Recent developments in binary black hole evolutions

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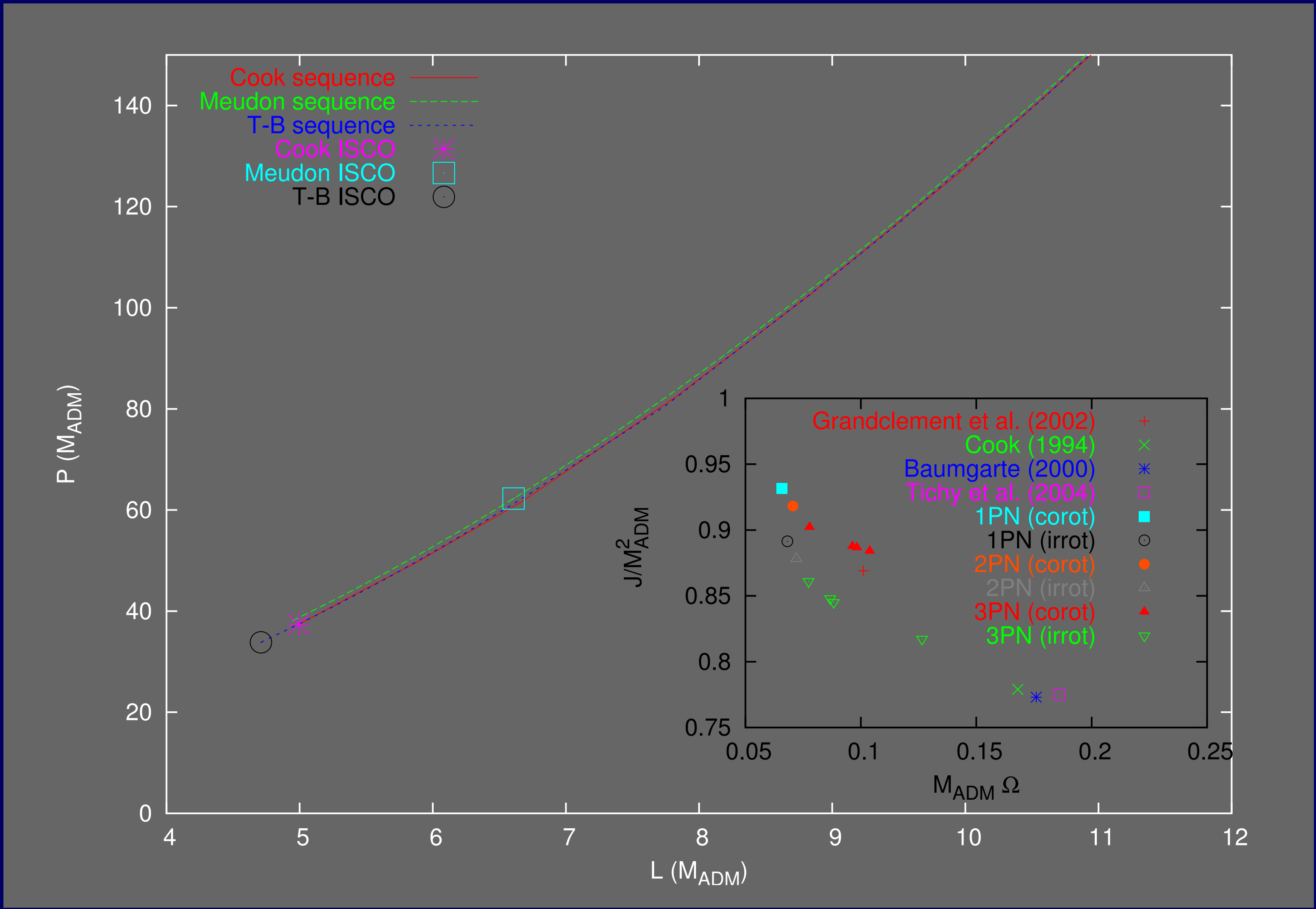
Quasi-circular binary black hole sequences



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Numerical evolutions of the Cook-Baumgarte quasi-circular sequence (QC-0: $L = 4.99M$ to QC-4: $L = 7.84M$.

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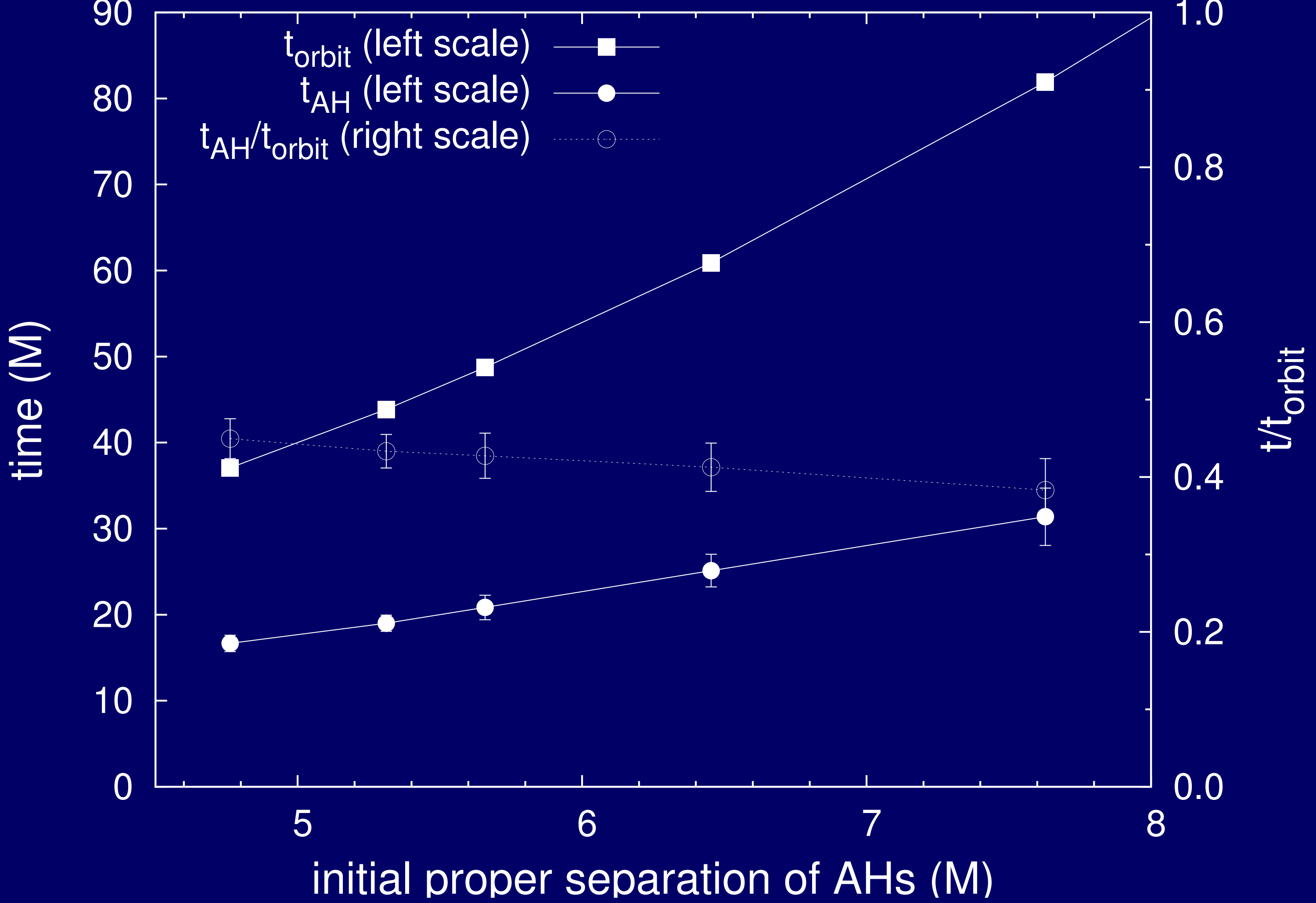
- Apperent-, event- and isolated-horizon analysis.

Unigrid simulations II



Unigrid simulations III

Initial values: $M_{\text{ADM}} = 1.01$ and $(J/M^2)_{\text{ADM}} = 0.779$.

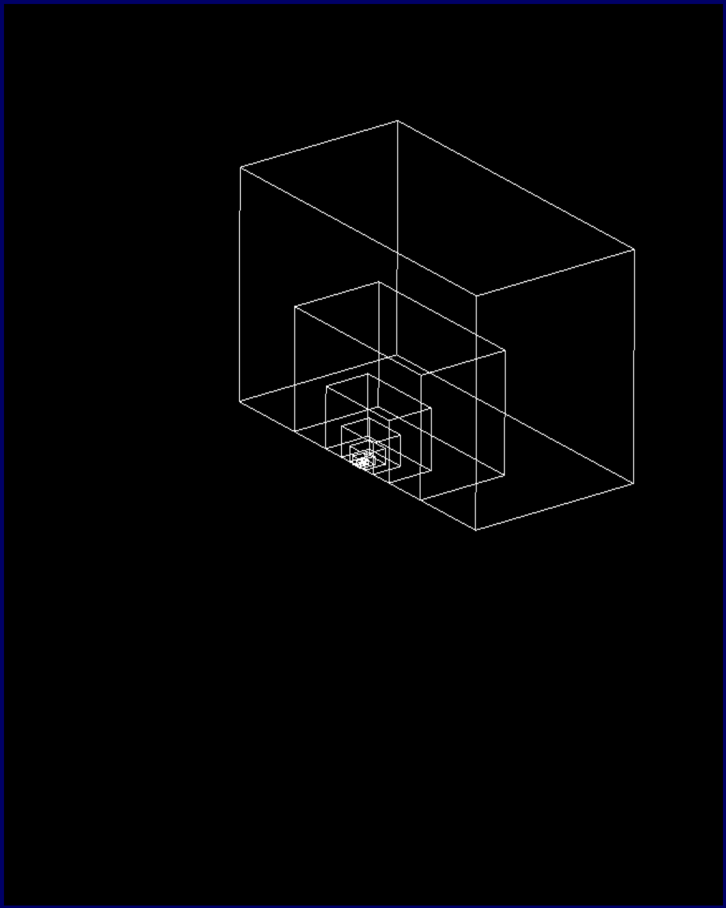
dx	0.080	0.060	0.048
a/m	0.450 ± 0.021	0.572 ± 0.025	0.632 ± 0.028
M_{irr}	0.947	0.933	0.923
M_{AH}	0.973 ± 0.003	0.978 ± 0.005	0.980 ± 0.006
J_{rad} (%)	45.3 ± 2.9	29.6 ± 3.7	22.1 ± 4.5
E_{rad} (%)	3.61 ± 0.25	3.12 ± 0.45	2.97 ± 0.59
T_{AH}	15.72	16.53	17.11

Lazarus results (Baker et. al. 2001, 2002): $E_{\text{rad}} = 3\%$ and $J_{\text{rad}} = 12\%$

FMR (Carpet)

Basic grid setup:

- 8 levels of refinement.
- Rotating quadrant symmetry.
- Boundaries at $96M$.
- Resolution on finest grid $0.025M$.
- Fourth order finite differencing (except shift advection terms).
- RK3 time integrator.
- 3rd order prolongation in space.
- 2nd order prolongation in time.
- Fixed size excision region.



New gauge parameter

Lapse:

$$\partial_t \alpha = -2\alpha\psi^n (K - K_0).$$

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The time derivative of the shift is adjusted dynamically based on the motion of the centroid of the apparent horizon.

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The time derivative of the shift is adjusted dynamically based on the motion of the centroid of the apparent horizon.

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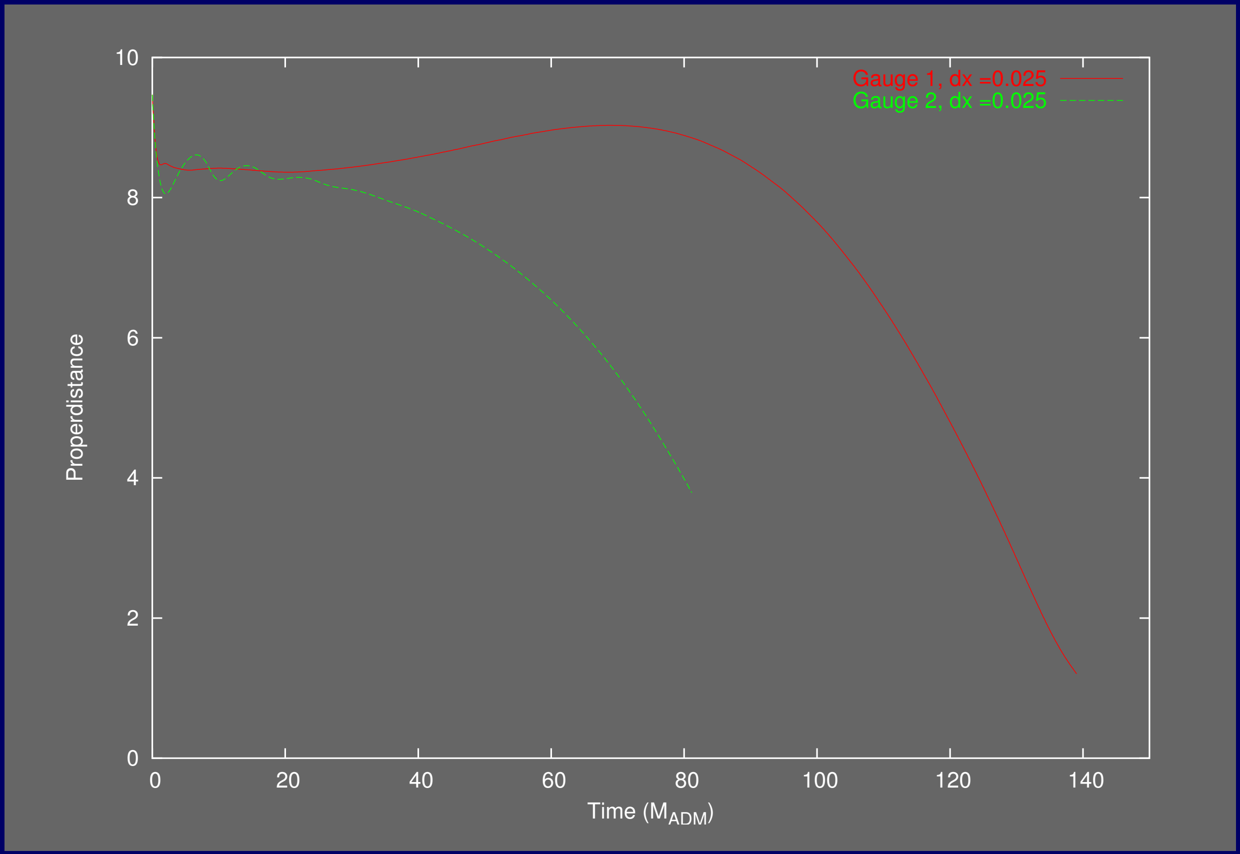
Gauge choice 1: $n = 4$, $\eta = 2$, $p = 4$, $m = 1$ and $k = 2$.

Gauge choice 2: $n = 0$, $\eta = 4$, $p = 1$, $m = 1$ and $k = 2$.

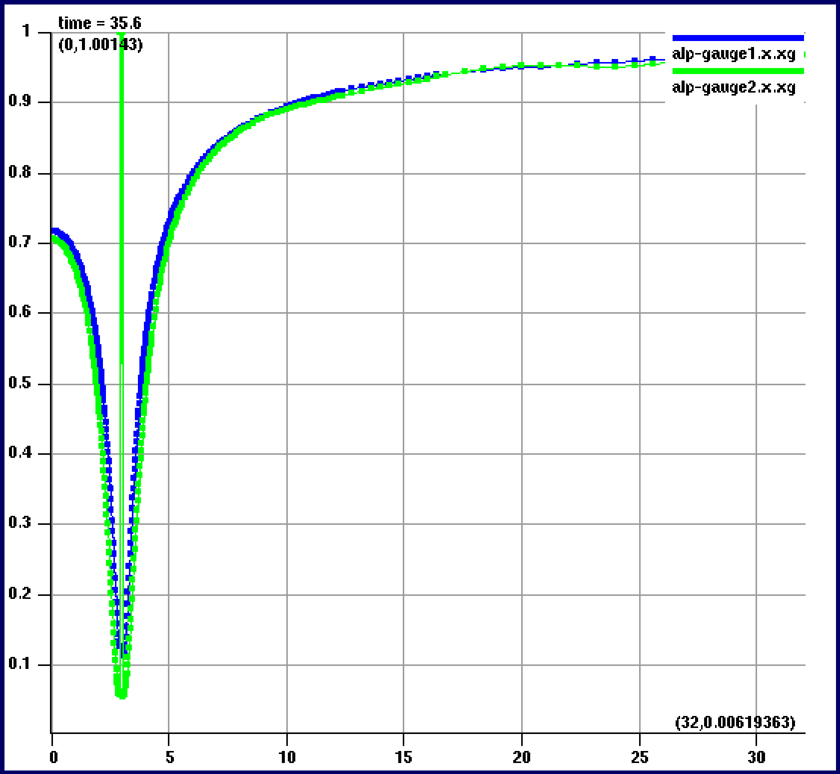
The battle of the gauges

For Tichy-Brügmann sequence ($D = 3.0$)



Found common AH at $T = 135M$.

The battle of the gauges II



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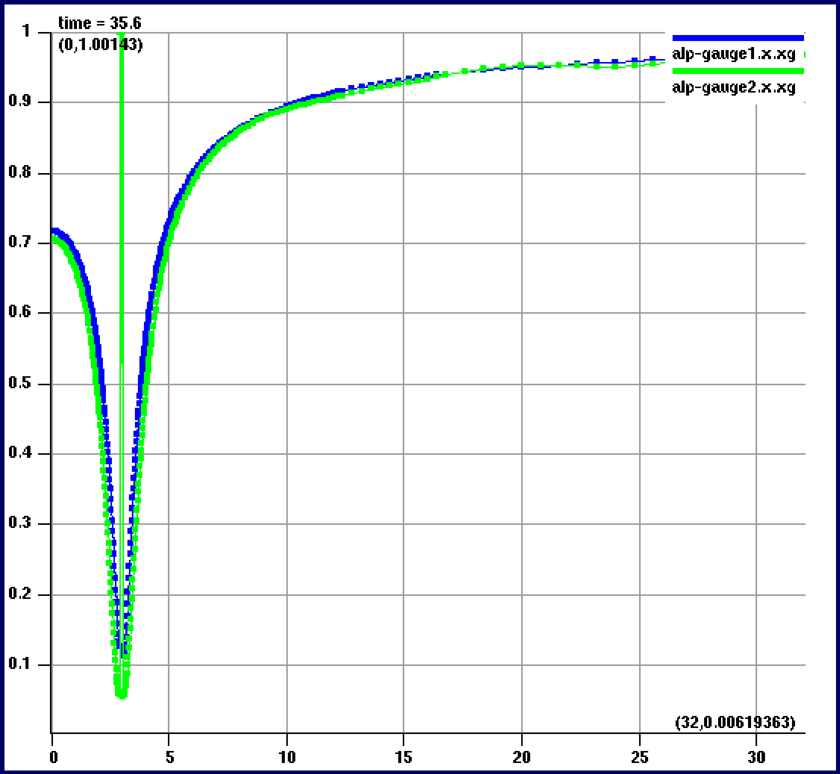

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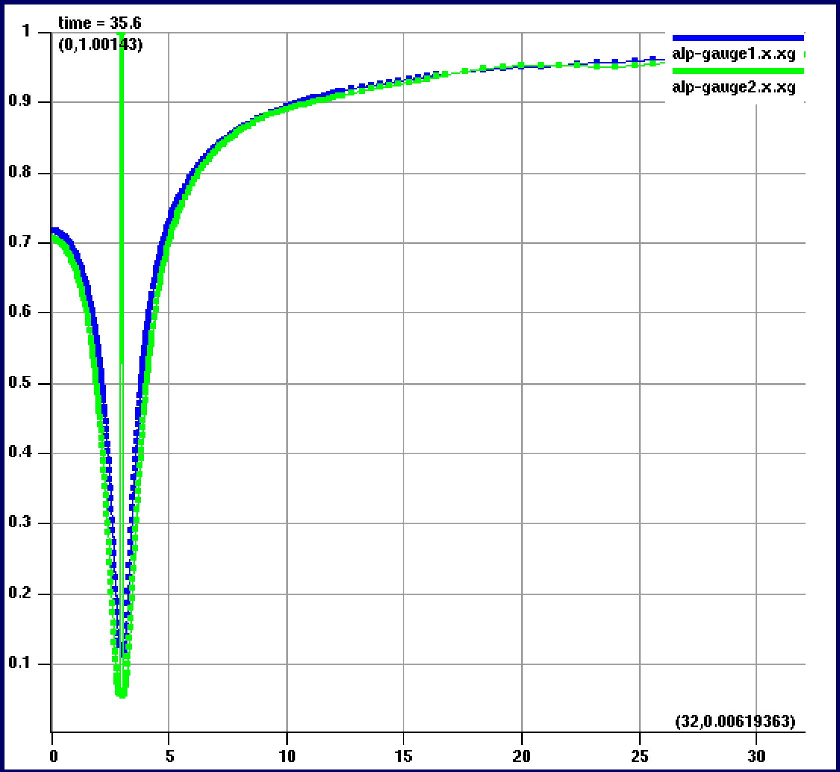

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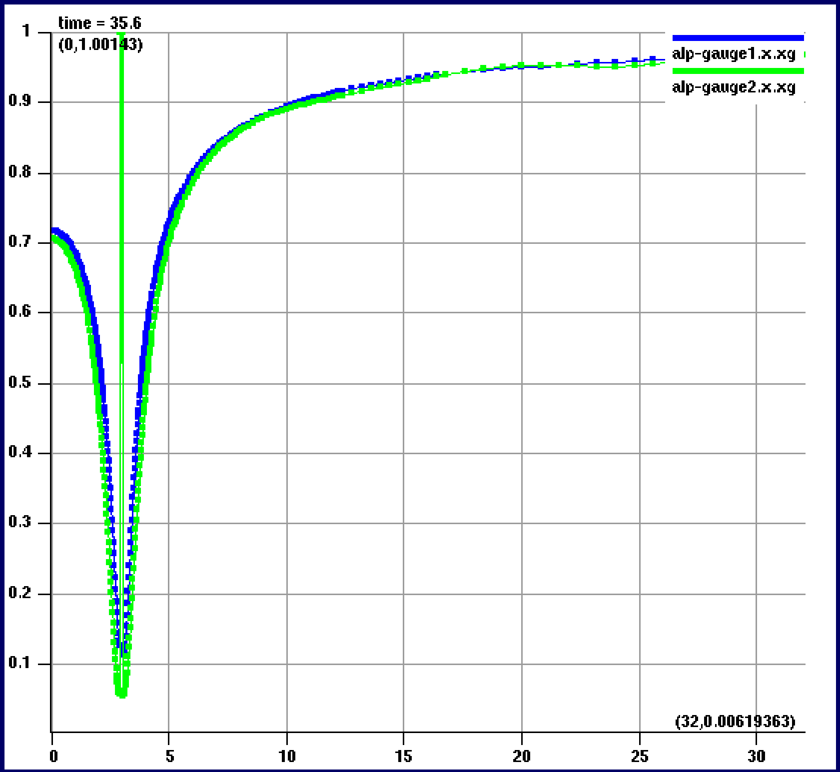

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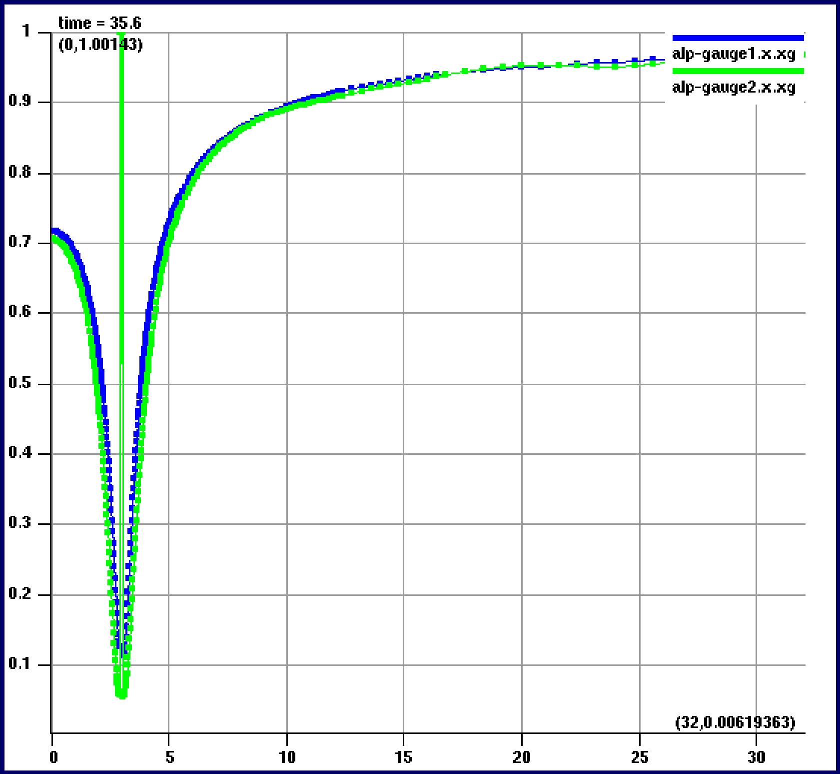

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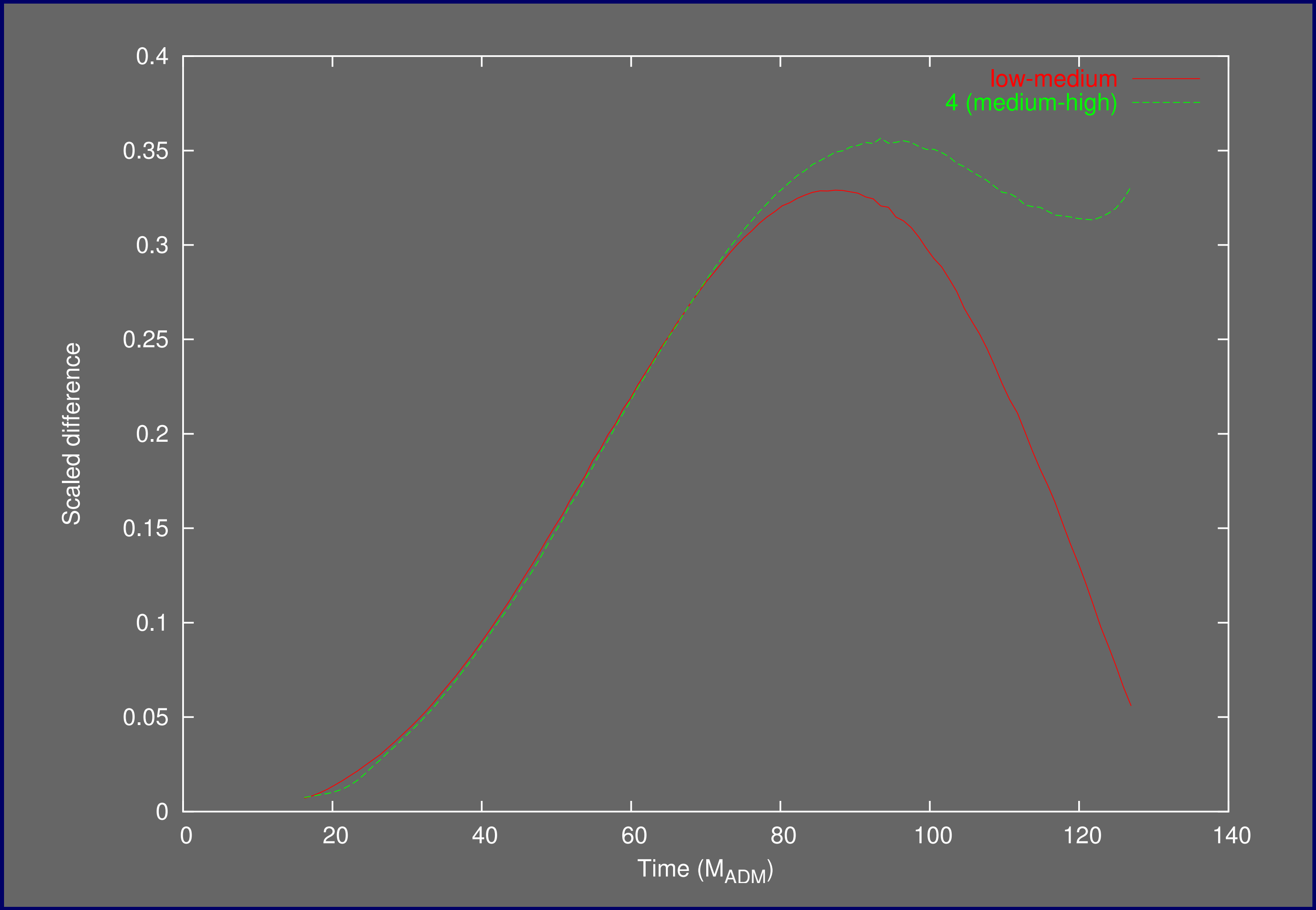
Can the results be reconciled?

To investigate this we decided to perform several runs for each gauge parameter set with different resolutions.

Convergence



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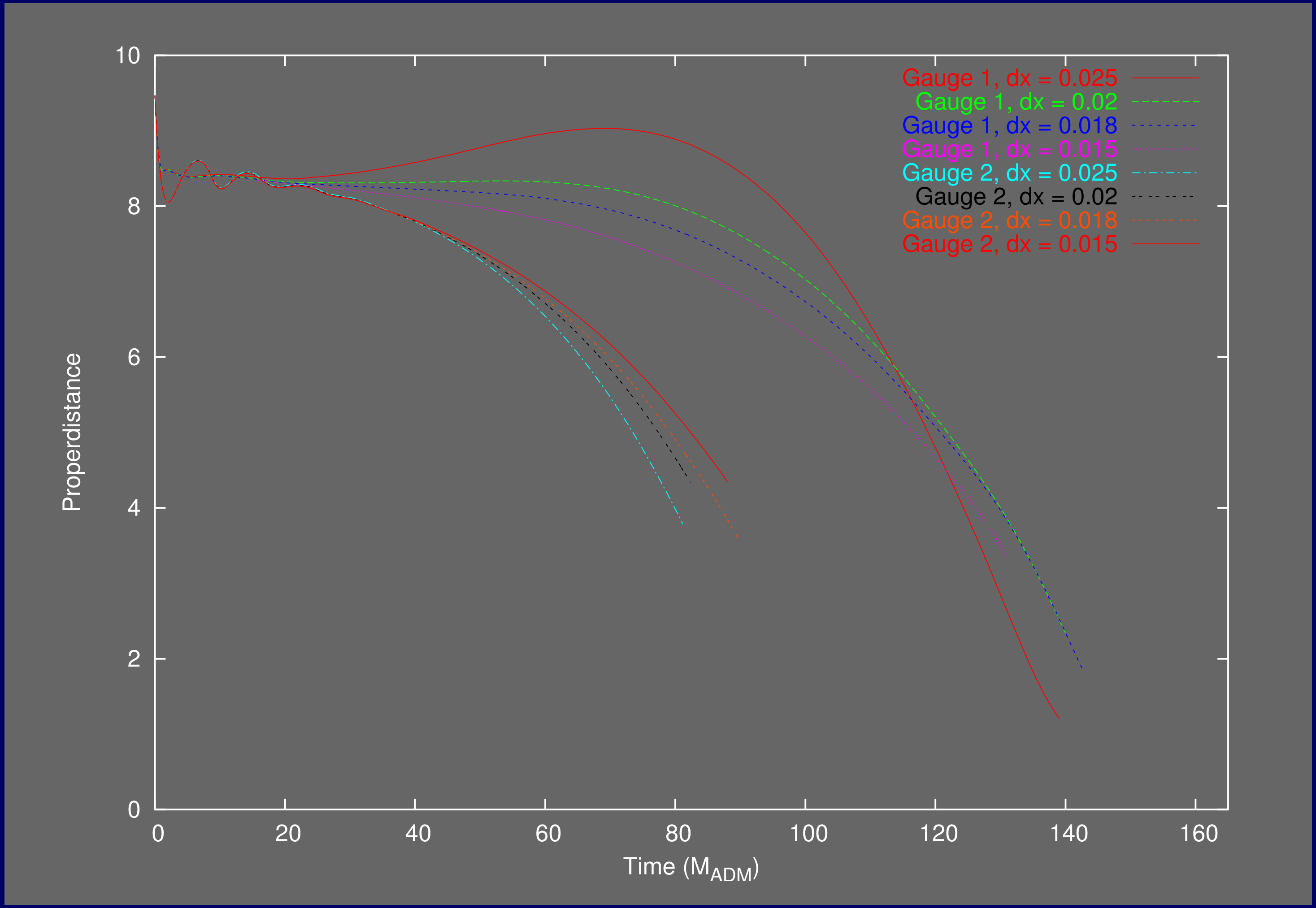

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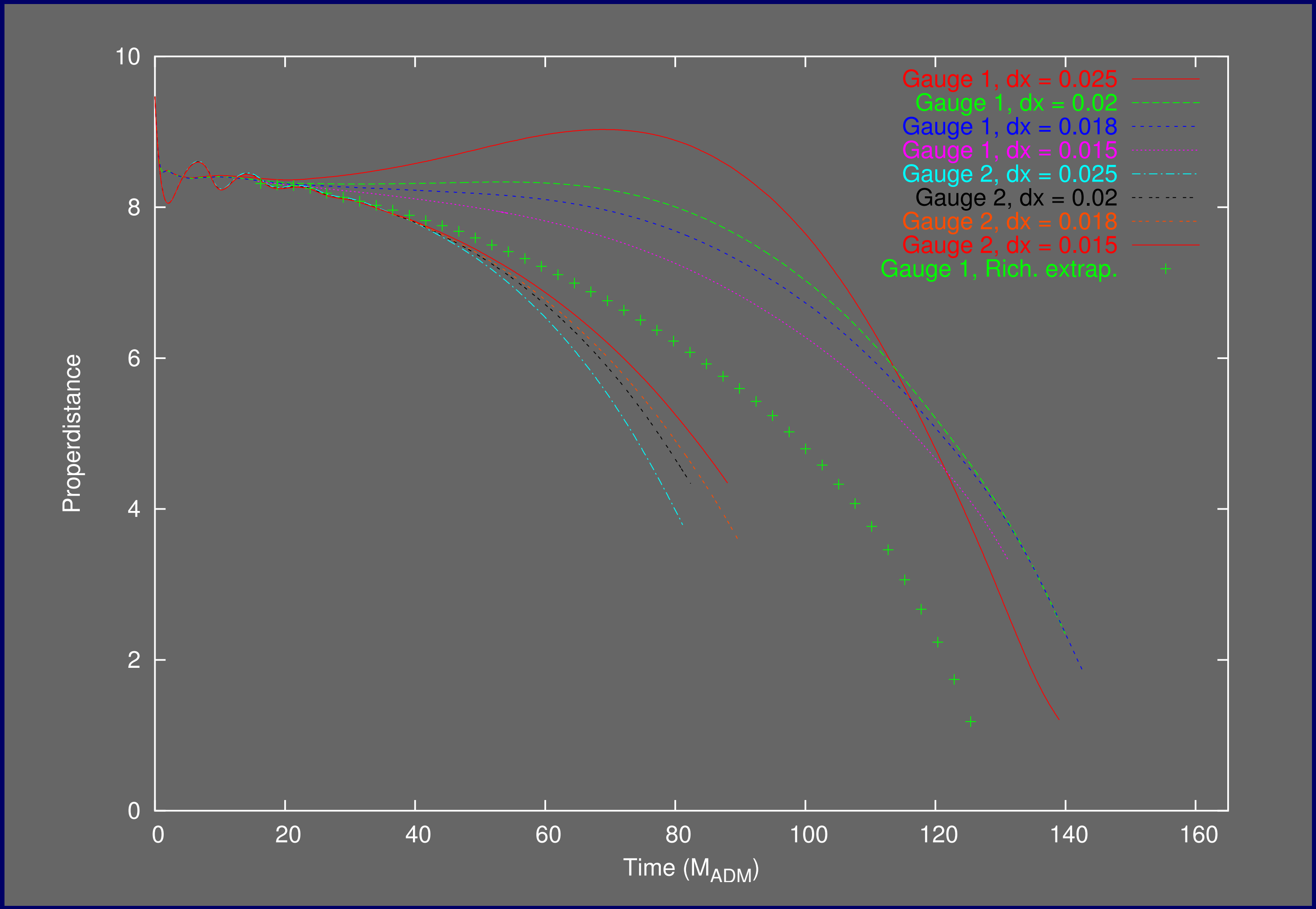

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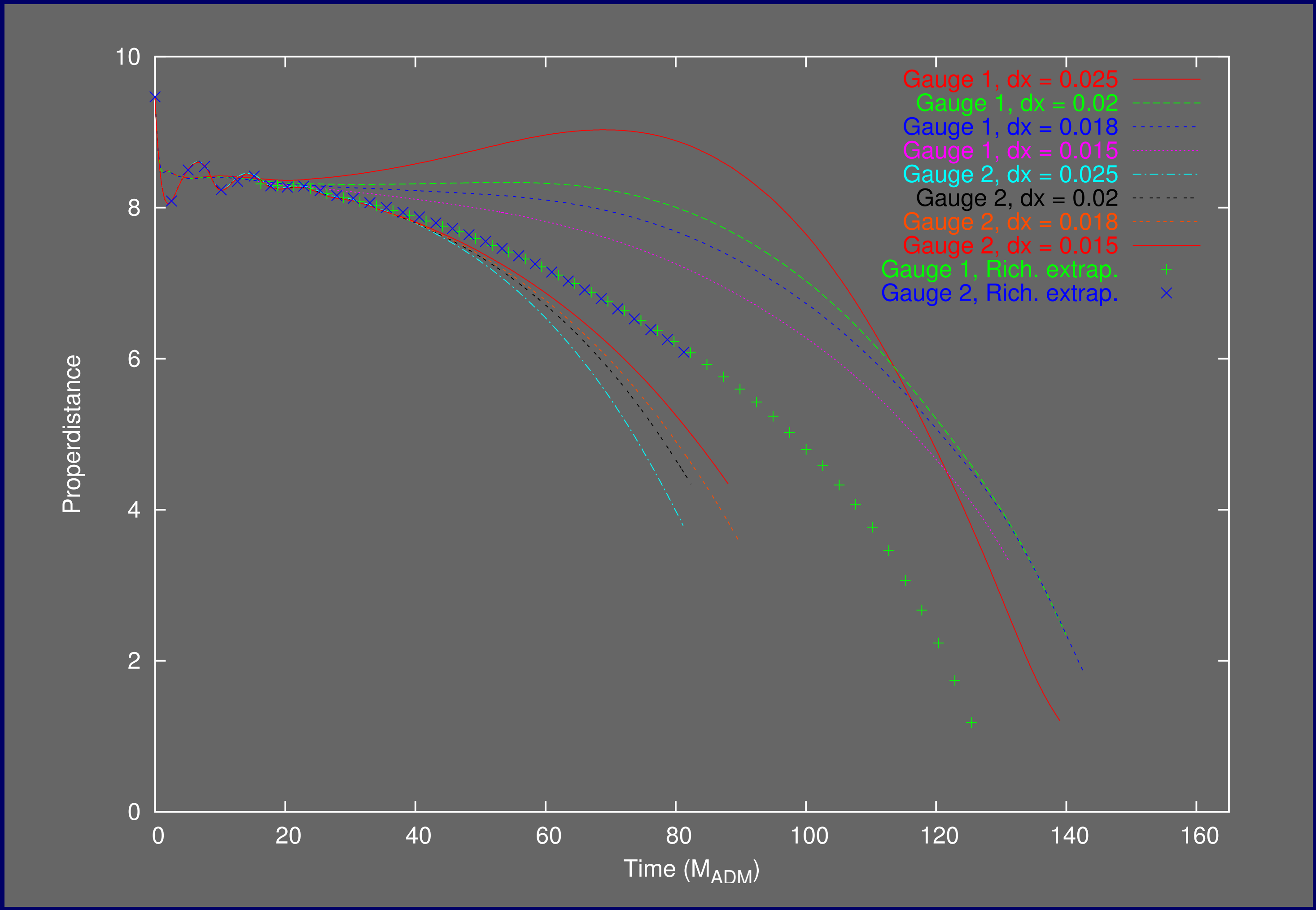


Convergence III

We assume that the properdistance as a function of time and resolution is given by:

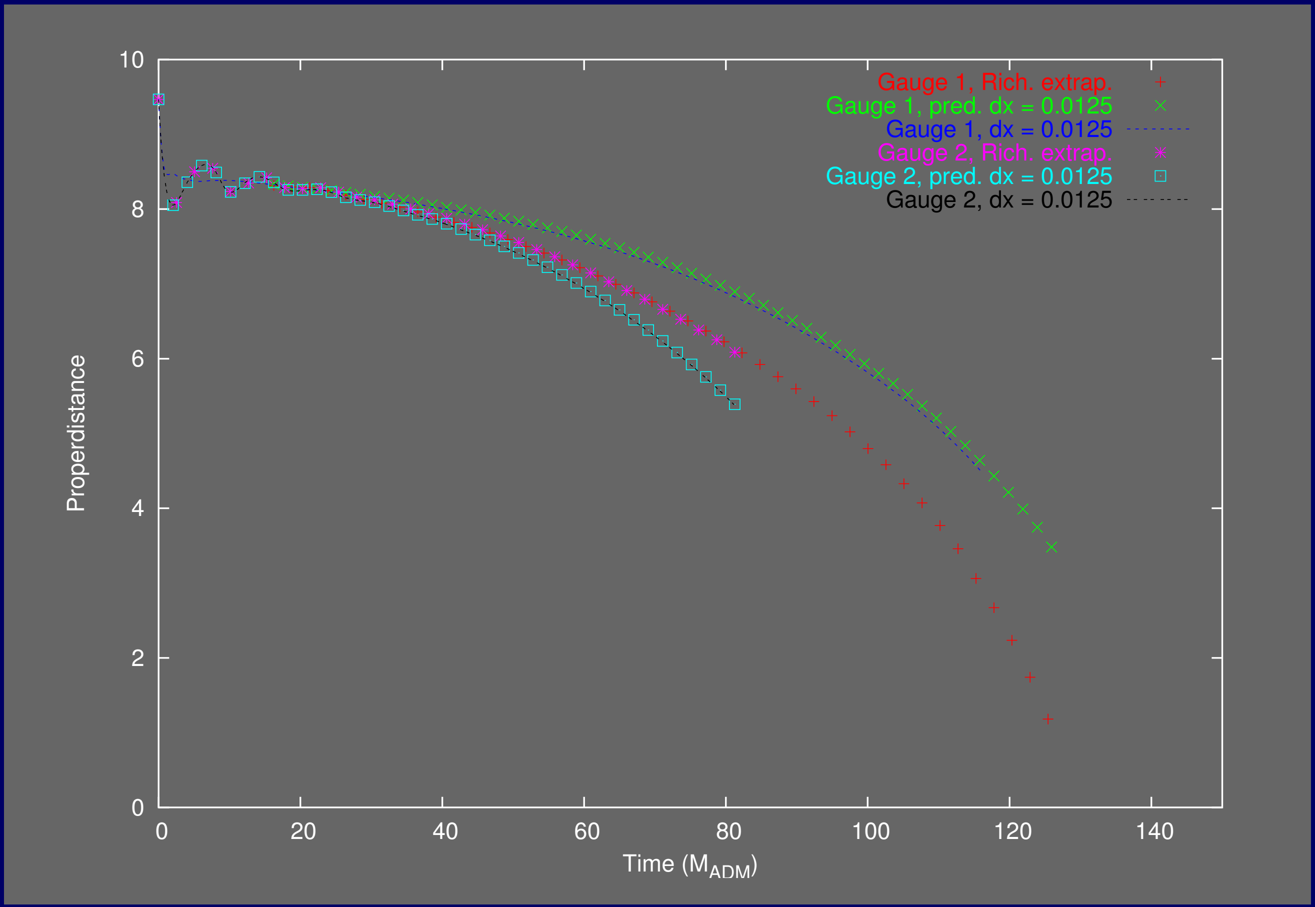
$$D(\Delta, t) = D(0, t) + a(t)\Delta^2 + b(t)\Delta^3.$$

Then given runs at 3 different resolutions, Δ_1, Δ_2, Δ_3, we get:

$$\begin{aligned} D(\Delta_1, t) &= D(0, t) + a(t)\Delta_1^2 + b(t)\Delta_1^3, \\ D(\Delta_2, t) &= D(0, t) + a(t)\Delta_2^2 + b(t)\Delta_2^3, \\ D(\Delta_3, t) &= D(0, t) + a(t)\Delta_3^2 + b(t)\Delta_3^3, \end{aligned}$$

which can be solved for $D(0, t)$, $a(t)$ and $b(t)$.

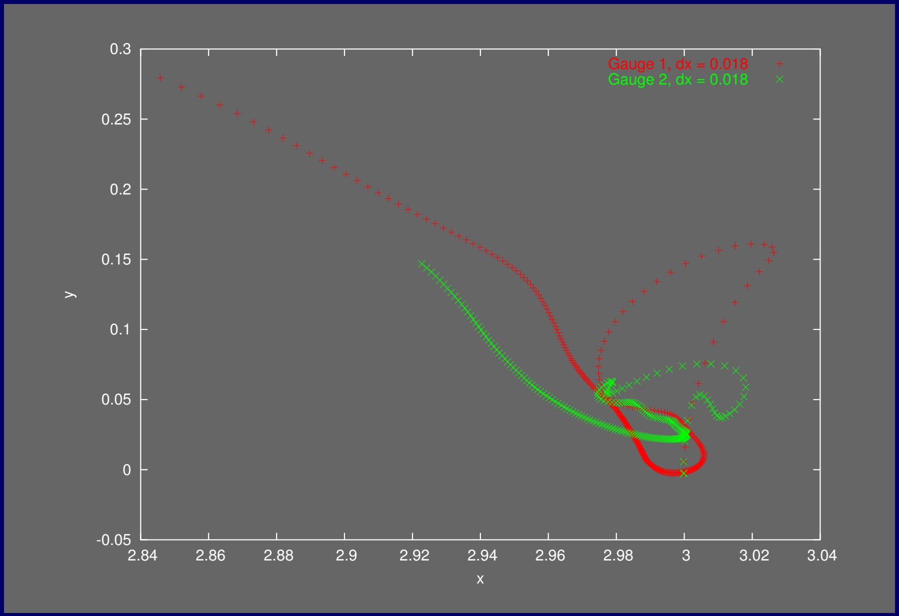
Convergence IV



To get less than 1% error we estimate $dx = M/200$.

Improving the gauge

We realized that the damping timescale for the drift correction was not chosen optimally. Recently Ryoji has experimented with different damping timescales (critical- or overdamping).



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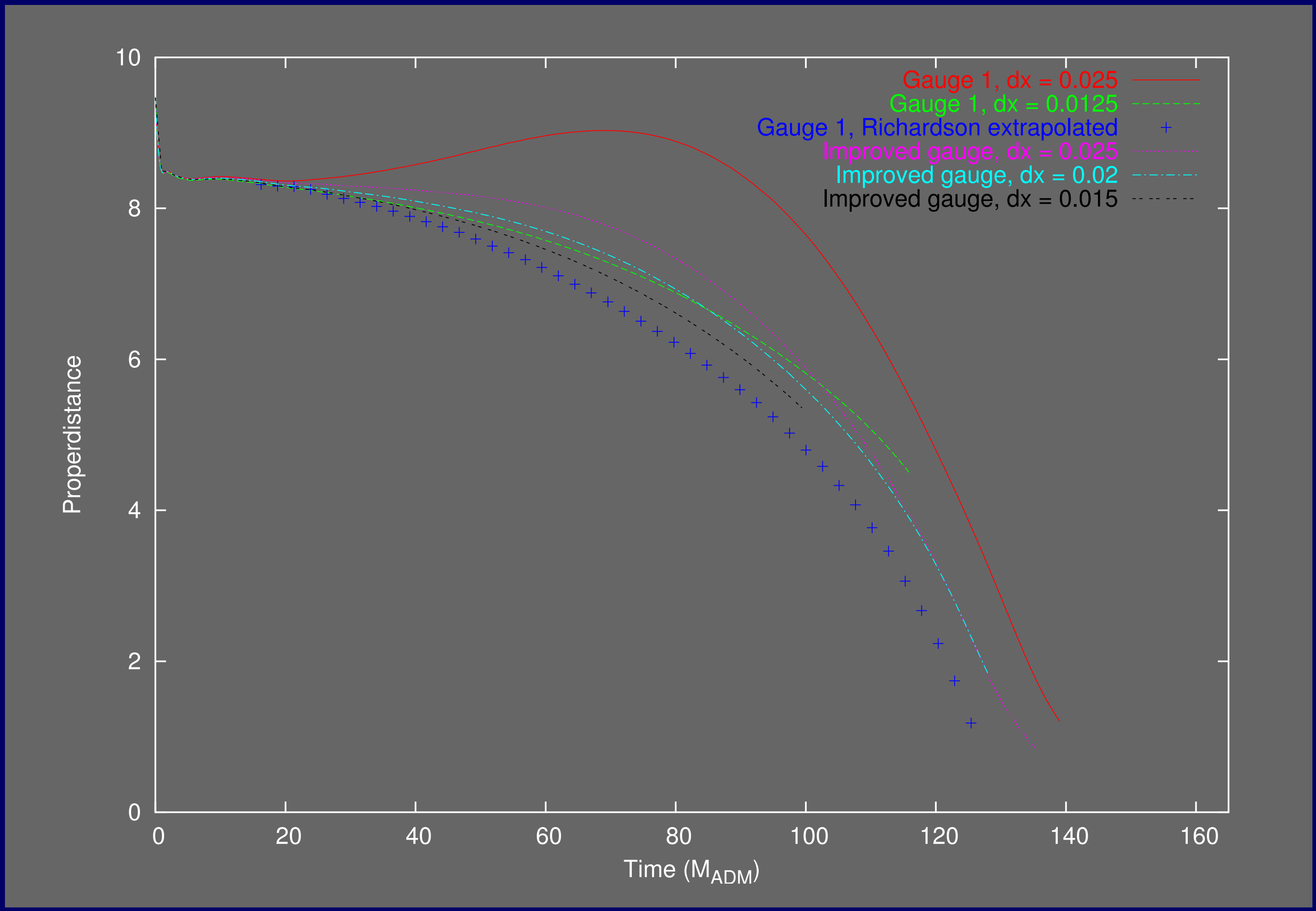

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- The data set from Brügmann et. al (2004) has been confirmed to actually perform more than an orbit before merger.
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- We need to get a better understanding of the new gauge parameter.