Binaries Containing Neutron Stars: The Merger Aftermath

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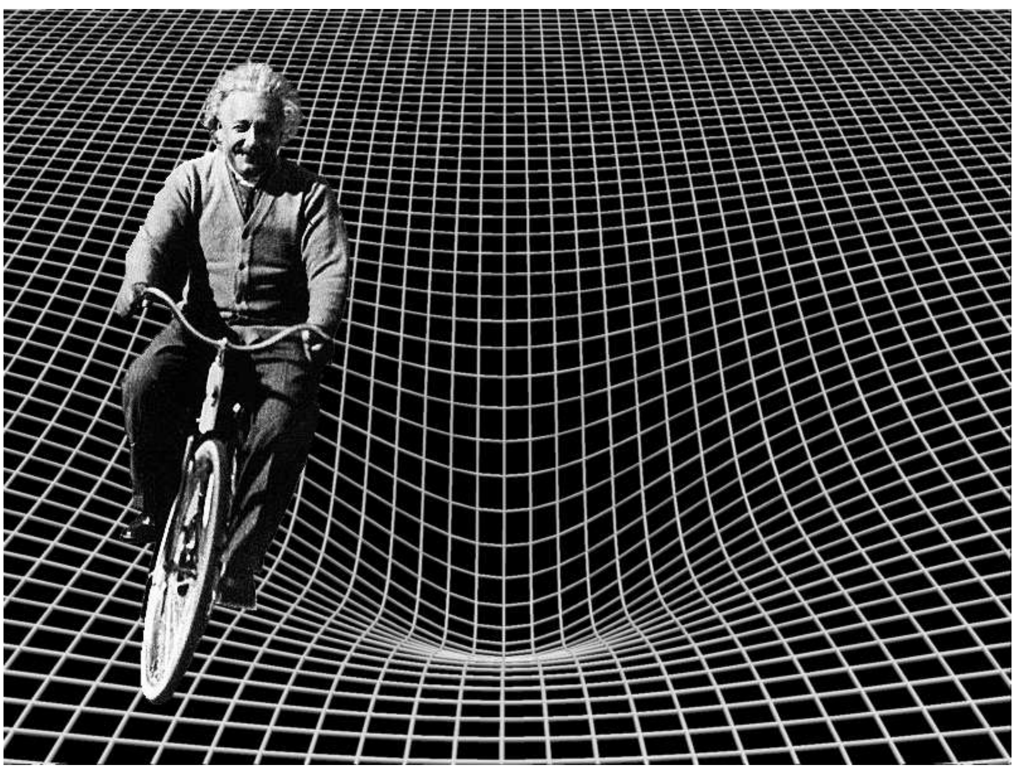


Numerical Relativity 2005: Compact Binaries

A Workshop at NASA Goddard Flight Center

2 November - 4 November, 2005

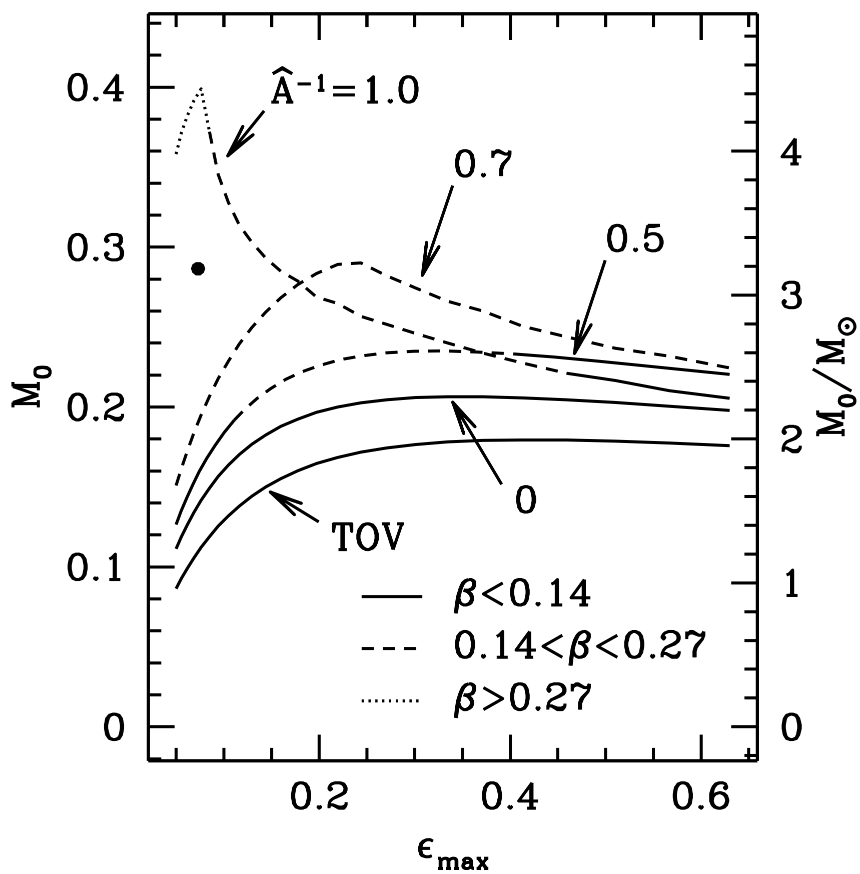
Hot Topics in Numerical Relativity: Partial List

- Binary BHs: initial data & evolution
- Binary NSs: initial data & evolution
- Binary BH-NSs: initial data & evolution
- Rotating stars: radial instab. & collapse;
  nonaxisym. instab. (dyn);
  viscous evolution (secular)
- Collisionless clusters: radial instab. & collapse;
  binary BHs
- Scalar fields: evolution & collapse;
  binary BHs
- Critical phenomena
- Cosmic censorhip
- GR MHD

Differentially Rotating ‘Hypermassive’ Stars

Baumgarte, Shapiro & Shibata (2000);

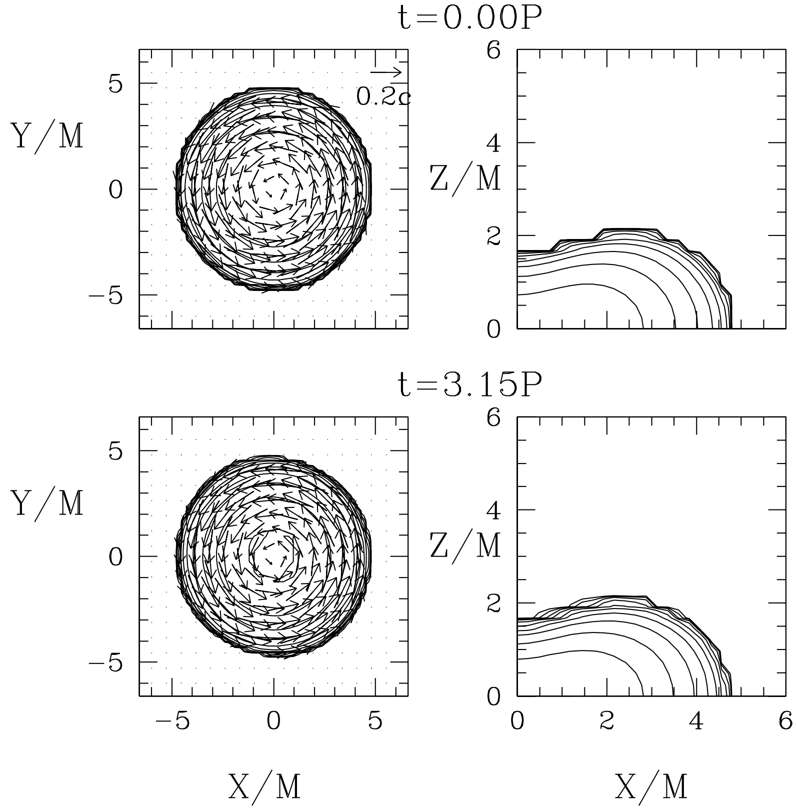
Morrison, Baumgarte & Shapiro (2004)



- Example: $\Gamma = 2$ polytrope; $\quad \Omega/\Omega_c \approx \frac{1}{1+(\varpi/\hat{A})^2}$

- Conclude: stable ‘hypermassive’ stars significantly exceed the spherical TOV mass limit .

Test For Dynamical Stability



- Conclusions:
  - At least some hypermassive stars are dynamically stable (e.g., $\beta \gtrsim 0.24 \rightarrow$ bar).
  - They are secularly unstable to J-redistribution via turbulent viscosity, magnetic braking, neutrinos &/or GWs ;

    $\rightarrow$ delayed collapse to BHs;

    $\rightarrow$ delayed GW burst.

Binary Neutron Star Merger

- Initial Data:
  - Symmetry: helical Killing vector: QE circular binaries;
  - Field: conformal thin-sand., $\tilde{\gamma}_{ij} = f_{ij}$;
  - Matter: integrated Euler eqn;
  - Spins: corotational (Baumgarte et al. 1997) & irrotational (Bonazzola et. al. 1999; Uryu & Eriguchi 2000; Taniguchi & Gourghoulon 2002).

- Evolution in 3+1 GRT:
  - State-of-the-art: $\Gamma = 2$ polytropes & realistic nuclear EOS (Shibata et al. 2003; 2005)
  - Rest-mass ratios: $0.9 < Q_M < 1$;
  - Key Finding: $M_{\text{crit}} \sim 2.5 - 2.7 M_{\odot}$ (ADM)

    $\rightarrow M_{\text{tot}} > M_{\text{crit}}$: prompt collapse to BH;

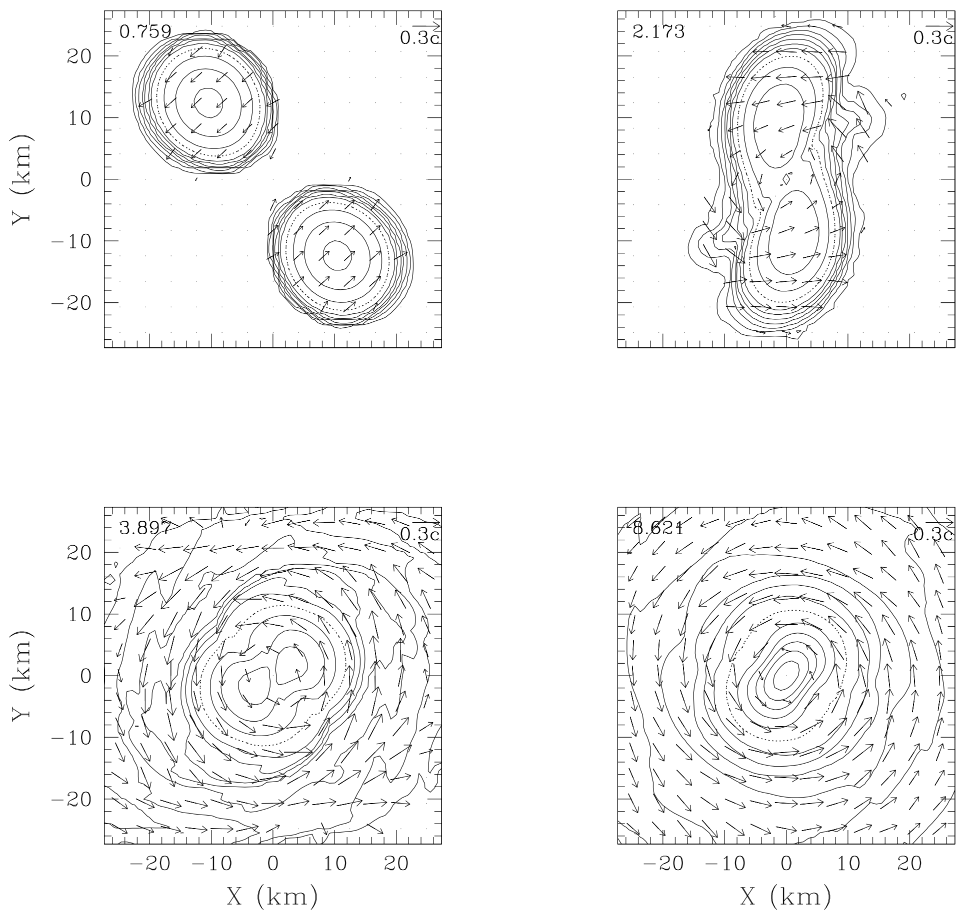
    $\rightarrow M_{\text{tot}} < M_{\text{crit}}$: hypermassive remnant + delayed collapse (~ 100 ms) & GW burst.
  - GWs: hypermassive star emits quasi-per. emission ($f \sim 3 - 4$ kHz) detectable by Advanced LIGO; may constrain EOSs.

NS-NS Merger: Formation of a Hypermassive NS

Rasio & Shapiro (1994), Zhuge et al. (1996): Newtonian

Faber & Rasio (2000): PN; Faber et al. 2004: CFGRT

Shibata et al. 2003; 2005*: GRT



*Model:

- $M_{\rm ADM} = 2.7 M_{\odot}, P = 2.11$ ms, realistic EOS;
- $\Gamma > 2.24 \rightarrow$ triaxial ellipsoid $\rightarrow$ GWs.

Black Hole Excision

- Basic Idea (Unruh , as cited in Thornburg 1987)

  • a nasty spacetime singularity resides inside the BH event horizon, a region casually disconnected from the BH exterior;

  • by causality, one can do anything inside the horizon that will produce a stable evolution outside;

$\longrightarrow$ one can 'excise' a region inside the horizon containing the singularity & replace it with suitable b.c.'s at its outer surface

- Early Feasibility Studies:

  • spherical scalar field collapse;

  (Seidel & Suen 1992; Marsa & Choptuik 1996; Gomez et al. 1997);

  • spherical collisionless gas collapse;

  (Scheel, Shapiro & Teukolsky 1995, in Brans-Dicke theory)

  • single stationary BHs: standard 3+1 ADM .

  (Anninos et al. 1995; BBH Grand Challenge 1998)

Black Hole Excision (cont.)

- Recent Work: Vacuum BH Spacetimes
  - single BHs: 3+1 BSSN;

(Alcubierre & Brügmann 2001; Yo, Baumgarte & Shapiro 2002)

  - single BHs: 3+1 hyperbolic formalism;

(Scheel et al. 2002; Calabrese et al. 2003; Tiglio et al. 2004)

  - single BHs: 3+1 characteristic formalism;

(Gomez et al. 1998)

  - binary BHs: 3+1 BSSN.

(Brandt et al. 2000, grazing collision; Brügmann et al. 2004, one orbit)

- Recent Work: GR Hydrodynamic Spacetimes
  - perfect gases: 3+1 BSSN;

(Duez, Shapiro & Yo 2004; Baiotti et al. 2005)

  - imperfect gases (shear viscosity): 3+1 BSSN;

(Duez et al. 2004)

  - MHD plasmas: 3+1 BSSN.

(Duez et al. 2005; Shibata & Sekiguchi 2005)

Black Hole Excision (cont.)

- Recent Work: Scalar Wave Spacetimes
  - binary BHs: generalized harmonic coords

(Pretorius 2005)

BH Excision in Action: Collapse of Rapidly Rotating Stars

- Key Question: Fate vs. $q \equiv J/M^2$
  - original work: stellar collapse (axisym.)

  (Nakamura 1981; Stark & Piran 1985: rigid rot. & stiff EOSs)
  - collisionless tori collapse (axisym.)

  (Abrahams et al. 1995)

- Find: "cosmic censorship protection"
  - $q < 1$: collapse to Kerr; $q > 1$: bounce

- Qualitative Explanation
  - bounce: centrifugal force $\sim$ grav force, i.e.

$$\frac{M}{R_b^2} \sim \frac{J^2}{M^2 R_b^3},$$

hence $$\frac{R_b}{M} \sim q^2 .$$

- Complexities: soft EOSs, diff. rot. & disks
  - recent work: stellar collapse (axisym. & 3+1)

  (e.g., Shibata 2002; 2004; Duez et al. 2004; Baiotti et al. 2005)

GR Hydro With Viscosity: Collapse & Disk Formation in Hypermassive Stars

Duez, Liu, Shapiro & Stephens (2004)

- Stress-Energy Tensor for GR Navier-Stokes Eqns:

$$T^{ab} = T^{ab}_{\rm ideal} + T^{ab}_{\rm visc} \ , \quad \text{where}$$

$$T^{ab}_{\rm visc} = -2 \underbrace{\eta}_{shear} \sigma^{ab} - \underbrace{\zeta}_{bulk} \theta P^{ab} \ .$$

set $\eta \propto P, \quad \zeta = 0$.

- Timescale Hierarchy: $t_{\rm visc} \gg t_{\rm dyn}$

scaling : $[\eta_1, t_1] \to [\eta_2, t_2 = t_1(\eta_1/\eta_2)]$.

- Viscous Heating: $\rho_0 T(ds/d\tau) = 2\eta\sigma^{ab}\sigma_{ab}$

limiting cases:

$t_{\rm cool} \gg t_{\rm visc}$ (no cooling) ,
$t_{\rm cool} \ll t_{\rm visc}$ (rapid cooling) .

GR MHD: Motivation

B-fields play a crucial role in determining the evolution of many relativistic objects:

- B-fields are present in most astrophysical plasmas;
- In any highly conducting plasma, the frozen-in B-field can be amplified appreciably by gas compression or shear.

$\Longrightarrow$ Even if the initial seed B-field is weak it can grow to:

(1) influence significantly the gas dynamical flow via MHD stresses acting on the matter;

(2) affect the spacetime geometry directly via $E\&M$ energy-momentum source terms in the Einstein field equations.

Astrophysical Scenarios Involving Relativistic, Dynamical Spacetimes Where MHD May Be Decisive

- Merger of Binary Neutron Stars
  magnetic braking + viscous damping of differential rotation in hypermassive NS remnants on Alfvén timescales.

  $\Longrightarrow$ delayed collapse & GW burst?
  mass loss?

- Rotating Core Collapse in Supernovae
  collapse-induced differential rotation will wind-up a frozen-in B-field.

  $\Longrightarrow$ delayed collapse of hypermassive NS?
  magnetic jet? enhanced bounce shock?
  'magnetar'?

Astrophysical Scenarios (cont)

- Gamma-ray Burst Sources (GRBs)
  GRB models: 'collapsars' (long-soft);
  NS-NS or NS-BH mergers (short-hard).

$\Longrightarrow$ extraction of disk or BH rotational energy via strong B-fields? jets? bursts? polarization?

- Supermassive Star Collapse
  possible origin of supermassive BHs in early universe.

$\Longrightarrow$ does magnetic braking or viscosity enforce uniform rotation, driving the star to relativistic radial instability?

- R-Mode Instability
  possible NS spin-down mechanism.

$\Longrightarrow$ suppressed by B-fields?

GR Maxwell + MHD Fluid Eqns

Duez, Liu, Shapiro & Stephens (2005a,b);
Shibata & Sekiguchi (2005); Antón et al. (2005)

- Evolution Equations

$$\begin{aligned}
\nabla_a(T^{ab}_{\rm fluid} + T^{ab}_{\rm em}) &= 0, \quad \text{(energy – mom conserv.)} \\
\nabla_a(\rho_0 u^a) &= 0, \quad \text{(rest – mass conserv.)} \\
\partial_t(\gamma^{1/2} B^i) &= \partial_k[\gamma^{1/2}(v^i B^k - v^k B^i)] \ . \quad \text{(induction)}
\end{aligned}$$

- Implementation: *conservative* HRSC scheme

$$\partial_t \mathbf{U} + \nabla \cdot \mathbf{F} = \mathbf{S} \ ,$$

where

$$\begin{aligned}
\mathbf{P} &= (\rho_0, P, v^i, B^i), \quad \text{(“primitive” variables)} \\
\mathbf{U}(\mathbf{P}) &= (\rho_\star, \tilde{\tau}, \tilde{S}_i, \tilde{B}^i), \quad \text{(“conserved” variables)} \\
\mathbf{F}(\mathbf{P}) &= \ldots, \quad \text{(flux variables)} \\
\mathbf{S}(\mathbf{P}) &= \ldots, \quad \text{(source variables)} \ .
\end{aligned}$$

- *no* derivs of primitive variables in **U** & **F** ;
- *no* artificial viscosity is necessary;
- magnetic induction via *constrained* transport: $\partial_i(\gamma^{1/2} B^i) = 0$ guaranteed .

GR MHD in 3+1: Metric Eqns

- Constraint Equations

$$
\begin{aligned}
R + K^2 - K_{ij}K^{ij} &= 16\pi(\rho_{fluid} + \rho_{em}), \quad \text{(Ham)} \\
D_j(K^{ij} - \gamma^{ij}K) &= 8\pi(S^i_{fluid} + S^i_{em}). \quad \text{(mom)}
\end{aligned}
$$

- Evolution Equations

$$
\begin{aligned}
\partial_t \gamma_{ij} &= \cdots \; , \\
\partial_t K_{ij} &= \cdots - 8\pi\alpha[S^{em}_{ij} - \frac{1}{2}\gamma_{ij}(S_{em} - \rho_{em})] \; .
\end{aligned}
$$

- E & M Source Terms

$$
\begin{aligned}
4\pi\rho_{em} &= \frac{1}{2}(E_kE^k + B_kB^k), \quad \text{(energy density)} \\
4\pi S^{em}_i &= \epsilon_{ijk}E^jB^k, \quad \text{(energy flux)} \\
4\pi S^{em}_{ij} &= -E_iE_j - B_iB_j + \frac{1}{2}\gamma_{ij}(E_kE^k + B_kB^k), \quad \text{(stress)} \\
4\pi S_{em} &= \frac{1}{2}(E_kE^k + B_kB^k). \quad \text{(trace)}
\end{aligned}
$$

Stabilizing the Field Solver

- Adopt BSSN Scheme

$$\tilde{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}, \quad \text{where} \quad e^{4\phi} = \gamma^{1/3} ,$$
$$\tilde{A}_{ij} = \tilde{K}_{ij} - \frac{1}{3}\tilde{\gamma}_{ij}K ,$$
$$\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\tilde{\Gamma}^i{}_{jk} = -\partial_j\tilde{\gamma}^{ij} .$$

- Evolve: $\tilde{\gamma}_{ij}$, $\tilde{A}_{ij}$, ϕ, K, & $\tilde{\Gamma}^i$

- Further Stabilize & Improve Accuracy

(Yoneda & Shinkai 2001, 2002; Yo, Baumgarte & Shapiro 2002; ...)

$$\partial_t\phi = \cdots + c_{H1}\Delta T\alpha\mathcal{H}$$
$$\partial_t\tilde{\gamma}_{ij} = \cdots + c_{H2}\Delta T\alpha\tilde{\gamma}_{ij}\mathcal{H}$$
$$\partial_t\tilde{A}_{ij} = \cdots - c_{H3}\Delta T\alpha\tilde{A}_{ij}\mathcal{H} ,$$

$$0 = \mathcal{H} \equiv \tilde{\gamma}^{ij}\tilde{D}_i\tilde{D}_j e^{\phi} - \frac{e^{\phi}}{8}\tilde{R} + \frac{e^{5\phi}}{8}\tilde{A}_{ij}\tilde{A}^{ij} - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}\rho .$$

•
•

Adopted Gauge Conditions

Alcubierre et al. 2001; Duez, Shapiro & Yo 2004; ...

- Hyperbolic shift driver:

$$\partial_t^2 \beta^i = b_1(\alpha \partial_t \tilde{\Gamma}^i - b_2 \partial_t \beta^i) \ ,$$

$\Longrightarrow$ drives $\partial_t \tilde{\Gamma}^i \to 0$, ($\approx$ minimial distortion) .

- Hyperbolic lapse driver:

$$\begin{aligned} \partial_t \alpha &= \alpha \mathcal{A} \\ \partial_t \mathcal{A} &= -a_1(\alpha \partial_t K \\ &\quad + a_2[\partial_t \alpha + e^{-4\phi} \alpha (K - K_{\mathrm{drive}})]) \ , \end{aligned}$$

$$K_{\mathrm{drive}} = 0 \ , \ K_{KS}(\alpha, \beta^i) \ , \quad \text{or} \quad K(t_{\mathrm{excis}}) \ .$$

$\Longrightarrow$ ensures $\alpha > 0$ and horizon penetration;

$\Longrightarrow$ drives $\partial_t K \to 0$ & $K \to K_{\mathrm{drive}}$;

$\Longrightarrow$ systems which settle into equilibrium appear stationary in the adopted coordinates.

Collapse of A Magnetized Hypermassive NS

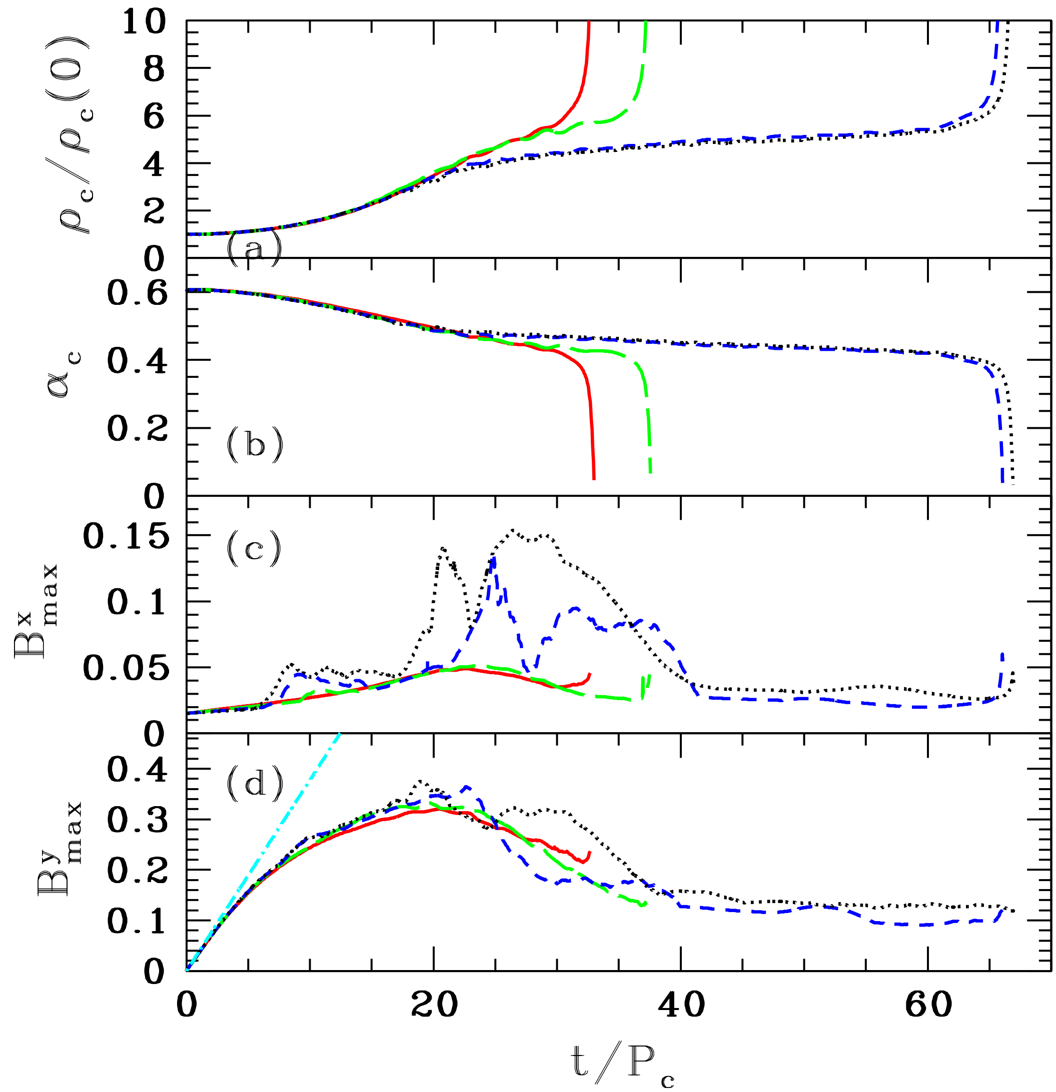
Duez, Liu, Shapiro, Shibata & Stephens (2005): axisymmetry

- Initial Seed B Field
  - Topology: purely poloidal
  - Strength: $C \equiv \max\left[\frac{B^2_{\text{fluid}}}{4\pi P}\right] = 2.5 \times 10^{-3}$
- B-field Amplification:
  - Winding: $\tau_A = R/v_A$
  - MRI: $\tau_{\text{MRI}} \sim P_c \ll \tau_A$ (Balbus & Hawley 1991)
- Computational Challenge
  - Wavelength: $\lambda_{\text{MRI}} = 2\pi v_A/\Omega \sim R/10$
  - Resolution Requirement: $\Delta \lesssim \lambda_{\text{MRI}}/10$

$\Longrightarrow$ To follow collapse, the evolution time must exceed $t_A \sim 75 P_c \sim 3000M$.

$\Longrightarrow$ To resolve the fastest growing MRI mode, we require N^2 zones with $N \gtrsim 400$.

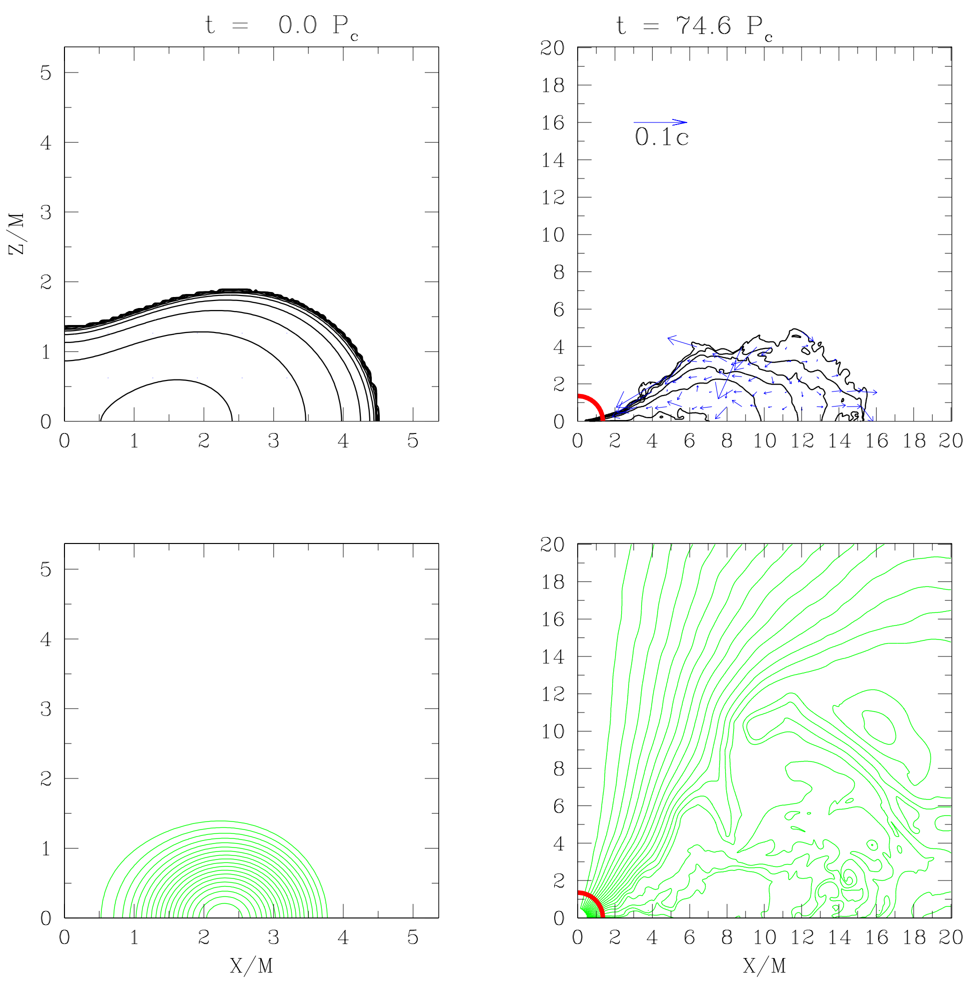
Evolution at Different Resolutions



- Resolutions: $N =$ 250, 300, 400, 500.

- B-Amplification: $B^x =$ radial poloidal (MRI)
  $B^y =$ toroidal (winding) .

Initial & Final Profiles: Rest-Mass Density and Poloidal B-Field



- End State:

A rotating BH ($M_h/M \sim 0.9$, $J_h/M_h^2 \sim 0.8$) surrounded by a hot, torus ($M_{\rm tor}/M \sim 0.1$), with a collimated B field along the rotation axis.

Central Engine For Short-Hard GRBs?

Shibata, Duez, Liu, Shapiro, & Stephens (2005)*

- GRBS: 2 Classes (BATSE, Swift, HETE, Chandra, HST)
  - Long-Soft GRBs:
    - $\tau \sim 2 - 1000$ sec;
    - in star-forming regions (spirals);
    - associated with SNs;
    - massive star collapse: 'collapsars' ?
  - Short-Hard GRBs:
    - $\tau \sim 10$ ms -2 sec;
    - in low star-form. regions (ellipticals);
    - SN associations excluded;
    - BNSs $\Rightarrow$ HMNSs*? BBHNS?

- Exciting Implications for LIGO!
  - Coincidence Detection:
    - GW burst + GRB;
    - reasonable event rate.
- Simulations in Full GR: Required & Underway!

Binary BH-NSs

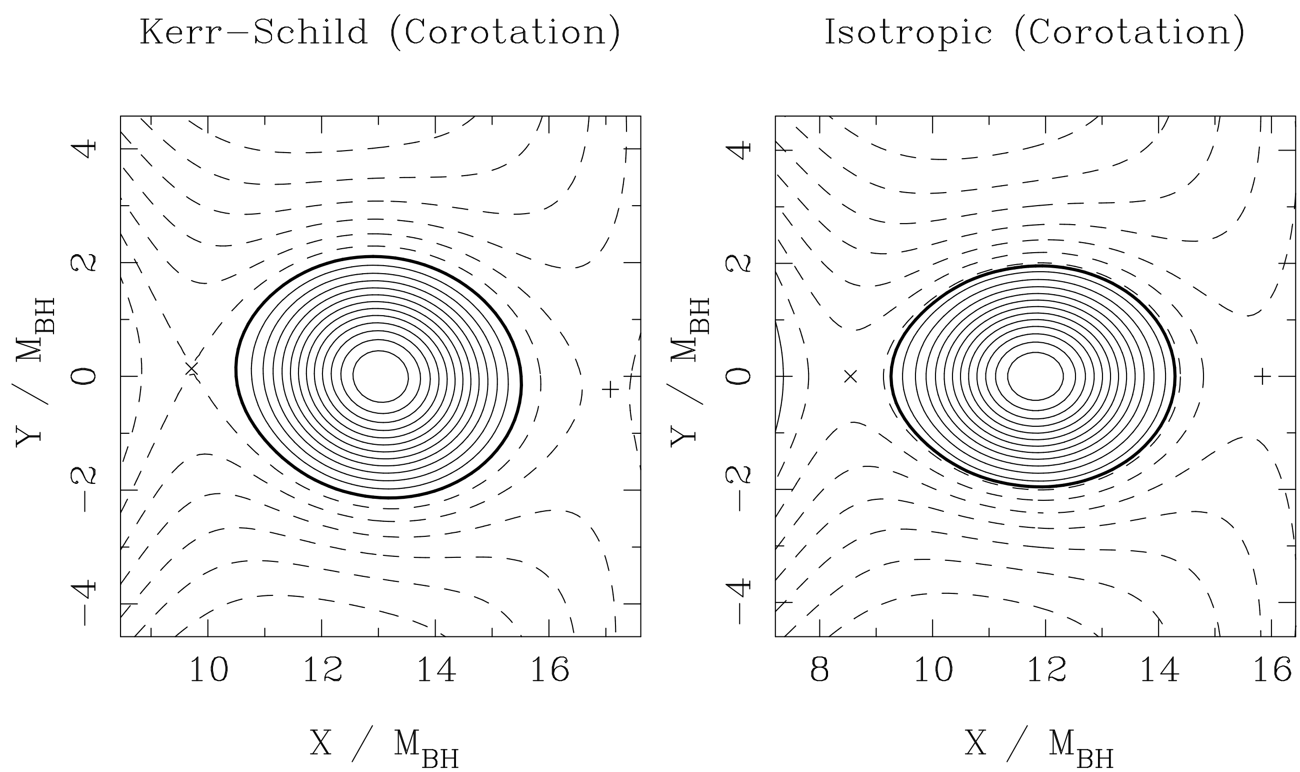
Baumgarte, Skoge & Shapiro (2004);

Taniguchi et al. (2005); Faber et al. (2005)

- Initial Data:
  - Symmetry: helical Killing vector (quasiequilibrium circular binaries);
  - Field: conformal thin-sandwich;
  - Matter: integrated Euler eqn for polytropic EOS;
  - Conformally-Related Background $\tilde{\gamma}_{ij}$: Kerr-Schild *or* isotropic ($\tilde{\gamma}_{ij} = f_{ij}$) BH;
  - NS spin: corotational *or* irrotational;
  - Current Implementation: $M_{\text{NS}}/M_{\text{BH}} \ll 1$.

- $3 + 1$ Evolution:
  - Case: corotating binary, isotropic BH
  - Method: conformally flat GRT & SPH

BH-NS Initial Density Profile Near Roche Limit



- Input:

NS is a $\Gamma = 2$ polytrope near Roche limit;
$M_{\mathrm{NS}}/R_{\mathrm{NS}} = 0.042$, $M_{\mathrm{NS}}/M_{\mathrm{BH}} = 0.1$;

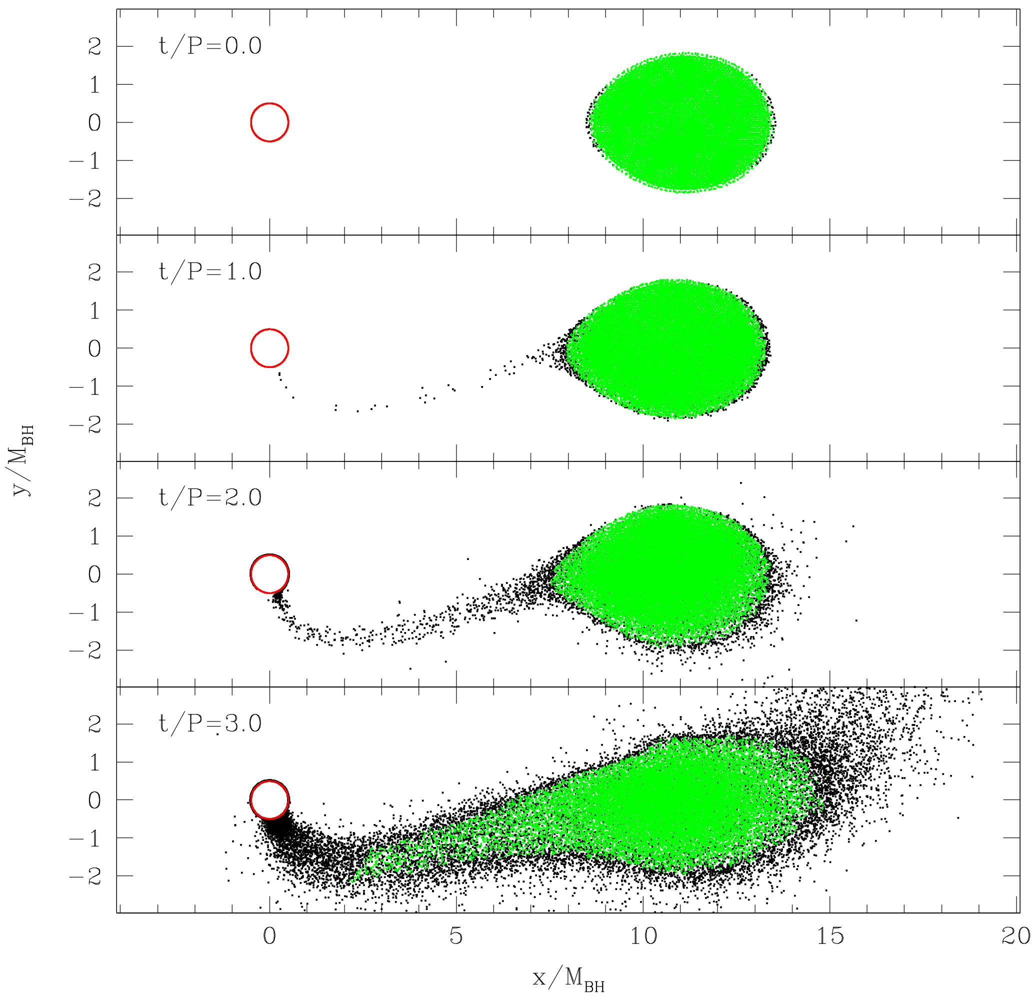
- Contours:

Lines of constant $\ln h \to C - \Phi_{\mathrm{eff}}$ (Newtonian), where $\Phi_{\mathrm{eff}} = \Phi_{\mathrm{NS}} - M_{\mathrm{BH}}/r_{\mathrm{BH}} - \frac{1}{2}(\mathbf{\Omega} \times \mathbf{x})^{\mathbf{2}}$.

- Comparison: (invariants)

$\Omega_{\mathrm{KS}} \approx \Omega_{\mathrm{iso}}$ ($\approx \Omega_{\mathrm{Kep}}$); $\rho^{\mathrm{max}}_{\mathrm{KS}} \approx \rho^{\mathrm{max}}_{\mathrm{iso}}$ ($< \rho^{\mathrm{max}}_{\infty}$).

Tidal Disruption & Mass Transfer



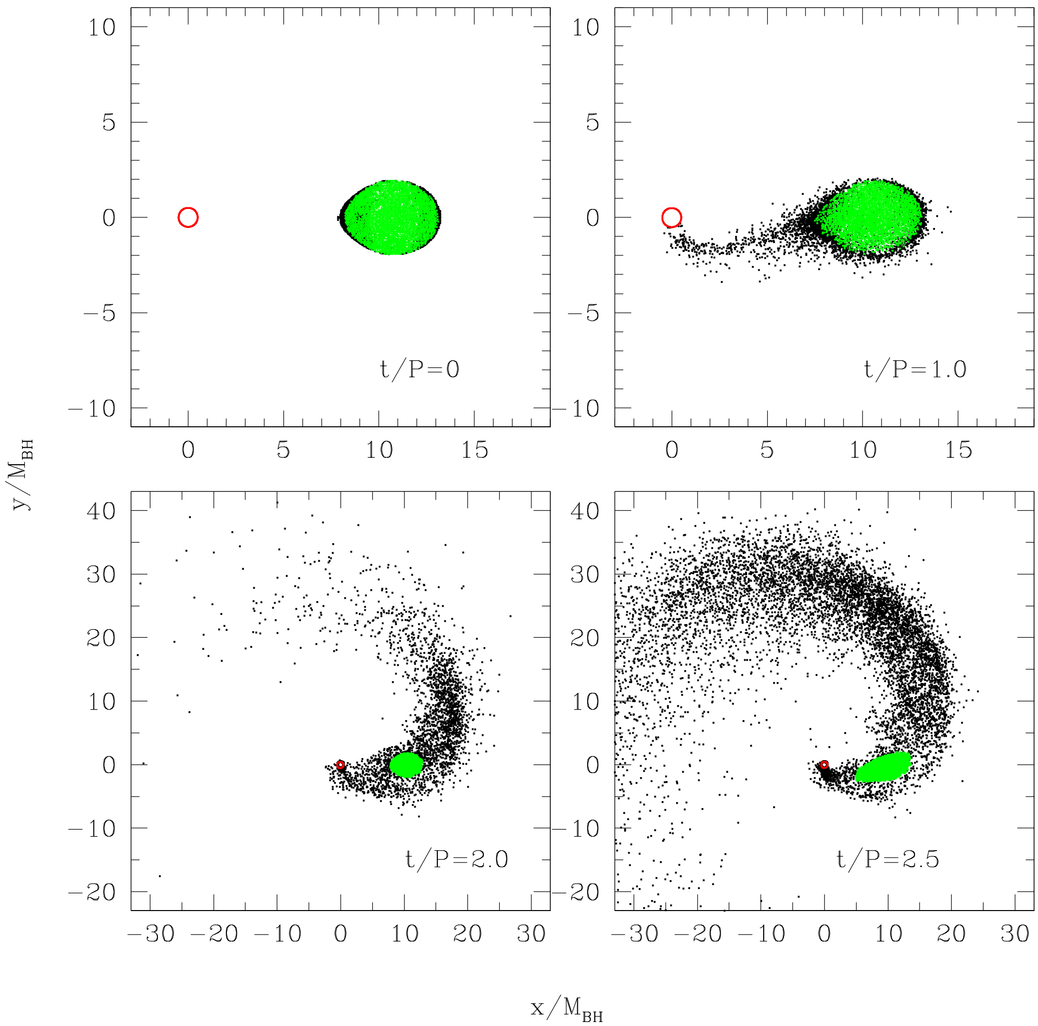
- Input:

NS is a $\Gamma = 2$ polytrope at Roche limit $>$ ISCO;
$M_{\rm NS}/R_{\rm NS} = 0.042$, $M_{\rm NS}/M_{\rm BH} = 0.1$;

- Dynamical Behavior:

NS tidal disruption: $dM_{\rm NS}/dt \gg M_{\rm NS}/t_{\rm GW}$.

Tidal Disruption & Mass Transfer



- Input:

NS is a $\Gamma = \frac{3}{2}$ polytrope at Roche limit > ISCO;
$M_{\rm NS}/R_{\rm NS} = 0.042$, $M_{\rm NS}/M_{\rm BH} = 0.1$;

- Dynamical behavior:

NS tidal disruption: $dM_{\rm NS}/dt \gg M_{\rm NS}/t_{\rm GW}$.