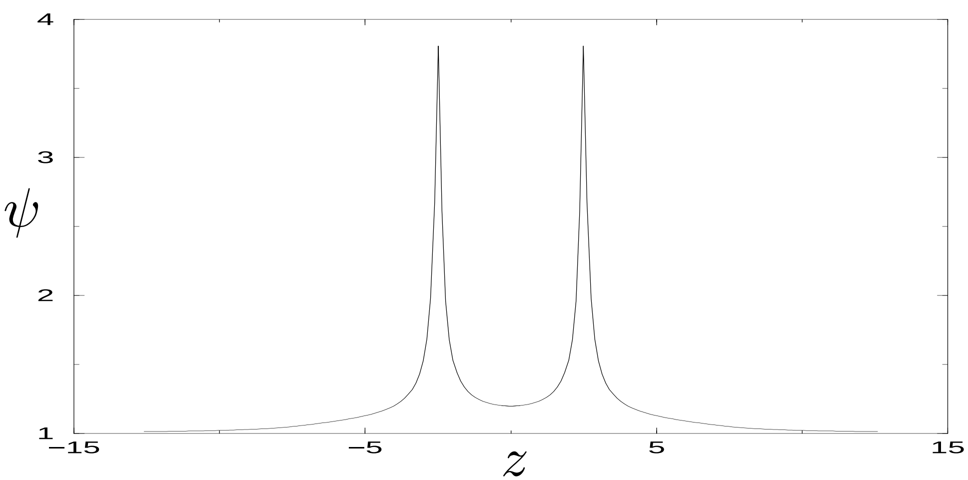
The LazEv Framework

- MOL integrator (RK2, RK3, RK4, ICN2)
- Mathematica scripts which convert PDEs to finite difference algorithms
- Supports arbitrary FD order
- Supports arbitrary FD stencils (e.g. upwinded, centered, mixed
- Quickly implement new evolution system as they become available
- Quickly implement new gauge conditions as they become available

Higher order FD

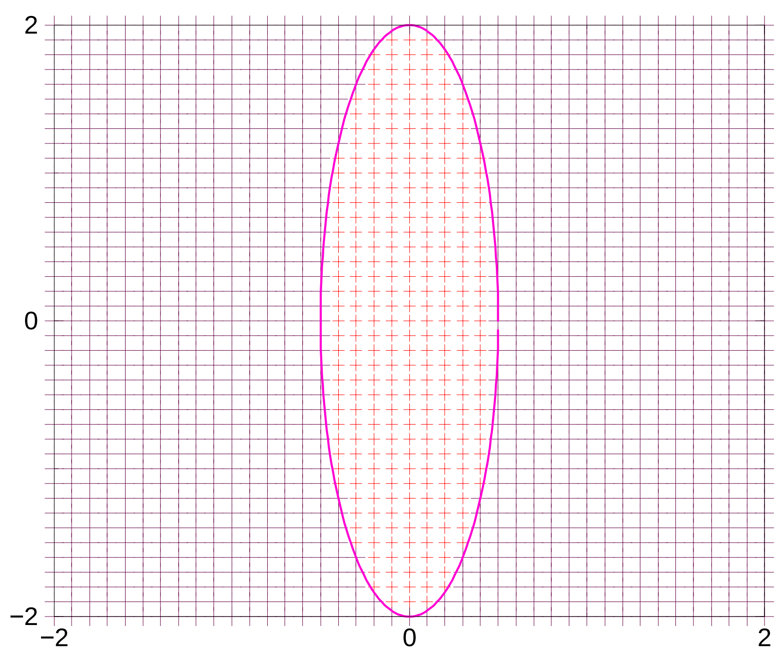
- Why would you use higher order methods.
  - Error scale as h^n, computational expense as $1/h^4$. Break even at fourth-order. Second-order (W/O FMR,AMR) is too expensive.
  - Higher effective resolution for the same number of gridpoints.

- Problems and Alternatives
  - Harder to stabilize
  - More complicated boundaries.
  - 2nd order AMR and FMR can produce better results at lower cost (FishEye)



Techniques

- Shift / No Shift (Gamma driver)
- Upwinding / no upwinding ($[\partial_t - \beta^i \partial_i]F = RHS$)
- 'LOR' (2nd order evolution inside AH)
- Second-order upwinding (4th order unstable near punctures)
- Dissipation (Kreiss Oliger)

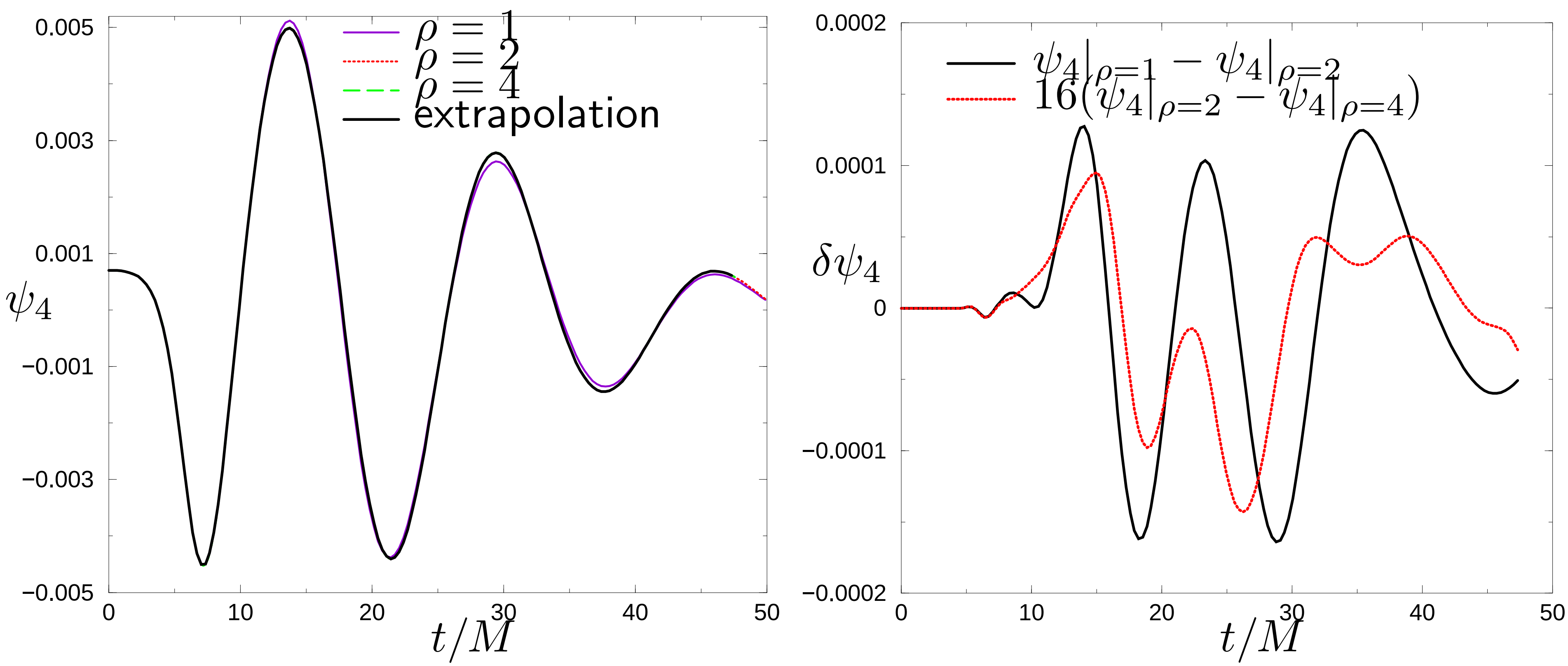


BBH data

We evolved Brill Lindquist data for Headon collision of two equal mass, non-spinning black holes.
We chose these data because:

- Symmetries allow us to evolve using 1 octant (cheap)
- No initial data solver required, and no ID error.
- Waveforms known from 2D codes, Lazarus, AEI 3d runs
- Can use published parameters for gauge conditions.

BBH Waveforms



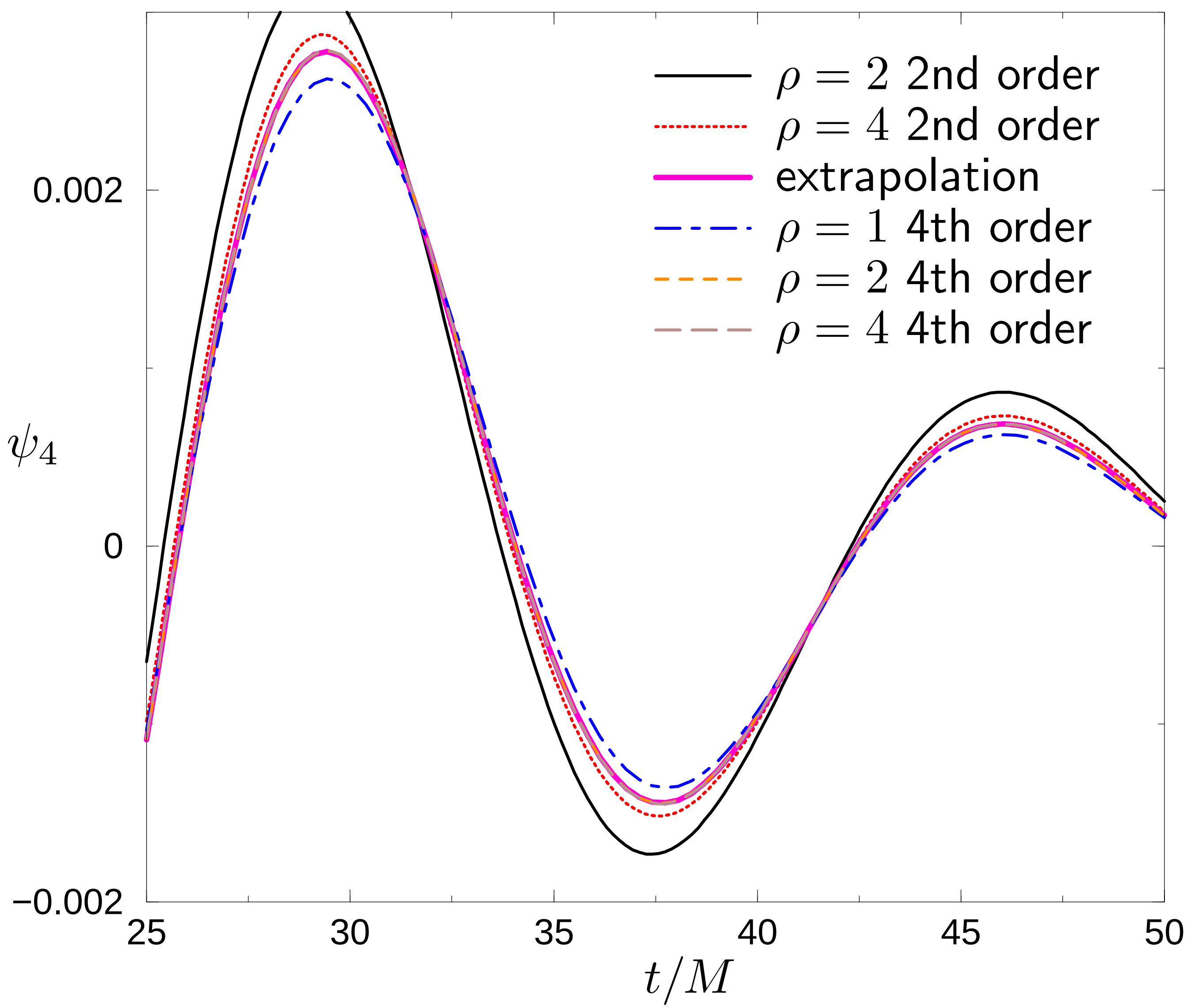


Figure 1: $(\ell = 2, m = 0)$

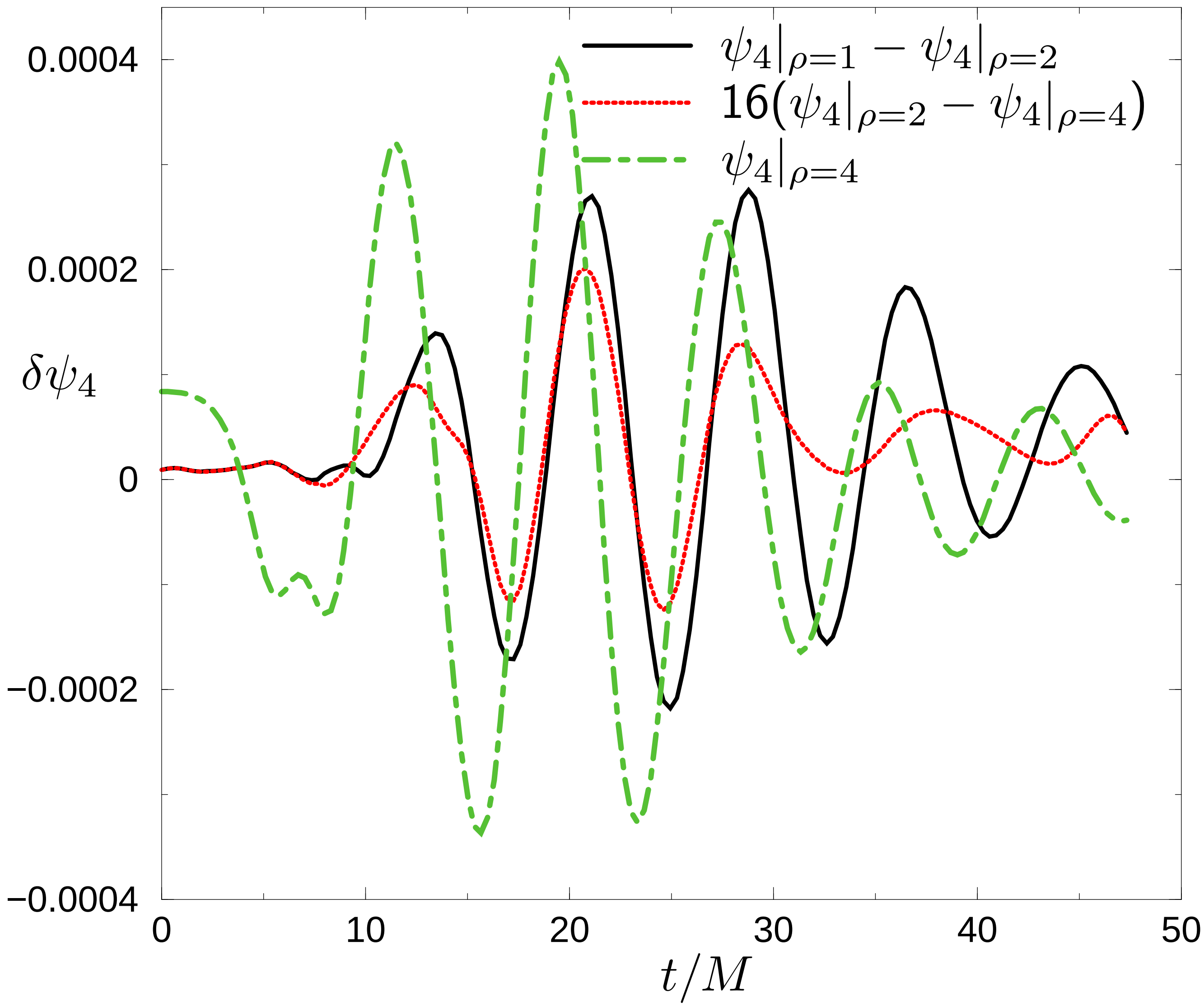


Figure 2: $\ell = 4, m = 0$

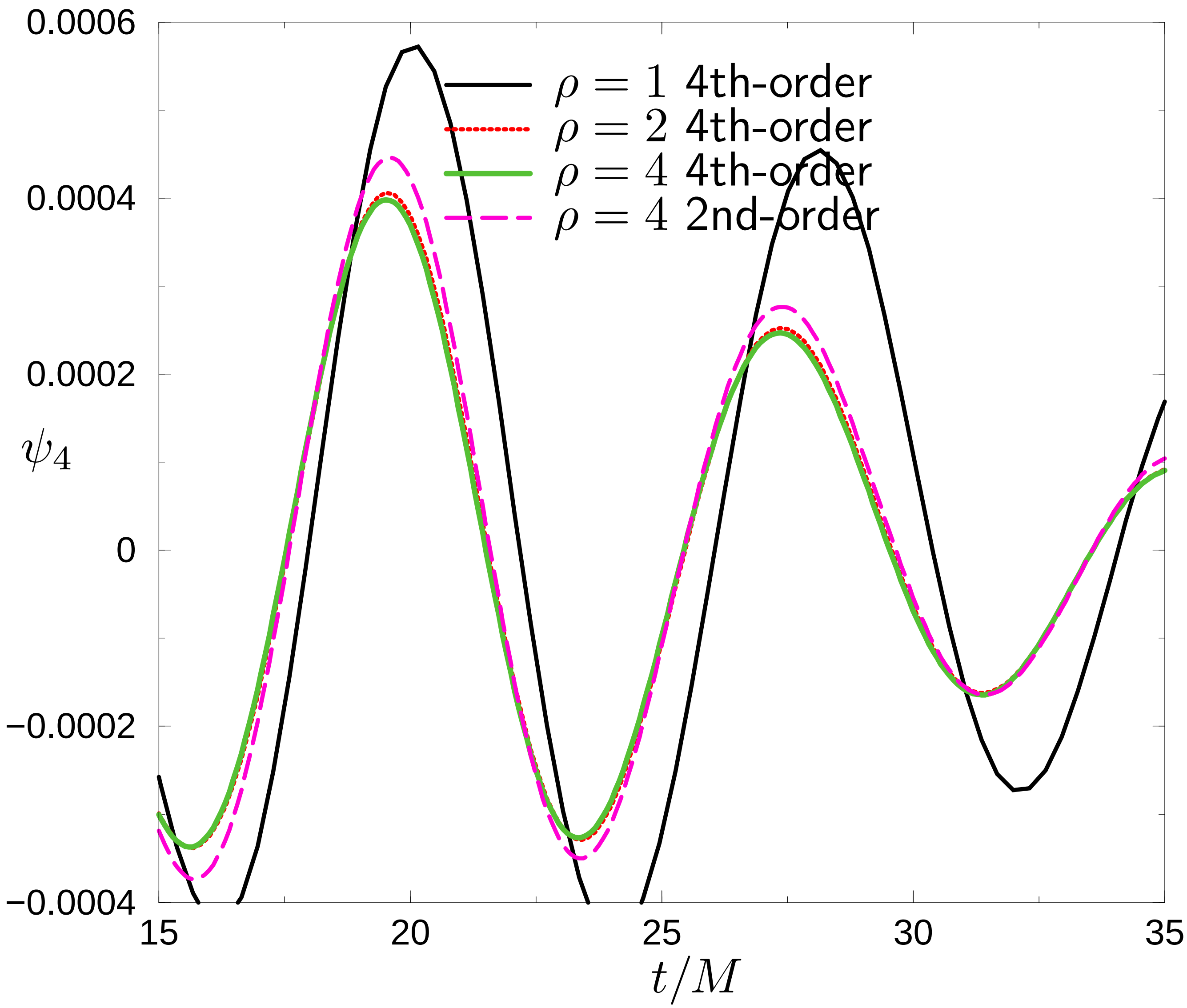
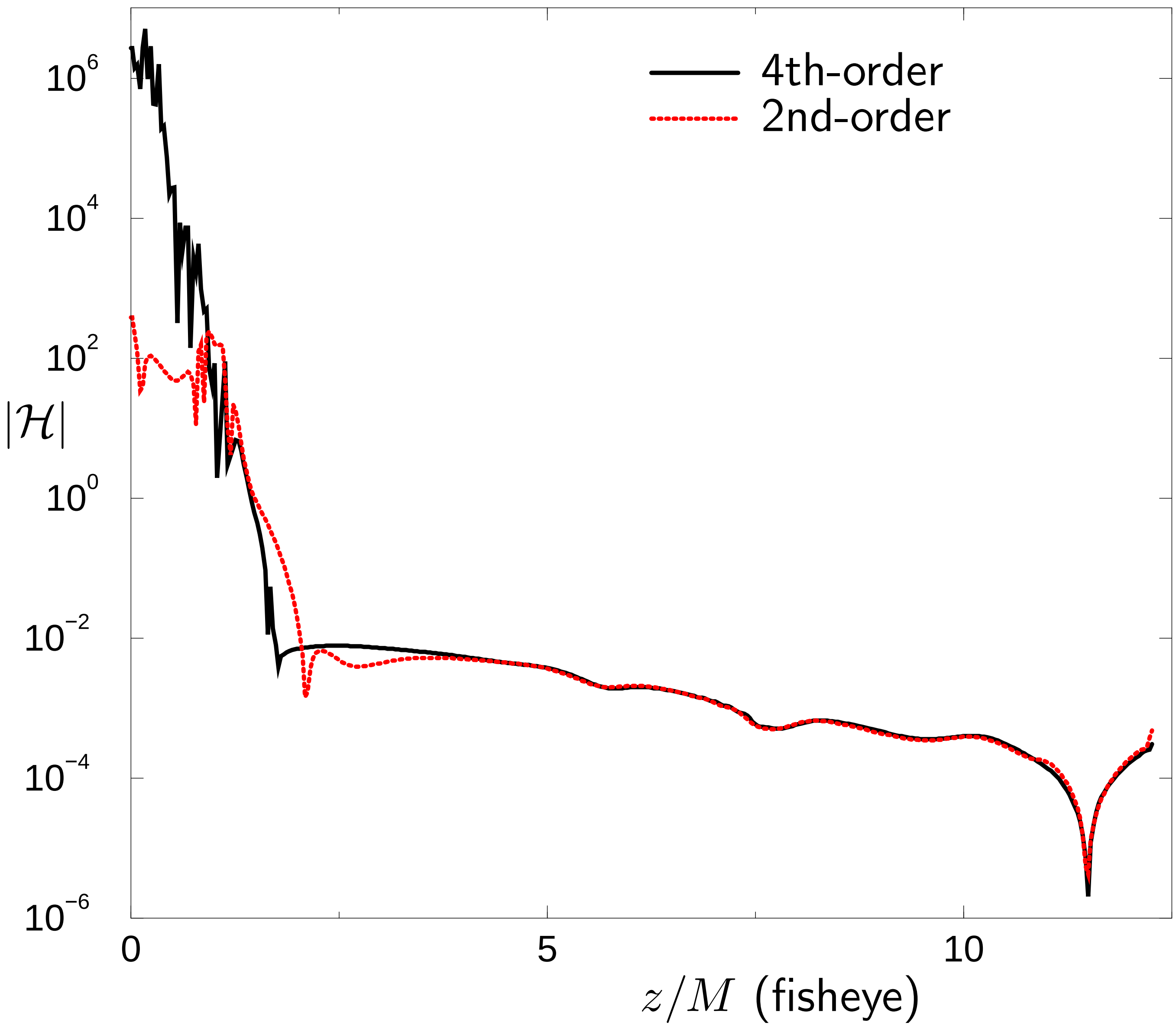


Figure 3: $(\ell = 4, m = 0)$


4th-order
2nd-order
10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
$|\mathcal{H}|$
0
5
10
z/M (fisheye)

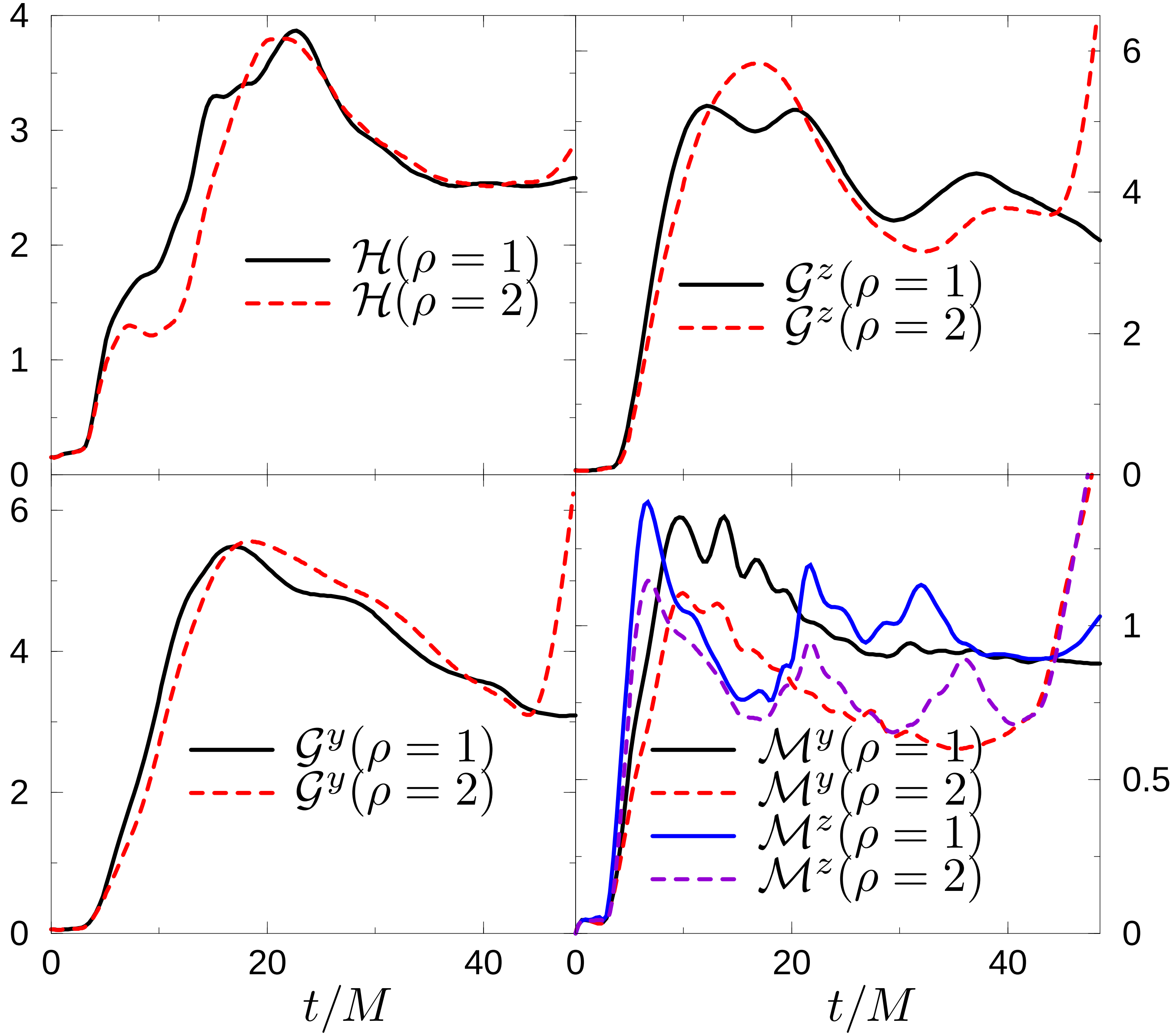


Figure 4: Rescaled constraint violation ($\rho = 1$ by 10^{-5}, $\rho = 2$ by 16×10^{-5})

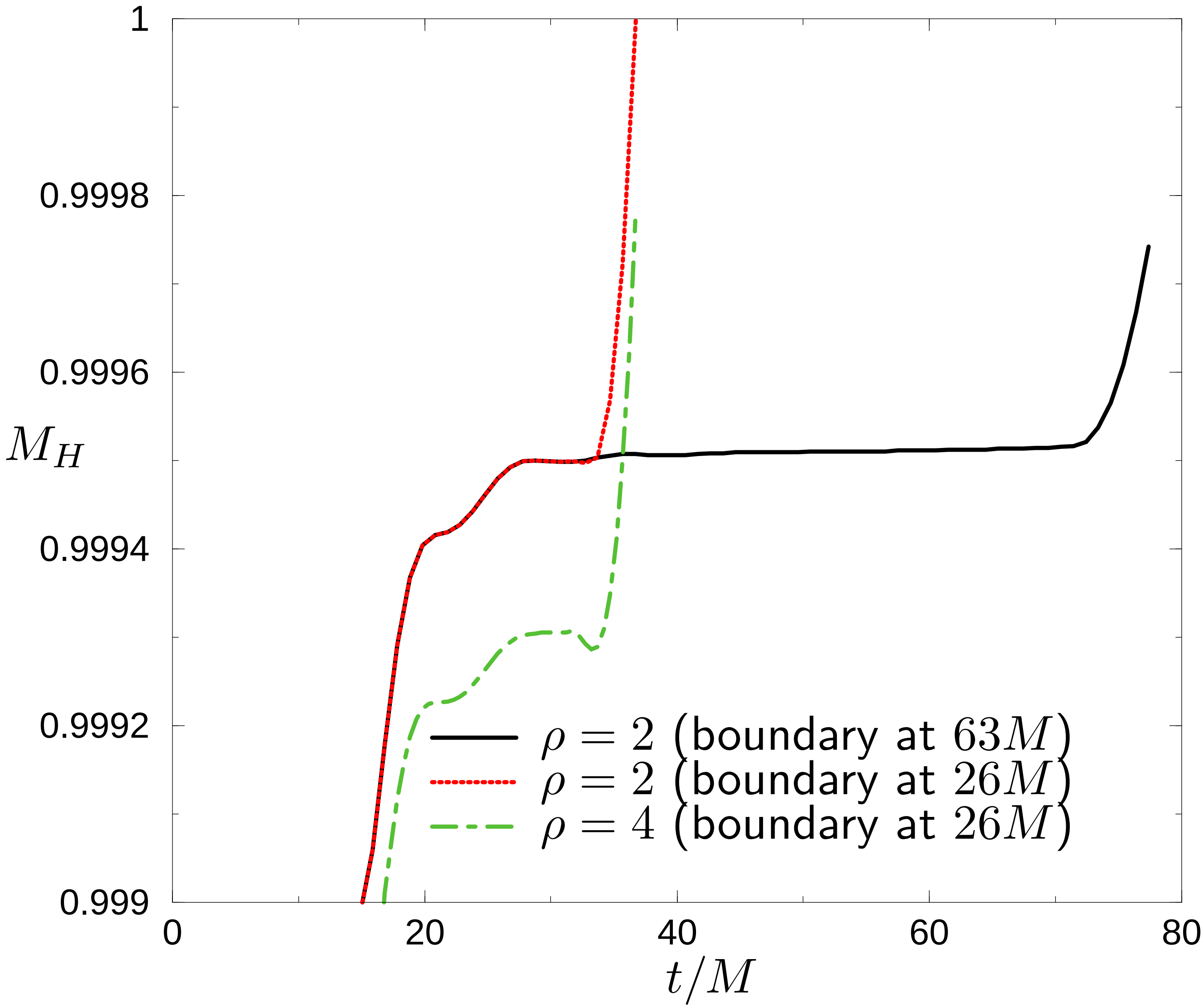


Figure 5: Horizon mass

QC0 via moving punctures

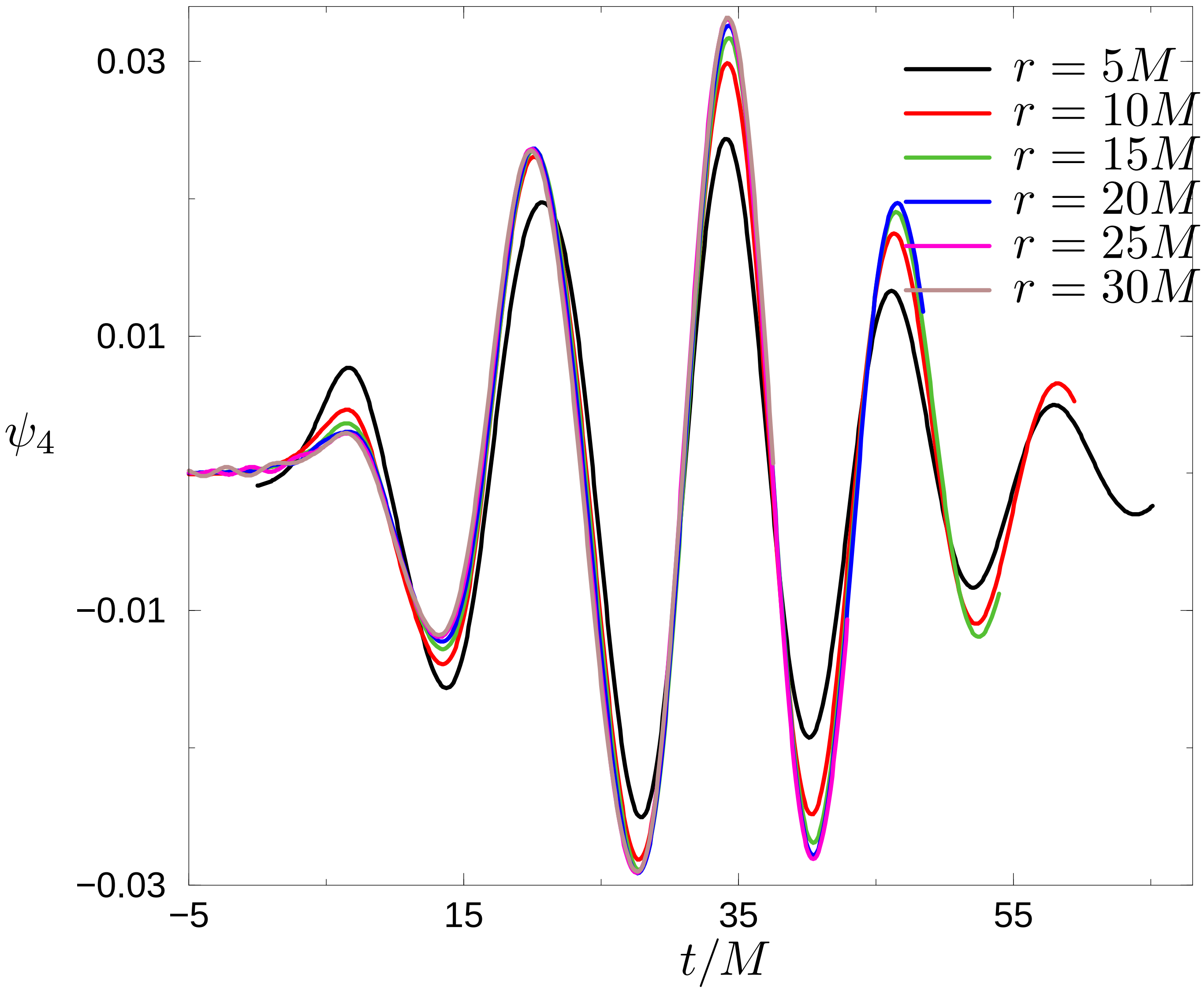


Figure 6: Real part of $(\ell = 2, m = 2)$ mode of ψ_4

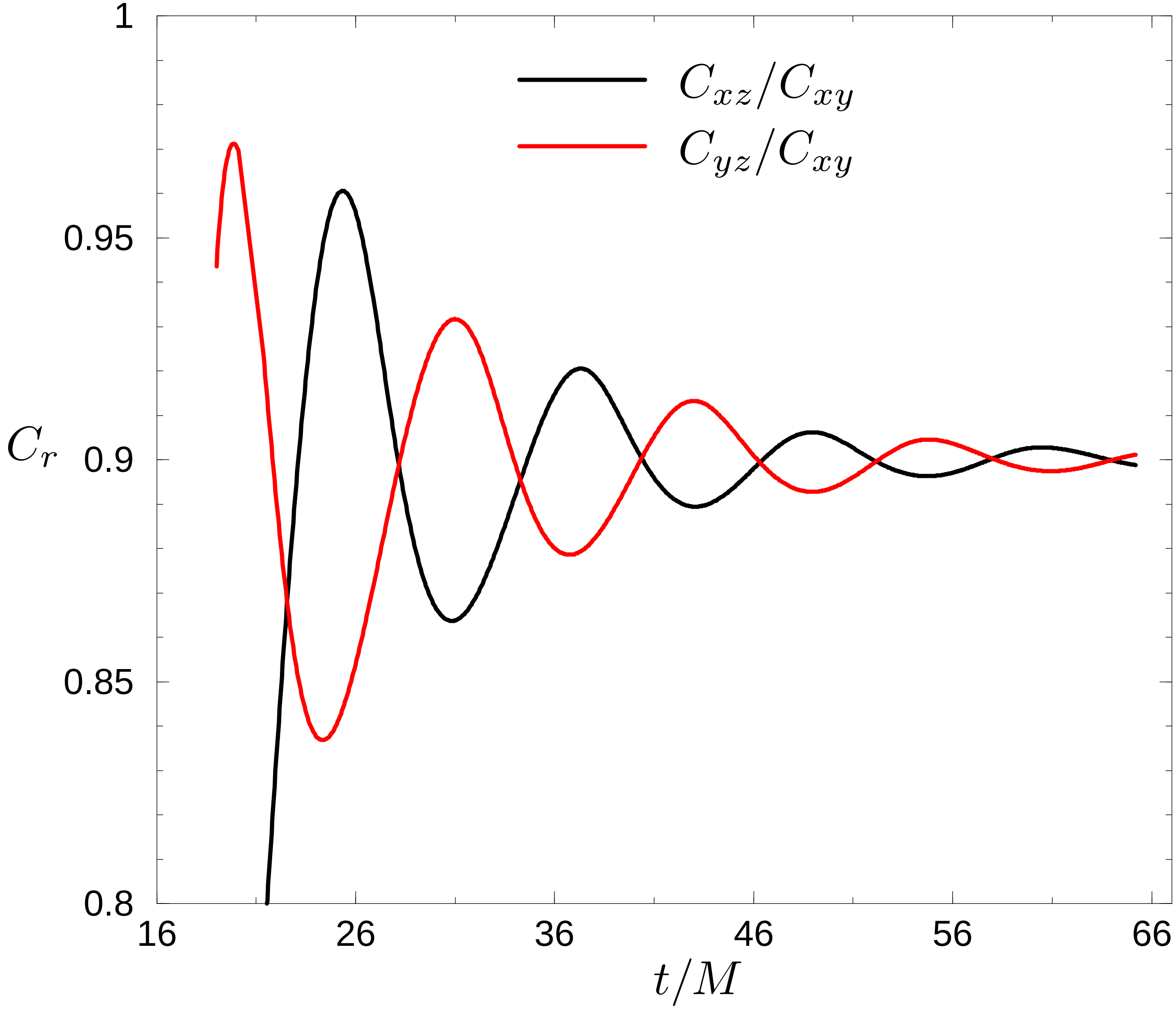


Figure 7: Horizon circumference ratios

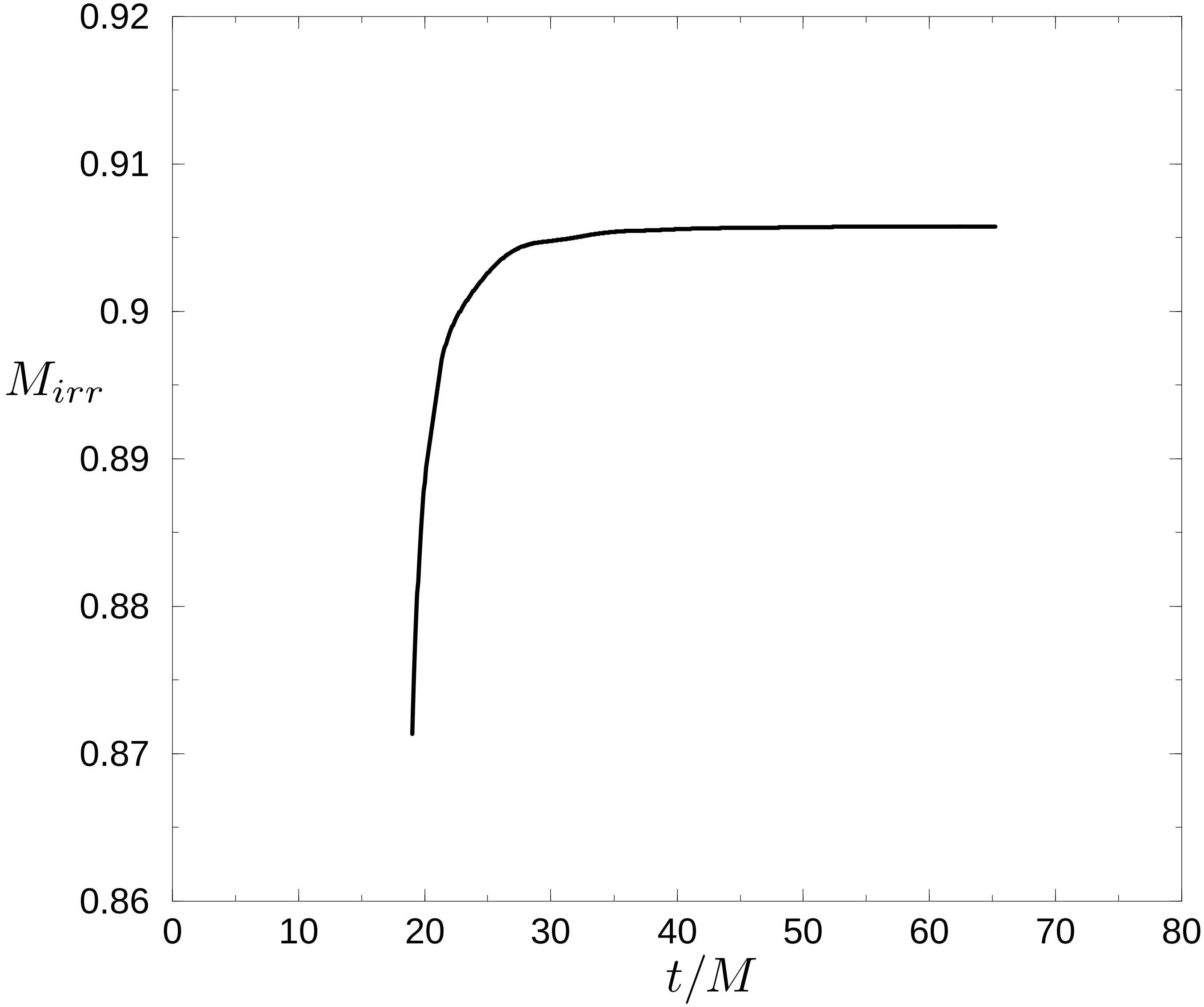


Figure 8: Irreducible horizon mass

QC0 results

$$C_r = \frac{1+\sqrt{1-(a/M)^2}}{\pi} E\left(-\frac{(a/M)^2}{\left[1+\sqrt{1-(a/M)^2}\right]^2} \right)$$

$$E_{rad} = \sum_{\ell,m} \frac{(r-2M)^2}{16\pi} \int_0^{t_f} dt \left[\int_0^t C_{\ell,m}(x)dx\right] \left[\int_0^t \bar{C}_{\ell,m}(x)dx\right]$$

- $h = M_{adm}/24$
- $M_{irr} = .91 M_{adm}$ (approx)
- $C_r = (.897, .9027)$
- $M_{\mathcal{H}} \simeq .972 M_{adm}$
- $\frac{a}{M_{\mathcal{H}}} = .678 \pm .008$ (Lazarus: .7)
- $E_{rad} \simeq .026\, M_{adm}$ (Lazarus 2.4% - 2.6%)

movie

Conclusion

- The LazEv framework allows for fast development of new codes based on $3 + 1$ decompositions.
- We obtained very accurate waveforms using puncture style BSSN evolution with fourth-order stencils.
- We can evolve orbiting black holes using the puncture approach with moving punctures.
- Future Work
  - Explore QC sequence
  - Evolve CTSP data
  - Unequal mass mergers