Simulations of orbiting black holes

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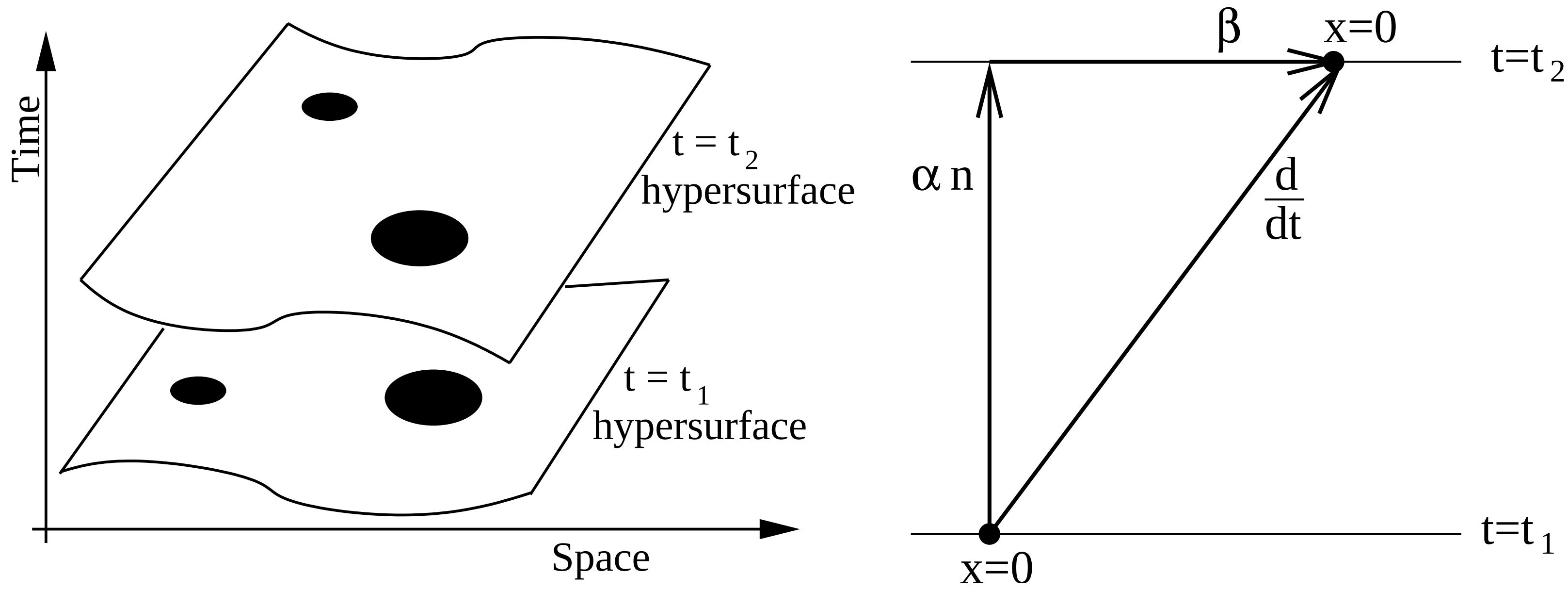
Plan of the talk:

- The 3+1 split of the spacetime
- Important ingredients necessary for numerical evolutions
- Binary black holes & quasi-circular orbits
- Comoving coordinates
- Results from the first binary black hole orbit simulation
- Summary

The 3+1 Split of spacetime



- Spacetime is foliated by $t = const$ slices
- Einstein's equations then split into evolution equations and constraint equations
- The evolution equations tell us how to evolve forward in time, from one slice to the next.
- The relation between the coordinates on the different slices is described by lapse α and shift β^i.

Ingredients for numerical evolution

- as before:
  - Puncture initial data for two orbiting black holes
  - Modified BSSN evolution system (i.e. replace all undifferentiated $\tilde{\Gamma}^i$ by derivatives of the metric, subtract trace of $\tilde{A}_{ij}$ from $\tilde{A}_{ij}$ after each ICN step)
  - Outer boundary of the shape of a "lego sphere", with Sommerfeld type outer boundary conditions for all evolved quantities:

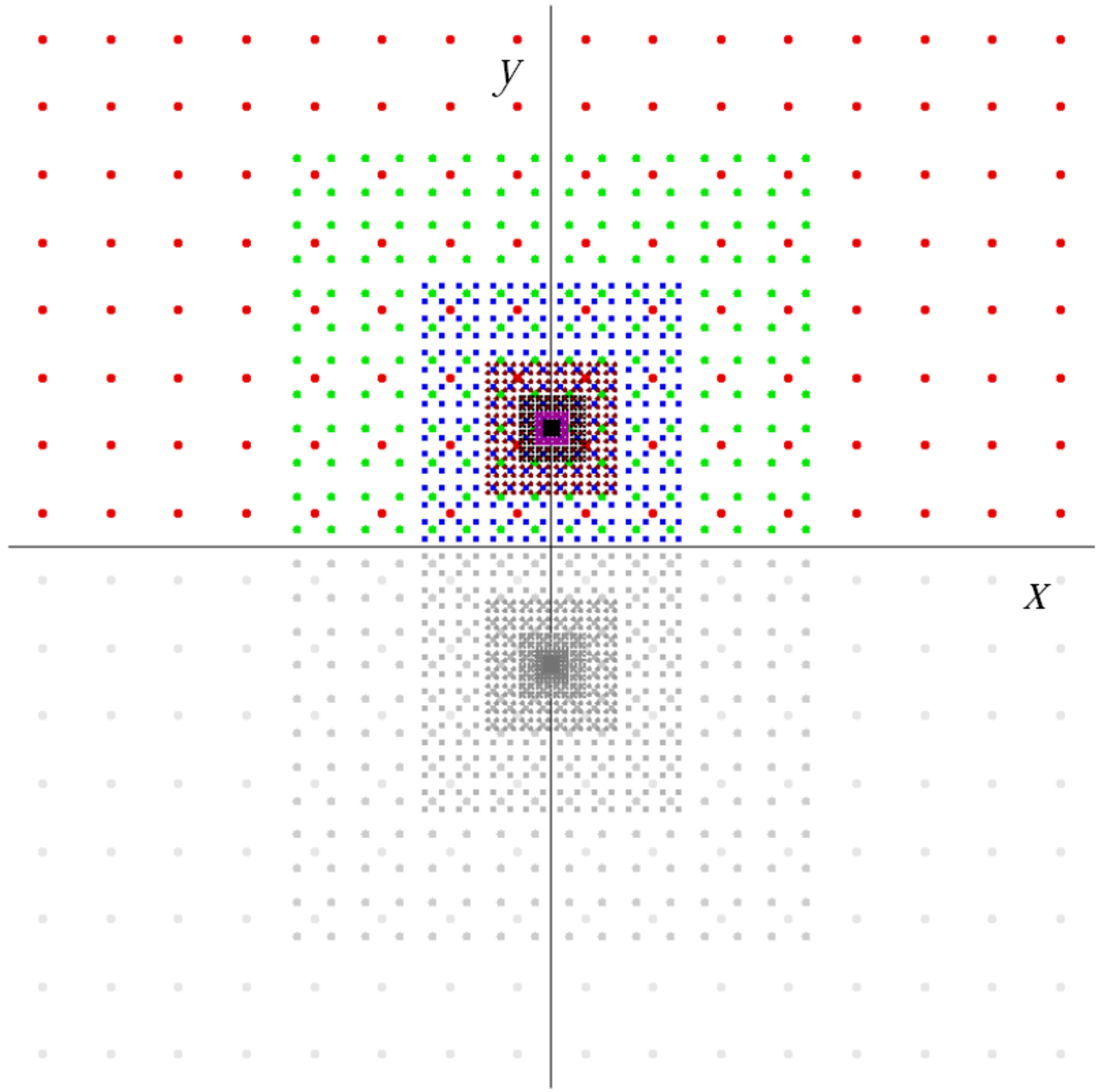
$$\partial_t F = \pounds_\beta F - v\frac{x^k}{r}\left(F - F_\infty\right)_{,k} - v\frac{F - F_\infty}{r},$$

  - Simple excision of the black holes inside the horizon (i.e. simply copy time derivative at next interior point onto excision boundary) extended to "lego spheres"
  - Singularity avoiding gauge (i.e. prevent slice from running into physical singularities)

- new:
  - our BAM code uses fixed mesh refinement (FMR) for efficiency
  - comoving coordinates, which compensate for black hole orbital motion

About FMR in BAM

- 7 nested boxes around each black hole
- For 48 points in x-direction:
  - resolution between $2M$ and $0.03125M$
  - outer boundary at $R = 48M$
- 3D quadrant symmetry for non-spinning equal mass black holes
- AMR not needed, because we use corotating coordinates
- ICN time stepping scheme similar as in Carpet (Schnetter, Hawley and Hawke 2003), but with lowered Courant factor on coarser grids, due to superluminal corotation



$\Rightarrow$ Runs can be done on a workstation!

Quasi-circular orbits

In principle, we want initial data, which represent a black hole binary that has slowly been inspiraling already for a long time, due to the emission of gravitational waves.

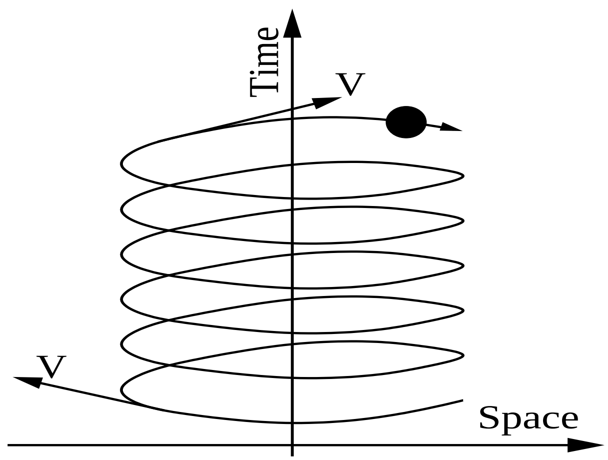
Post-Newtonian calculations predict that the black holes are moving on quasi-circular orbits with slowly shrinking radius, i.e. there are the two timescales:

$$T_{orbit} \ll T_{inspiral}$$

$\Rightarrow$ a comoving coordinate system exists in which

$$\partial_t g_{ij} \approx \partial_t K_{ij} \approx 0$$

- we should be able to find a lapse α and shift β^i which realize these comoving coordinates, so that the time evolution of the system is minimized.

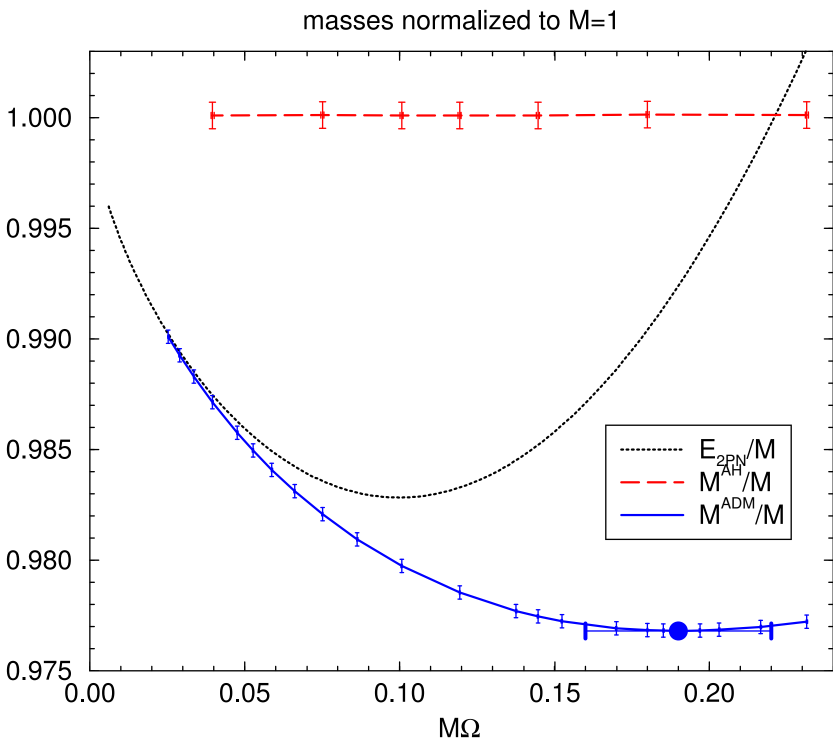


Questions:

- How fast do the black holes rotate?
- How fast do they drift toward each other?

Black hole puncture initial data, for quasi-circular orbits

- We use initial data from a binary black hole sequence (WT, B. Brügmann, P. Laguna, 2003), which tells us the angular velocity Ω for circular orbits at any given black hole separation.



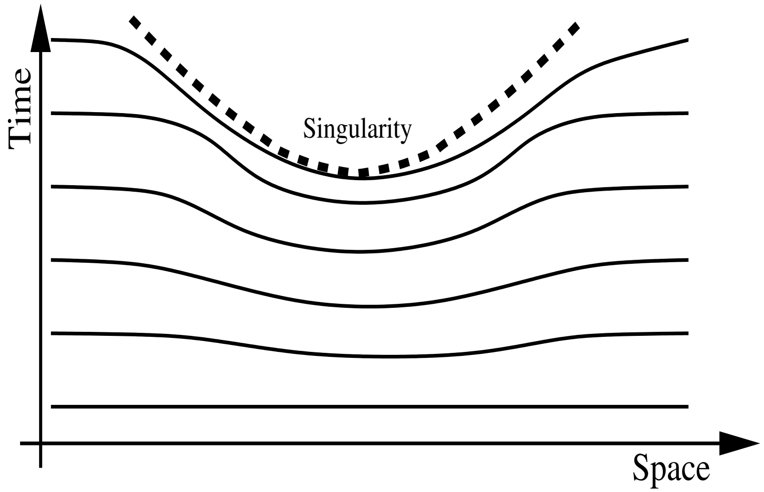
- ADM mass agrees with post-Newtonian results for low $\Omega \Leftrightarrow$ large separations
- but disagreement for large $\Omega \Leftrightarrow$ small separations

- We choose initial data in the regime where numerical and post-Newtonian predictions still agree.
- We focus on: $\Omega = 0.055/M \quad \Leftrightarrow \quad T_{orbit} = 114M$ and $R = 3M$
- The goal is to evolve for about one orbit, i.e. for at least $114M$.

Gauge or coordinate choice for numerical evolution

- Initial lapse and shift: $\alpha = 1$, $\beta^i = 0$

- Choose a singularity avoiding local gauge:



  - "1+log" lapse:

    $\partial_t \alpha = -2\alpha K \psi^4$

  - "Gamma-driver" shift:

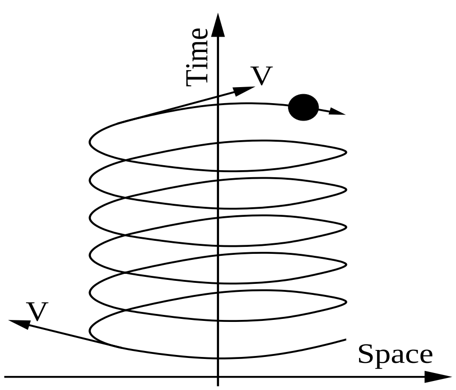
    $\partial_t \beta^i = \frac{3}{4}\alpha\psi^{-2}B^i$, $\partial_t B^i = \partial_t \tilde{\Gamma}^i - (2/M)B^i$

- This local gauge works well for a single black hole, but it knows nothing about the orbital motion and does not lead to comoving coordinates.

- With this alone the run crashes after $\sim 8M$.

- Since the black holes are in quasi-circular orbits, comoving coordinates should exist in which time evolution is minimized.

- We should be able to shift β^i which realizes these comoving coordinates.

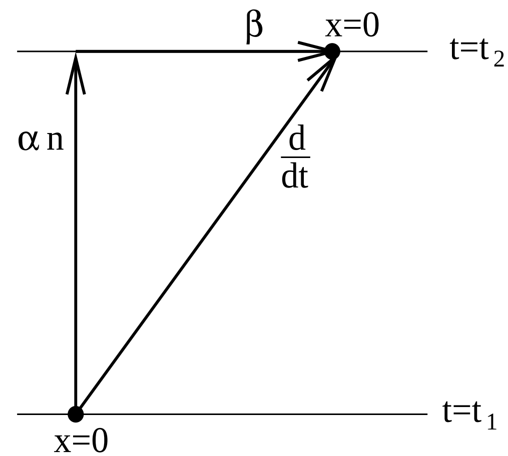


Global gauge choice - comoving coordinates

- Add a comotion shift which counters the global rotation and also the drift of the two holes toward each other, i.e. $\beta^i \rightarrow \beta^i + \beta^i_{com}$ with

$$\beta^i_{com} = \psi^{-3}\left[(\Omega \times x)^i - AV_r x^i\right]$$

- For point particles this would work perfectly.



How well does this work for black holes?

- We have several parameters in the attenuation functions ψ and A, which determine the form of the shift near the black holes and also far away (i.e. zero at puncture $\rightarrow$ rigid rotation far away).

- With our best choice of parameters and with Ω taken from our initial data sequence:
  - the apparent horizon stays near its initial location for a while, but then starts drifting away
  - the simulation lasts up to $\sim 60M$ and dies when the apparent horizon drifts too far

Using the lapse to find approximate black hole horizons

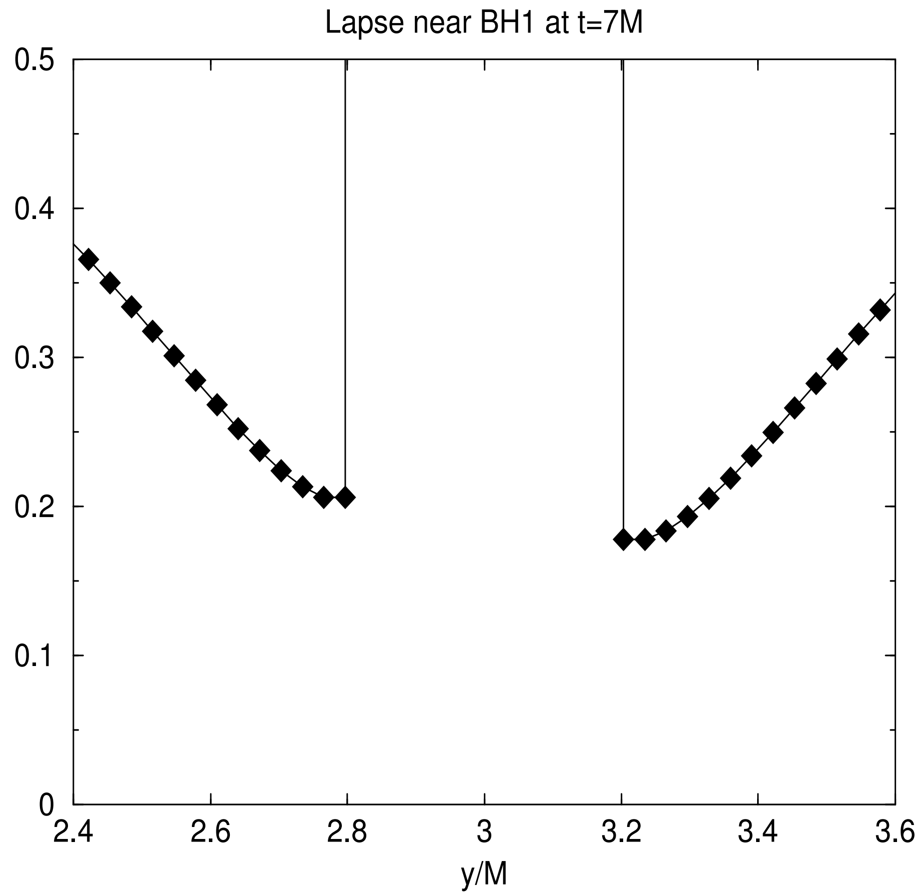
- When we use a "1+log" lapse, α is a good indicator of the location of the black hole horizons:
  apparent horizon is located roughly at $\alpha \approx 0.3$

- If we add the comotion shift:

  - initially the lapse α near the black holes is quite symmetric

  $\Rightarrow$ initially the apparent horizon is centered on excision region

  - the run lasts up to $\sim 60M$ and dies when the lapse becomes too asymmetric, i.e. when the apparent horizon starts drifting away



- Note: excision was used here, but up to $\sim 60M$ it is not needed

Dynamically adjusted comoving coordinates

- Dynamically adjust Ω and V_r in the comotion shift $\beta^i_{com} = \psi^{-3}\left[(\Omega \times x)^i - AV_r x^i\right]$
  - Define the asymmetry in the lapse α by its "center of mass"

$$d^i := \sum_{x^i \in \text{ exc. B.}} (x^i_{\text{BH, initial}} - x^i)\alpha \Big/ \left(\sum_{x^i \in \text{ exc. B.}} \alpha \right).$$

    This asymmetry indicates if and in which direction the black hole is moving.

  - From time to time (every $\Delta t = 2M$) we change Ω and V_r in β^i_{com} by

$$\Delta\Omega = \Delta v_t / R \qquad \Delta V_r = \Delta v_r$$

    where Δv^i is computed from the estimated coordinate distance d^i by which the black hole has moved with respect to our coordinates.

  - We use a damped harmonic oscillator equation

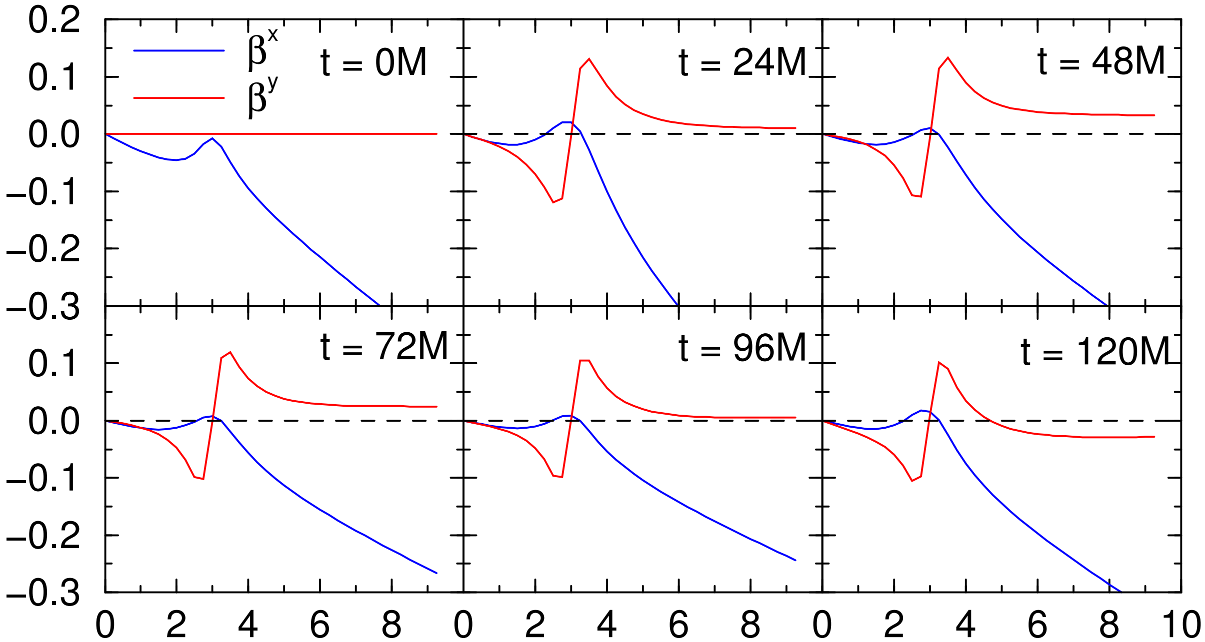
$$\Delta v^i = (-kd^i - \gamma \partial_t d^i)\Delta t$$

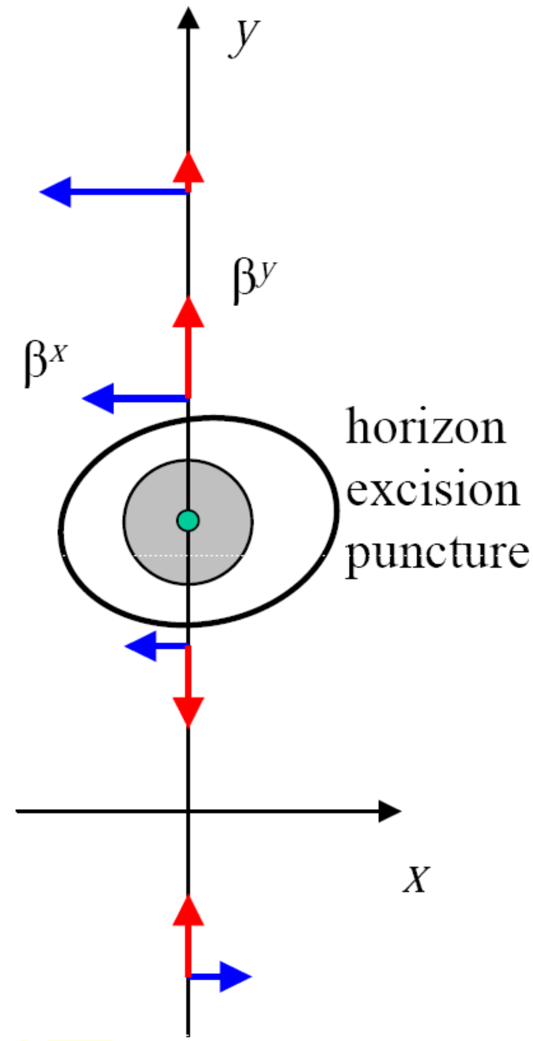
    to compute the changes in the shift.

- Now we can evolve to around $125M$, which is more than the orbital timescale of $T_{orbit} = 114M$, inferred from the initial data.

Evolution of the shift

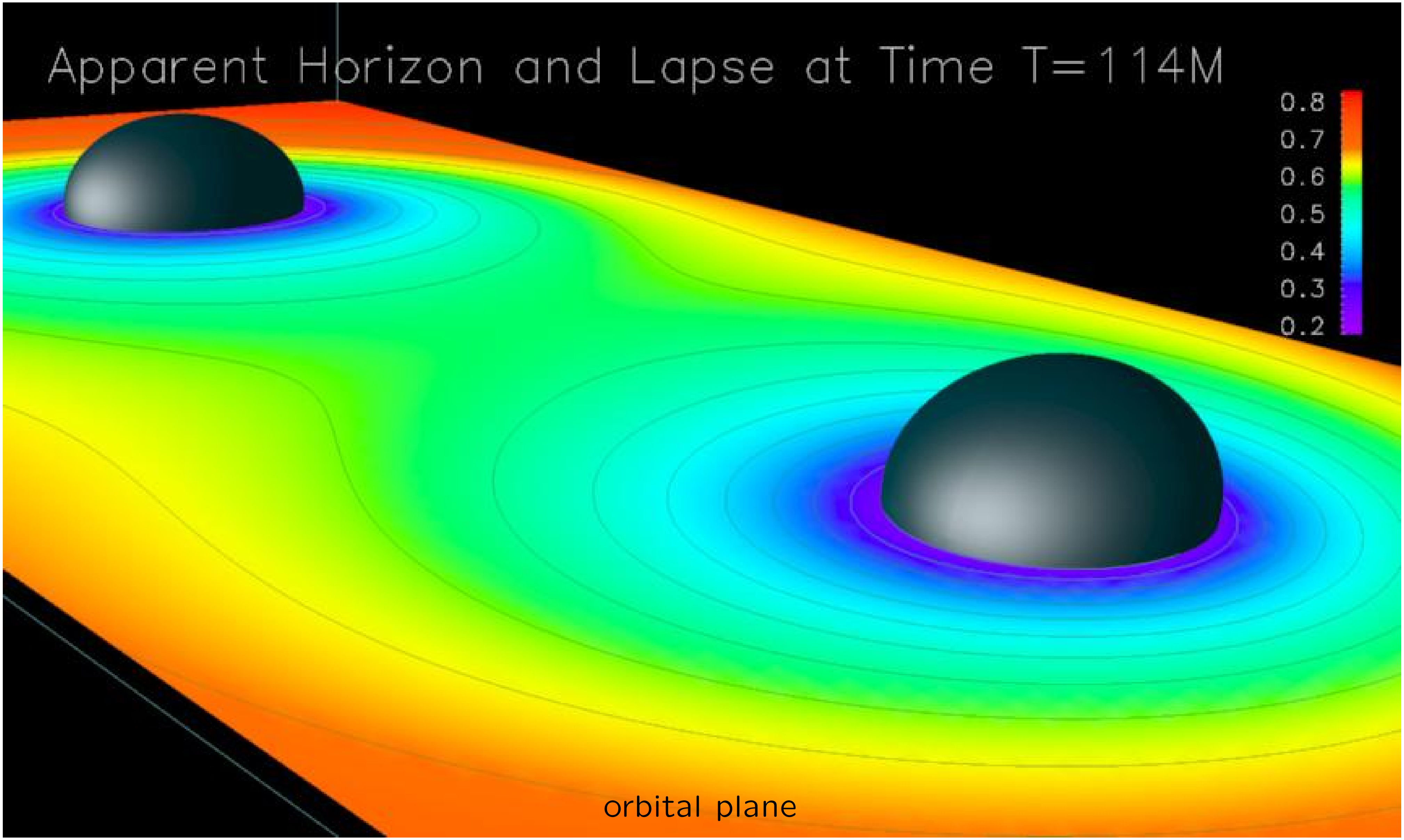
x- and y-components of the shift along the y-axis





- shift is dynamically adjusted in order to keep the BHs from moving with respect to our coordinates
- $|\beta_x|$: first increase, then slow decrease, then increase toward end
- β_y: first becomes positive, then negative again

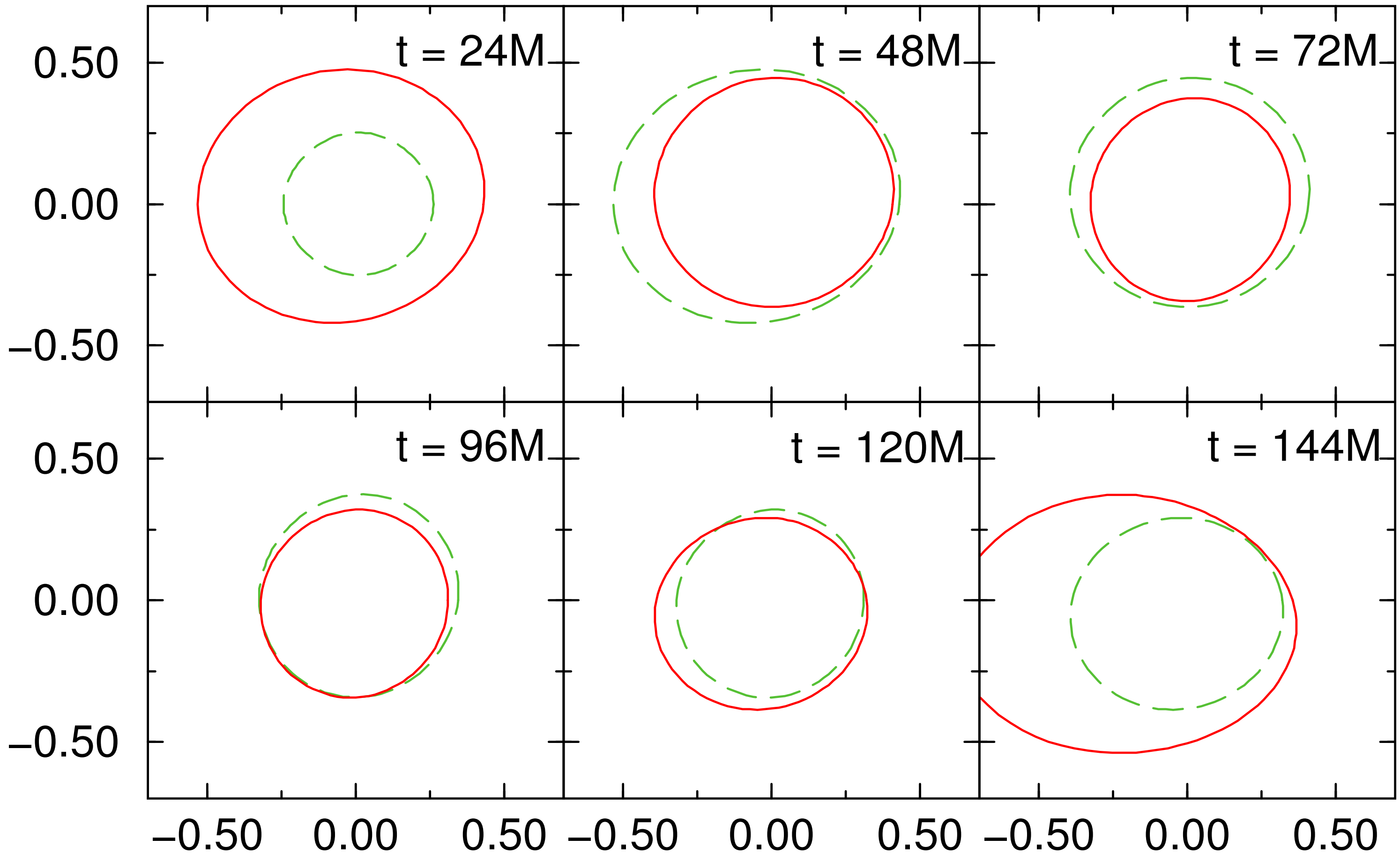
Apparent horizons and lapse after about one orbit



- comoving coordinates keep the BHs centered at their initial coordinate locations
- location of apparent horizon is where $\alpha \sim 0.3$

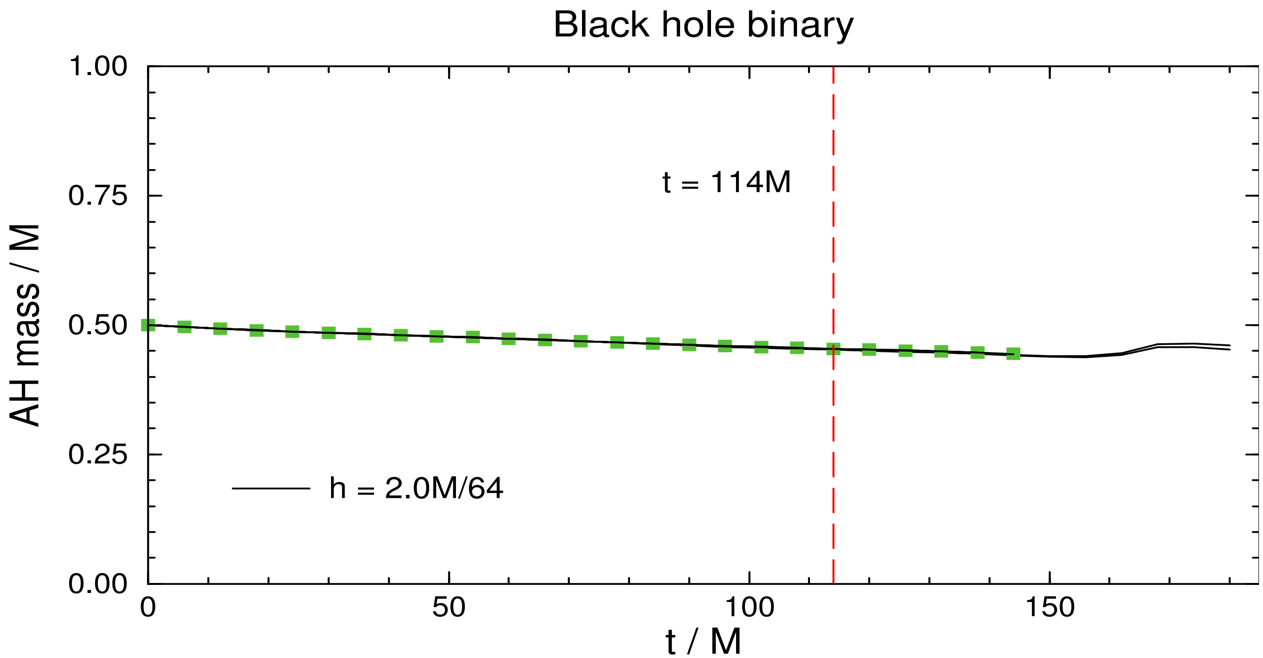
Residual motion of the apparent horizon

apparent horizon (AH) of one of the two black holes in the orbital plane



- due to our comoving coordinates the AH and thus the black hole stays more or less in place
- the coordinate size of the AH changes over time
- the AH shape becomes non-spherical in the chosen coordinates
- until the end of the simulation, no common apparent horizon was found

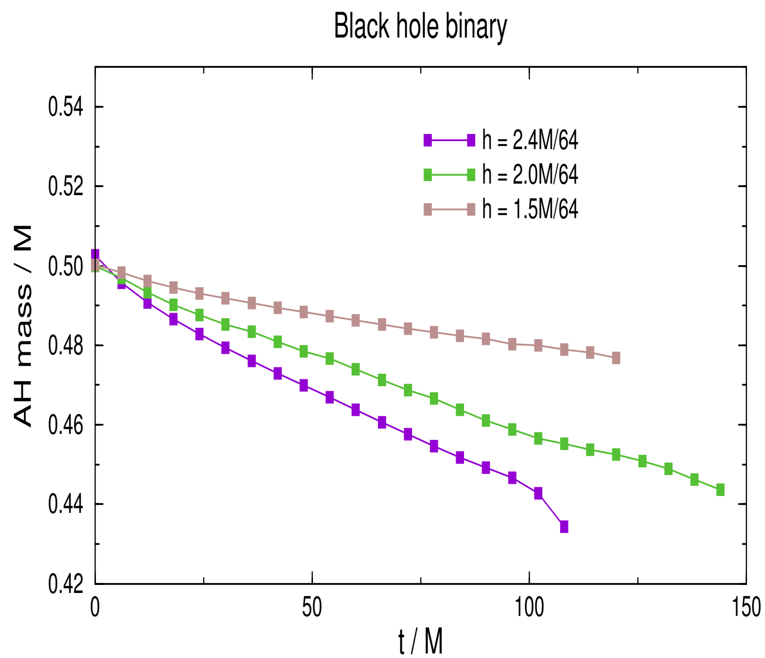
Apparent horizon area and mass

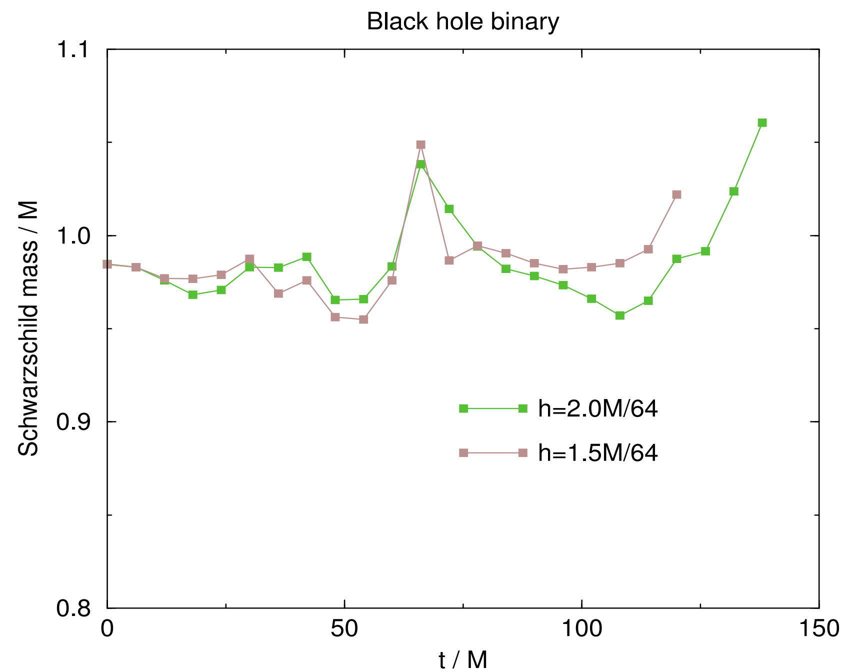


- 7 nested boxes
- resolution between $2M$ and $0.03125M$
- several runs with: cubical and spherical outer boundaries at $24M$, $48M$, $96M$

- The proper horizon area A and the black hole mass defined by $M_{AH} = \sqrt{A/16\pi}$ remain approximately constant during the evolution.
- Our evolution time is longer than one orbital period (as predicted by our initial data sequence).
- We obtain similar but shorter lived results without excision.
  $\rightarrow$ excision seems OK

Numerical accuracy and current limitations





- The apparent horizon mass and area stay approximately constant. The slow downward drift decreases for finer resolutions h.

- The ADM mass at infinity as estimated by assuming a Schwarzschild background fluctuates on the order of 5%.

$\Rightarrow$ Further improvements are needed before accurate gravitational waves can be extracted.

Summary

- We have found a dynamic gauge choice which for the first time allows us to evolve a black hole binary for about one orbit.
- The initial separation is large enough to expect the black holes to really orbit and not to just plunge toward each other.
- Until the end of our numerical simulation, no common apparent horizon was found.

$\Rightarrow$ Likely, the 2 black holes have not merged until then.

- It seems that the gauge alone was the ingredient necessary to achieve this, even though there were many other suspects (such as: the BSSN evolution system or inner and outer boundary conditions)
- Our dynamic gauge is far from perfect, since the apparent horizons still drift around, which could be the reason for the crash in the end.

END