EVOLUTIONS OF BINARY BLACK HOLE SPACETIMES IN THE LAST ORBIT

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Outline

1. Binary black hole evolutions of helical Killing vector data.
   - Physical model.
   - Mesh refinement, Excision, Gauge conditions.
   - Results from evolutions of thin sandwich data.

2. Evolutions of single BHs using a multipatch code.
   - Recent implementation of interpolating patches.
   - BSSN evolution system with modified gauges.
   - Test cases: Kerr-schild, hydro, distorted BHs.

Initial data

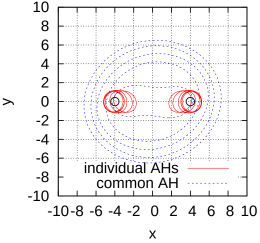
- We require a solution to the constraint equations representing a pair of black holes at an instant in time.
- Standard solution procedures:
  - Puncture data with Bowen-York angular momentum.
  - Conformal thin sandwich.
- Solution should be astrophysically motivated: In “quasi-circular” orbit.
- Two commonly used methods for choosing orbital parameters:
  - Chose by searching for a minima in an effective potential.
  - Impose existence of a helical Killing vector (HKV) on the initial data solution.
- We have concentrated on two types of initial data:
  - Punctures with parameters along an effective potential sequence developed by [Cook 1994] .
  - Thin sandwich data using the HKV condition, constructed by [Grandclement-Gourgoulhon-Bonazolla 2002] – “Meudon” data.

The AEI evolution code

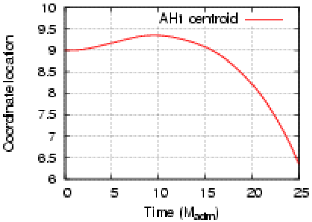
- Uses BSSN formulation of Einstein's equations – 1st-order in time, 2nd-order in space. [Nakamura-Kojima-Oohara 1987, Shibata-Nakamura 1995, Alcubierre et al. 2002]
- Free evolution – constraints are not actively enforced during the evolution.
- Dynamic gauge conditions: Bona-Massó slicing, Γ-driver shift, co-rotating frame.
- Implemented on a cubical grid with timelike outer boundary faces.
- Artificial radiative outer boundary condition – leads to loss of accuracy and potential stability problems.
- Straightforward finite differencing in space, typically 2nd or 4th order.
- Time integration via method of lines integrator (eg. iterated Crank-Nicholson, Runge-Kutta).
- Mesh refinement, concentrate resolution in strong field regions.

BLACK HOLE EVOLUTIONS OF "MEUDON" DATA

- Gourgoulhon et al. (2002) generated binary data by solving the thin-sandwich equations under the additional assumption of a helical killing vector (HKV) within the slice.
- Data imported onto finite difference grid from Meudon spectral code.
- Evolved using standard BSSN evolution code and gauges.
  [Koppitz PhD 2004]
- Known inconsistencies in the inner boundaries (due to the construction procedure) are only apparent at extremely high resolutions.



Horizon evolution.



Separation between apparent horizons within a slice.

Black hole evolutions of "Meudon" data

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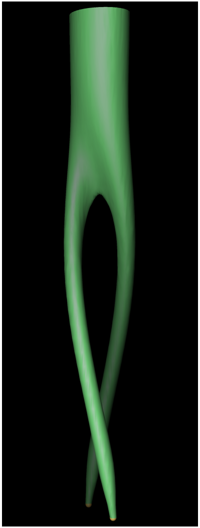
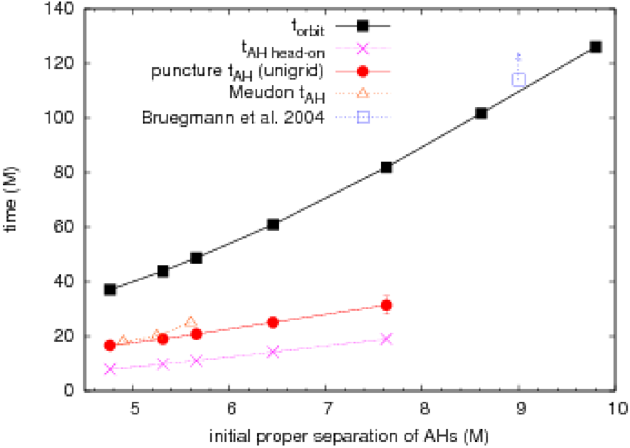


Event horizon evolution during BBH inspiral – using finder by Diener (2003).

Black hole evolutions 2. "Meudon" data



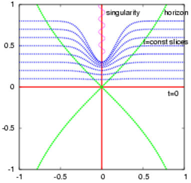
Coordinate conditions

- We have found that dynamical gauge conditions are crucial to long term black hole evolutions.

Lapse condition:

- Bona-Massó family of slicings [Bona et al. 1994] :

$$\partial_t \alpha = -\alpha^2 f(\alpha)(K - K_0)$$



- Typically choose "1+log" variant: $f(\alpha) = 2/\alpha$
- Singularity avoiding, not prone to gauge shocks.
- Prone to "slice stretching". [Reimann et al. 2003, 2004]

Shift condition:

- Hypberbolic "$\tilde{\Gamma}$-driver" shift [Alcubierre et al. 2002] :

$$\partial_t \beta^i = F B^i,$$

$$\partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta B^i$$

- Parameters $F(x)$, $\eta(x)$ used to tune the shift evolution.

Coordinate conditions

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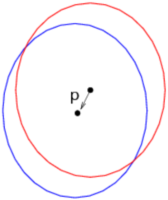


Co-rotating coordinates

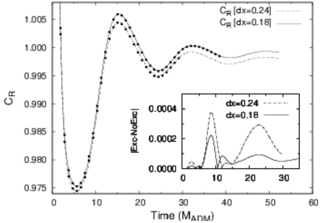
- Initial data for the shift vector incorporates a rotational component to slow the motion of BH horizons.
- The horizon location is monitored during evolution.
- The shift evolution is periodically adjusted to keep the horizons in place.
- The correction is applied as the solution of a damped harmonic oscillator:

$$T^2\ddot{p} + 2T\dot{p} + p = 0 \qquad \text{with} \qquad p = r - r_0$$

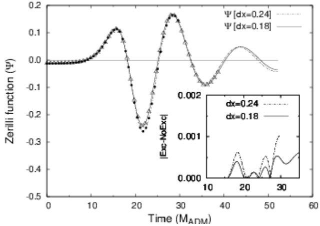
$$\dot{\beta}^a \rightarrow \dot{\beta}^a + F(x)\ddot{p}^a.$$

EXCISION

- It has become conventional to treat the singularity at the centre of a BH by cutting it from the evolution domain.
- This is an inflow boundary → in principle not technically difficult to apply a BC if you know the characteristic structure of your evolution system.
- Our usual technique is a simple 1st-order extrapolation of the update terms of the evolution variables – “simple excision” [Alcubierre-Brügmann 2001] .
- On a cartesian grid, it is not possible to excise on a smooth surface. However, stable finite differencing is difficult in the presence of corners and edges.



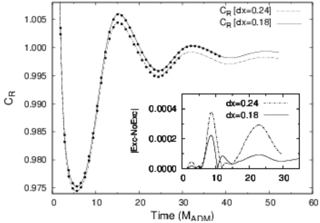
Horizon oscillations.



Asymptotically extracted waveforms.

EXCISION

- We have evolved distorted puncture BHs, and head-on collisions, both with and without excision and extracted the wave-forms at a large radius.
  [Alcubierre et al. (2004)]
- Even in the near zone, eg. at the horizon, physical differences in the spacetimes are small, and decrease with resolution.
- However, growth of error on the irregular inner boundary motivates smooth inner boundaries via multipatch.



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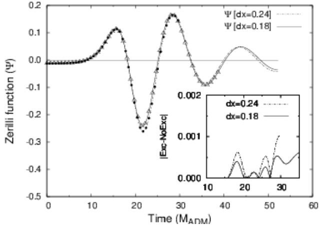
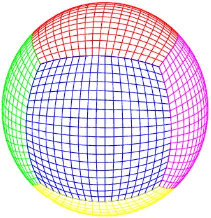


Asymptotically extracted waveforms.

MULTIPATCH METHODS FOR BLACK HOLE SPACETIMES

- We would like a smooth inner boundary for excision
  - cubic excision has causality problems at corners
  - it is difficult to develop stable finite difference schemes for "Lego" excision, in particular with a shift
  - the z-axis for spherical coordinates is difficult to treat in 3D
- We would like a smooth outer boundary
  - well-posed outer BC are easier to implement without corners/edges
  - asymptotic compactification
  - matching to characteristic outer boundary code
- A number of groups have developed multipatch infrastructures for their codes – Meudon, Cornell-Caltech, LSU, AEI.
- Codes differ in choice of spatial discretisation (spectral, finite differencing), and communication between patches.

6-PATCH "INFLATED CUBE" COORDINATE SYSTEM



- Thornburg has implemented a multipatch infrastructure based on interpolation between adjacent grids – currently aimed at single hole topologies.
- Construction of angular coordinates: Draw *xyz* grid lines on the faces of a cube, then inflate the cube to a sphere $\rightarrow$ 6 angular patches around a sphere at a given constant r.
- Patches have ghost-zones which overlap – interpolation from the neighbouring patch is used to fill ghost zone values.
- Angular coordinates are chosen so that adjacent patches share angular coordinate perpendicular to their mutual boundary $\rightarrow$ only need 1D interpolations

GENERAL RELATIVITY

- Write Einstein equations in a 3-covariant form
- Each patch uses a local coordinate basis
- Coordinate transform field variables when interpolating between neighbouring patches
- Non-tensorial quantities (eg. BSSN Γ^i require special care).
- Currently implementated within Cactus, using Carpet driver for multipatch support, Whisky for hydrodynamics.

Gauge conditions for multiple patches

- Commonly used Cartesian shift conditions for BSSN are based on $\tilde{\Gamma}^i$ (Γ-freezing, Γ-drivers).
- These are not covariant due to the nature of the Γ^i variables.
- Bona and Palenzuela have proposed a driver condition based on the distortion tensor:

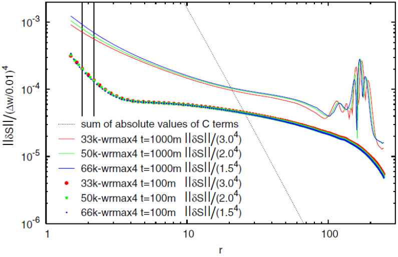
$$\partial_t \beta^a = hB^a,$$
$$\partial_t B^a = 2(D_i \Sigma^{ia}) - \eta B^a$$

  where Σ_{ab} is the distortion tensor.
- This is a covariant condition whose principle part is similar to $\tilde{\Gamma}$-driver.

TEST CASES 1: SINGLE BH

Kerr-Schild evolution

- Rotating ($a = 0.6$) BH in Kerr-Schild coordinates
- Thornburg 2004 demonstrated long term stability and convergence, evolved using BSSN, static shift
- Eventual problems due to outer boundary, not excision or interpatch BC.



Convergence of state vector after $t = 100M$, $t = 1000M$ (Thornburg 2004)

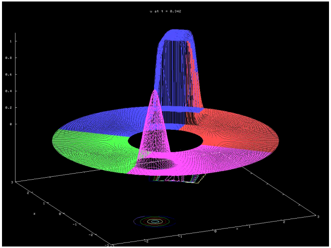
TEST CASES 2: HYDRO

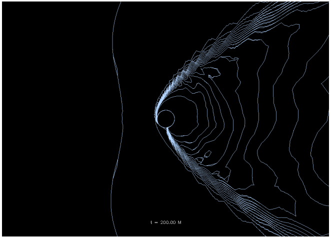
Shock propagation across boundaries
[Thornburg, Hawke].

- Test case involving discontinuous initial data.
- Use high-resolution shock capturing.
- Shocks are able to cross patch boundaries.

Relativistic test-fluid accretion

- Cactus+Carpet+Whisky.
- Test problem of Font, Ibanez, Papadopoulos gr-qc/9810344.
- 5th order HRSC spatial differencing.
- 4th order ENO interpatch interpolation.
- $\Delta\theta = 4.5 deg$, $\Delta r = 0.08 m$ at BH, $1 m$ at outer bdy.



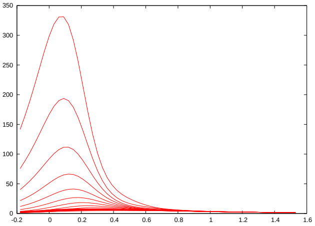


[Test cases by Ian Hawke]

DISTORTED BLACK IN FULL GR

- BH in isotropic coordinates, distorted by a Brill wave
- BSSN evolution
- 4th order in space differencing, RK4 time integrator, 5th order lagrange interpolation between patches

- Excision implemented via lagrange extrapolation
- $1 + \log$ slicing, minimal-distortion driver shift
- Interpatch effects remain well below FD accuracy
- Eventual problems due to classical grid stretching



$\tilde{\gamma}_{zz}$ on the z-axis.

SUMMARY

- For puncture data, use of higher resolution is leading to a systematic understanding of trajectories in the last orbit for such BHs (see talk by Diener).
- We have the same techniques to thin sandwich data, such as that generated by Grandclement et al.
- Details of the gauge condition can have an important effect on the accuracy of the evolution, and thus the physical interpretation.
- Multipatch techniques are making good progress evolving spacetimes
  - Nonlinear distorted black holes without interpatch instability.
  - Still need work on gauges.

End.

REFERENCES I