Modelling and measuring the Universe

Summary

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UMBC, November 2nd, 2006
The first part of the lecture

- Universe is expanding (Hubble relation)
- Newton’s not enough: Einstein’s idea about space-time
- General relativity for curved space-time
- Four equations to describe the expanding/contracting universe
The second part of the lecture

- How to model the Universe
- the Friedmann equation as a function of density parameters
- Matter-, radiation-, lambda-, curvature-only universe
- mixed-component universes
- the important times in history: $a_{r,m}$ and $a_{m,\Lambda}$
The second part of the lecture
- How to measure the Universe
- the Friedmann equation expressed in a Taylor series: $H_0$ and $q_0$ (deceleration parameter)
- luminosity distance, angular size distance
- distance ladder: parallax, Cepheids, SuperNova Type Ia
- results from the SuperNova measurements
The second part of the lecture

- What is the matter contents of the Universe?
- matter in stars
- matter between stars
- matter in galaxy clusters
- dark matter
Friedmann equation

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3c^2} \epsilon - \frac{\kappa c^2}{R_0^2 a^2} \]

Fluid equation

\[ \dot{\epsilon} + 3\frac{\dot{a}}{a} (\epsilon + P) = 0 \]

Acceleration equation:

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\epsilon + 3P) \]
Density parameter $\Omega$ and curvature

$$\Omega(t) \equiv \frac{\epsilon(t)}{\epsilon_c(t)}$$

$\Omega_0 > 1$

$\Omega_0 < 1$

$\Omega_0 = 1$
The effect of curvature

Scale factor $a(t)$ in an empty universe
Spatially flat Universe

Our key questions for any type of Universe:
Scale factor $a(t)$? What is the age of the Universe $t_0$?
Energy density $\varepsilon(t)$? Distance of an object with redshift $z$?
The effect of curvature

What happens in a flat universe?
One component only?

Friedmann equation:

\[
\dot{a}^2 = \frac{8\pi G}{3c^2} \sum \epsilon_{w,0} a^{-1-3\omega} - \frac{\kappa c^2}{R_0^2}
\]
Friedmann equation:

\[
\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_{\omega} \epsilon_{w,0} a^{\omega - 1 - 3\omega} - \frac{\kappa c^2}{R_0^2}
\]

Friedmann equation (flat, single-component):

\[
\dot{a}^2 = \frac{8\pi G \epsilon_0}{3c^2} a^{-1 - 3\omega}
\]

Flat, single component universe:

\[
H_0 \equiv \left( \frac{\dot{a}}{a} \right)_{t=t_0} = \frac{2}{3(1+\omega)} t_0^{-1}
\]
Friedmann equation (flat, single-component):
\[ \dot{a}^2 = \frac{8\pi G \epsilon_0}{3 c^2} a^{-1-3\omega} \]

Flat, single component universe:
\[ H_0 \equiv \left(\frac{\dot{a}}{a}\right)_{t=t_0} = \frac{2}{3(1+\omega)} t_0^{-1} \]

Flat, single component universe:
\[ t_0 = \frac{2}{3(1+\omega)} H_0^{-1} \]

Proper distance:
\[ d_P(t_0) = \frac{c}{H_0} \frac{2}{1+3\omega} \left[ 1 - (1 + z)^{-(1+3\omega)/2} \right] \]
Our key questions for any type of Universe:
Scale factor $a(t)$? What is the age of the Universe $t_0$?
Energy density $\epsilon(t)$? Distance of an object with redshift $z$?
Friedmann equation (flat, single-component):
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The effect of curvature in an empty universe.

Scale factor $a(t)$ in a matter-only (nonrelativistic) universe.
The effect of curvature
Matter (top) & empty (bottom) universe

Scale factor $a(t)$ in a radiation-only universe

Matter (top) & empty (bottom) universe
Spatially flat Universe with dark energy only

Our key questions for any type of Universe:
Scale factor $a(t)$? What is the age of the Universe $t_0$?
Energy density $\varepsilon(t)$? Distance of an object with redshift $z$?
The effect of curvature
Matter, empty, radiation
only universe

Scale factor $a(t)$ in a lambda-only universe
The effect of curvature scale factor $a(t)$ in a flat, single-component universe.
Universe with matter and curvature only

\[ \frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2} \]

\[ P = \omega \epsilon \]

Density parameter:
\[ \Omega(t) \equiv \frac{\epsilon(t)}{\epsilon_c(t)} \]
Universe with matter and curvature only
Universe with matter and curvature only
Curved, matter dominated Universe

\[ \Omega_0 < 1 \quad \kappa = -1 \quad \text{Big Chill (a } \propto t \text{)} \]

\[ \Omega_0 = 1 \quad \kappa = 0 \quad \text{Big Chill (a } \propto t^{2/3} \text{)} \]

\[ \Omega_0 > 1 \quad \kappa = -1 \quad \text{Big Crunch} \]
Universe with matter and curvature only

\[ \frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2} \]

What is \( a(t) \) ?
Universe with matter and $\Lambda$
(matter + cosmological constant, no curvature)

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

What values for $\Omega$ in order to get a flat Universe?
Possible universes containing matter and dark energy

Abbe George Lemaître (1894-1966) and Albert Einstein in Pasadena 1933
Universe with matter, $\Lambda$, and curvature

(matter + cosmological constant + curvature)

\[
\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}
\]

\[
\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{1-\Omega_{m,0}-\Omega_{\Lambda,0}}{a^2} + \Omega_{\Lambda,0}
\]

\[\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0}\]
Flat Universe with matter, radiation
(e.g. at $a \sim a_{rm}$)

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$$\Omega_0 = \Omega_{m,0} + \Omega_{r,0}$$
Describing the real Universe - the “benchmark” model

\[
\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}
\]

\[
\Omega_0 = \Omega_L + \Omega_{m,0} + \Omega_{r,0} = 1.02 \pm 0.02
\]

The “benchmark” model

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>photons</td>
<td>$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>neutrinos</td>
<td>$\Omega_{\nu,0} = 3.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>total radiation</td>
<td>$\Omega_{r,0} = 8.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>baryonic</td>
<td>$\Omega_{bary,0} = 0.04$</td>
</tr>
<tr>
<td>dark matter</td>
<td>$\Omega_{dm,0} = 0.26$</td>
</tr>
<tr>
<td>total matter</td>
<td>$\Omega_{m,0} = 0.30$</td>
</tr>
<tr>
<td>dark energy</td>
<td>$\Omega_{\Lambda,0} \approx 0.70$</td>
</tr>
</tbody>
</table>

$$\Omega_0 = \Omega_L + \Omega_{m,0} + \Omega_{r,0} = 1.02 \pm 0.02$$


“The Universe is flat and full of stuff we cannot see”
The “benchmark” model

Important Epochs in our Universe:

<table>
<thead>
<tr>
<th>Epoch</th>
<th>scale factor</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>radiation-matter</td>
<td>$a_{rm} = 2.8 \times 10^{-4}$</td>
<td>$t_{rm} = 47,000\text{ yr}$</td>
</tr>
<tr>
<td>matter-lambda</td>
<td>$a_{m\Lambda} = 0.75$</td>
<td>$t_{m\Lambda} = 9.8\text{ Gyr}$</td>
</tr>
<tr>
<td>Now</td>
<td>$a_0 = 1$</td>
<td>$t_0 = 13.5\text{ Gyr}$</td>
</tr>
</tbody>
</table>

“The Universe is flat and full of stuff we cannot see - and we are even dominated by dark energy right now”
The “benchmark” model

Some key questions:

- Why, out of all possible combinations, we have $\Omega_0 = \Omega_\Lambda + \Omega_{m,0} + \Omega_{r,0} = 1.0$ ?
- Why is $\Omega_\Lambda \sim 1$?
- What is the dark matter?
- What is the dark energy?
- What is the evidence from observations for the benchmark model?

“The Universe is flat and full of stuff we cannot see”
How do we verify our models with observations?

\[ H_0 \cdot t = \int_0^a \frac{da}{\sqrt{\Omega_{\tau,0} a^{-2} + \Omega_{m,0} a^{-1} + \Omega_{\Lambda,0} a^2 + (1-\Omega_0)}} \]
Taylor Series

A one-dimensional Taylor series is an expansion of a real function $f(x)$ about a point $x=a$ is given by

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots$$
Scale factor as Taylor Series

\[ f(x) = f(\alpha) + f'(\alpha)(x - \alpha) + \frac{f''(\alpha)}{2!}(x - \alpha)^2 + \frac{f^{(3)}(\alpha)}{3!}(x - \alpha)^3 + \ldots + \frac{f^{(n)}(\alpha)}{n!}(x - \alpha)^n + \ldots \]

\[ a(t) \simeq 1 + H_0(t - t_0) - \frac{1}{2}q_0H_0^2(t - t_0)^2 \]

qo = deceleration parameter
Deceleration parameter $q_0$

\[ a(t) \simeq 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2(t - t_0)^2 \]

Acceleration equation:

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\epsilon + 3P) \]

\[ - \frac{\ddot{a}_0}{a_0 H_0^2} = q_0 = \frac{1}{2} \sum \Omega_\omega (1 + 3\omega) \]
How to measure distance

Measure flux to derive the luminosity distance
Measure angular size to derive angular size distance

M101 (Credits: George Jacoby, Bruce Bohannan, Mark Hanna, NOAO)
Luminosity Distance

In a nearly flat universe:

\[ d_L \sim \frac{c}{H_0} \, z \left[ 1 + \frac{1-q_0}{2} \, z \right] \]

How to determine \( a(t) \):

- determine the flux of objects with known luminosity to get luminosity distance
- for nearly flat: \( d_L = d_p(t_0) \, (1+z) \)
- measure the redshift
- determine \( H_0 \) in the local Universe
  \( \rightarrow q_0 \)
Angular Diameter Distance

\[ d_A = \frac{\text{length}}{\delta \Theta} = \frac{d_L}{(1+z)^2} \]

For nearly flat universe:
\[ d_A = \frac{dp(t_0)}{(1+z)} \]
Angular Diameter Distance $d_A = 1 / \delta \Theta$
Measuring Distances - Standard Candles
The effect of curvature

Cepheids as standard candles

Data from a Well-Measured Cepheid

Henrietta Leavitt
Cepheids as standard candles

Large Magellanic Cloud (Credit: NOAO)

M31 (Andromeda galaxy) (Credit: Galex)
Cepheids as standard candles

Hipparcos astrometric satellite (Credit: ESA)

Hubble Space Telescope Credit: NASA/STS-82
For nearly flat Universe:

\[ d_L \sim \frac{c}{H_0} z \left[ 1 + \frac{1-q_0}{2} z \right] \]
Simulation of galaxy merging
Super Nova types - distinguish by spectra and/or lightcurves
Super Nova Type II - “core collapse” Super Nova
A white dwarf in NGC 2440 (Credits: Hubble Space Telescope)
The progenitor of a Type Ia supernova

Two normal stars are in a binary pair.

The more massive star becomes a giant...

...which spills gas onto the secondary star, causing it to expand and become engulfed.

The secondary, lighter star and the core of the giant star spiral inward within a common envelope.

The common envelope is ejected, while the separation between the core and the secondary star decreases.

The remaining core of the giant collapses and becomes a white dwarf.

The aging companion star starts swelling, spilling in a white dwarf...causing the companion
Electron degeneracy pressure can support an electron star (White Dwarf) up to a size of \( \sim 1.4 \) solar masses.

\[ M_{Ch} \approx \frac{3\sqrt{2\pi}}{8} \left( \frac{hc}{2\pi G} \right)^{3/2} \left[ \frac{Z}{A} \frac{1}{m_H} \right]^2 \]
Super Nova Type Ia lightcurves

Corrected lightcurves

light-curve timescale 
“stretch-factor” corrected

Kim, et al. (1997)
Perlmutter et al. 1999

No Big Bang

68%
90%
95%
99%

Flat Universe

$\Omega_\Lambda = 0$

expands forever
recollapses eventually

open
closed
flat
Primordial Nucleosynthesis: \( \Omega_{\text{bary},0} = 0.04 \pm 0.01 \)
Circular orbit: acceleration $g = \frac{v^2}{R} = \frac{G M(R)}{R^2}$
Andromeda galaxy M31 in X-rays and optical
Second midterm exam

- Tuesday, 8:30 a.m. - 9:45 a.m. (here)
- Prof. Ian George
- show up 5 minutes early
- bring calculator, pen, paper
- no books or other material
- formulas provided on page 3 of midterm exam
- start with ‘easy’ questions (50%)
Second midterm exam preparation

- Ryden Chapter 5 - 8 (incl.)
- look at homework (solutions)
- look at midterm #1
- check out the resources on the course web page (e.g. paper about benchmark model)
- careful with other web resources: different notation!