Laser requirements for a gravitational wave mission

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1. Introduction

This technical note contains some of the requirements for the laser that is envisaged for a future gravitaional wave mission. They should be considered an updated version of the requiremenst given for the laser for LISA and NGO.

The appendix summarizes the rationales for the different requirements to allow for bettr tracking and updating of the requirements.

2. Definitions

Throughout this technical note, the following definitions are used:

2.1. RMS and average

With t_0 denoting the start of the measurement and T its duration, the following definitions for the average and the "rms" (or standard variation) are used throughout this document.

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2.1.1. Average

The average of $ave_T x$ of a time-varying quantity x(t) over a measurement time T is given by

$$ave_{T} x = \frac{1}{T} \int_{t_{0}}^{t_{0}+T} x(t) dt$$
(1)

2.1.2. RMS

The "RMS"-value $rms_T x$ over a measurement time T of a time-varying quantity x(t) is given by

rms_T x =
$$\sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} (x(t) - ave_T x)^2 dt}$$
 (2)

The above definitions yield *estimators* for the *true* values of the average and the standard variation that are obtained in the limit of $T \rightarrow \infty$. The true values are denoted without the index T (i.e. ave x and rms x).

2.1.3. Testing

In the test of any requirement, the measurement time T must be chosen such that the *true* value y of the entity (e.g. y = rms x) is contained within an interval of $\pm 5\%$ centered on the estimated value \hat{y} (e.g. $\hat{y} = \text{rms}_T x$) with a probability of 99.73% (corresponding to 3σ in case of a gaussian distribution).

$$p(0.95\hat{y} \leqslant y \leqslant 1.05\hat{y}) \geqslant 0.9973 \tag{3}$$

2.2. Linear power spectral density

The single sided linear power spectral density (PSD) of a quantity x with unit [x] is written as $S_x(f)$ and carries the unit $[S_x(f)] = [x]/\sqrt{Hz}$.

2.2.1. Testing

In the test of any requirement based on a PSD $S_x(f)$ for a quantity x it must be ensured through a suitable choices of the estimating method and measurement time that the *true* value of $S_x(f)$ is at each frequency contained within an interval of ± 5 % of the estimated value of the PSD $\hat{S}_x(f)$, centered on the estimated value, with a probability of 99.73 % or larger (corresponding to 3σ in case of a gaussian distribution).

$$p(0.95 \hat{S}_x(f) \leq S_x(f) \leq 1.05 \hat{S}_x(f)) \ge 0.9973$$
 for all applicable frequencies f (4)

In each test, the method used for estimating $\hat{S}_x(f)$ has to be specified.

2.3. Excess noise

All the requirements in this document that are stated in terms of a maximal allowable PSD are complemented by a requirement on *excess noise*. This is necessary as PSD estimators are ill-suited to test the requirements in the case of the presence of single spectral lines or, in general, non-gaussian noise.

To limit the amount of excess noise, the estimated probability \hat{p} for a measurement $x_i = x(t_i)$ of the time varying quantity x(t) to lie *inside* the interval $[ave_T x - \delta_{pp}/2, ave_T x + \delta_{pp}/2]$ must be at least 99% for all measurement times t_i .

$$p(\operatorname{ave}_{\mathsf{T}} x - \delta_{\operatorname{pp}}/2 \leqslant x_{i} \leqslant \operatorname{ave}_{\mathsf{T}} x + \delta_{\operatorname{pp}}/2) \geqslant 99\%$$
(5)

Note that this requirement uses the *estimated* quantities that need to fulfill the requirements of (3).

If no other reasing is given, the value of δ_{pp} is calculated by evaluating the rms-noise of the quantity over the relevant frequency band and multiplying it by 6.

3. Assumptions

3.1. Proof mass interferometer

The laser power P_{PM} used in the interferomter that is used to read out the proof mass position is assumed to be

$$P_{\rm PM} = 1 \,\mathrm{mW} \tag{6}$$

throughout this technical note whenever numerical values are considered.

3.2. Signal power

The signal power P_s is the power received from the remote spacecraft and delivered to the optical bench and is taken *before* any beam splitters. Throughout this document, this power is taken to be $P_s = 270 \text{ pW}$

3.3. Local oscillator

The power of the local oscillator P_{LO} used in the interferometers is assumed to be

$$P_{LO} = 2 \,\mathrm{mW} \tag{7}$$

3.4. Modulation index

The modulation index M for the RF modulation is taken throughout this document to be

$$M = 0.671\,421 \tag{8}$$

which corresponds to a total power P_{sb} in the sidebands¹ of

$$P_{\rm sb} = 0.1 P_{\rm out} \tag{9}$$

3.5. Photodiode sensitivity

The sensitivity η of the photodiodes is assumed to be

$$\eta = 0.7 \frac{A}{W} \tag{10}$$

4. Requirements

4.1. Mode of operation

The laser must operate in continuous wave mode with a maximum intenisty noise given in section 4.9. The laser must emit a single longitudinal mode with a frequency stability as given in section 4.6 and a spectral purity as given in section 4.8.

4.2. Output power

The output power of the laser, delivered to the optical bench in the nominal spatial mode and the nominal polarisation state, shall be

$$P_{out} \ge 1.2 \, W \tag{11}$$

at the end of the nominal mission duration (2.5 years of operation).

A suitable requirement for the output power at BOL has to be derived.

4.3. Wavelength

The wavelength λ of the laser light must be

$$\lambda = (1064.6 \pm 0.8) \,\mathrm{nm} \tag{12}$$

¹This is the sum of the power in the upper and the lower sideband

4.4. Frequency tunability

The tunability of the laser frequency must be at least ± 2 GHz without any change of longitudinal mode.

4.5. Frequency controlability

The frequency controlability (or "fine tunability") is a requirement that has to be derived from the frequency stability requirements for the prestabilised laser in section 4.6.2.

4.6. Frequency stability

The requirements for the frequency stability of the laser are given for the freerunning laser and the prestabilised laser. The laser frequency is denoted by ν and is to be distinguished from the fourier frequency f. An illustration of the requirement is given in figure 1.

4.6.1. Free-running laser

The requirement for frequency stability for the free-running laser is given by

$$S_{\nu}(f) = \frac{60 \text{ kHz}}{f} \frac{\text{Hz}}{\sqrt{\text{Hz}}} \left\{ \begin{array}{l} \text{for} & 3 \times 10^{-5} \text{ Hz} \leqslant f \leqslant 1 \text{ Hz} \\ \delta_{pp}\nu = 65 \text{ MHz} \end{array} \right\} \quad \text{for} \quad 3 \times 10^{-5} \text{ Hz} \leqslant f \leqslant 1 \text{ Hz} \\ S_{\nu}(f) = \frac{30 \text{ kHz}}{f} \frac{\text{Hz}}{\sqrt{\text{Hz}}} \\ \delta_{pp}\nu = 360 \text{ kHz} \end{array} \right\} \quad \text{for} \quad 1 \text{ Hz} \leqslant f \leqslant 1 \text{ MHz}$$

$$(13)$$

4.6.2. Prestabilised laser

The requirement for the prestabilised laser is given by

$$S_{\nu}(f) = 300 \frac{\text{Hz}}{\sqrt{\text{Hz}}} \times \sqrt{1 + \left(\frac{2.8 \text{ mHz}}{\text{f}}\right)^{4}} \right\} \text{ for } 3 \times 10^{-5} \text{ Hz} \leqslant f \leqslant 1 \text{ Hz}$$

$$\delta_{pp}\nu = 50 \text{ kHz} \qquad (14)$$

$$S_{\nu}(f) = \frac{30 \text{ kHz}}{\text{f}} \frac{\text{Hz}}{\sqrt{\text{Hz}}} \left\{ \text{ for } 1 \text{ Hz} \leqslant f \leqslant 1 \text{ MHz} \right\}$$



Figure 1: Illustration of the requirement on the frequency stability of the laser.

As this requirement applies to a prestabilised laser, it implies a derived requirement for a *frequency controlability*.

4.7. Spectral linewidth

The spectral linewidth δv is *not* a primary requirement for the laser but can be derived from the required frequency stability of the *free-running* laser. Given the requirement in section 4.6, the linewidth on various timescales τ are

$$\begin{split} \tau &= 1\,s \qquad \delta\nu = 30\,kHz\\ \tau &= 1\,ms \quad \delta\nu = 1\,kHz\\ \tau &= 1\,\mu s \quad \delta\nu = 30\,Hz \end{split}$$

This requirement does not need to be individually tested and is given for information only.

4.8. Spectral purity

4.8.1. Broadband

The integrated power in a 10 nm bandwidth around the main laser line, excluding the laser line and a ± 5 GHz frequency band centered on it must be smaller than

4.8.2. Narrowband

The power excess noise $\delta_{pp}P$ in the frequency band $\pm 5 \text{ GHz}$ centered on the laser line must be smaller than

$$S_{P}(f) \leq 1.38 \frac{pw}{\sqrt{Hz}}$$

$$\delta_{pp} P \leq 8.3 \, nW$$
(16)

4.9. Output Power Stability (RIN)

The relative intensity noise (RIN) of the laser is a dimensionless number that gives the variation of the laser power in terms of the long-term average of the laser power (often referred to as the "DC"-power). The requirement for the RIN is given in equation (17), the graph in figure 2 is given for illustration purposes only.

$$\begin{split} & S_{\text{RIN}}(f) < 10^{-4} \,/ \sqrt{\text{Hz}} \\ & \delta_{\text{pp}} \text{RIN} < 0.02 \end{split} \qquad \text{for} \quad 3 \times 10^{-5} \,\text{Hz} \leqslant f \leqslant 1 \,\text{kHz} \\ & S_{\text{RIN}}(f) < 10^{-6} \,/ \sqrt{\text{Hz}} \\ & \delta_{\text{pp}} \text{RIN} < 0.01 \end{aligned} \qquad \text{for} \quad 1 \,\text{kHz} \leqslant f \leqslant 1 \,\text{MHz} \\ & \delta_{\text{pp}} \text{RIN} < 10^{-4} \qquad \text{for} \quad 1 \,\text{MHz} \leqslant f \leqslant 5 \,\text{MHz} \\ & S_{\text{RIN}}(f) < 10^{-8} \,/ \sqrt{\text{Hz}} \\ & \delta_{\text{pp}} \text{RIN} < 5 \times 10^{-4} \end{aligned} \qquad \text{for} \quad 5 \,\text{MHz} \leqslant f \leqslant 25 \,\text{MHz} \end{split}$$

4.10. RF modulation

The laser must allow a phase modulation of the laser light at (2.0 ± 0.2) GHz. The phase noise $S_{\varphi}(f)$ and the excess noise $\delta_{pp}\varphi$ in the frequency band 3×10^{-5} Hz to



Figure 2: Illustration of the RIN requirements

1 Hz must be smaller than

$$S_{\phi}(f) < 400 \times \sqrt{1 + \left(\frac{2.8 \text{ mHz}}{f}\right)^4 \frac{\mu \text{rad}}{\sqrt{\text{Hz}}}}$$
(18)
$$\delta_{\text{pp}} \phi < 66 \text{ mrad}$$

Figure 3 is given as illustration.

4.11. Polarisation

The laser light shall be linearily polarised with not more than 1% of the power in the orthogonal polarisation.

4.12. Spatial mode

The spatial mode of the laser light shall be TEM_{00} with less than 1 % of the power in higher modes.



Figure 3: Illustration of the requirement on the phase noise of the RF modulation

4.13. Spatial beam quality

The ratio of the beam parameter product of the laser to that of an ideal gaussian beam (M^2) shall fulfill

$$M^2 \leqslant 1.05 \tag{19}$$

4.14. Radiation

The laser shall withstand a total dose of ionising radiation of 50 Gy (equivalent to 5 krad).

A. Justifications

A.1. Mode of operation

Continous wave operation in a single longitudinal mode is needed for the interferometric measurement.

A.2. Output power

The requirement of the ouptput power is based on the allocated contribution of shot noise in the phase measurement noise budget of NGO and the assumed diameter D of the telescope and the efficiency ϵ of the optical chain:

$$S_{\rm SN}(f) = 25.7 \left(\frac{\lambda}{1.064\,\mu\rm m}\right)^{\frac{1}{2}} \left(\frac{0.25}{\varepsilon}\frac{1.2\,W}{P_{\rm out}}\right)^{\frac{1}{2}} \left(\frac{0.2\,m}{D}\right)^{2} \frac{\mu\rm rad}{\sqrt{\rm Hz}}$$
(20)

Note that the efficiency of the delivery system to the optical bench is included in the above *total* efficiency ϵ , so that P_{out} denotes the power *delivered* to the optical bench, in the nominal spatial mode and and the nominal polarisation state.

Note further that the above requirement for the measurement noise corresponds to a displacement noise of $4.35 \text{ pm}/\sqrt{\text{Hz}}$ at an armlength of 10^9 m as stated in the Yellow Book for NGO.

A.3. Wavelength

The wavelength is chosen from prior experience.

A.4. Frequency stability

A.4.1. Free-running laser

The requirement for the frequency stability of the free-running laser is chosen to enable the initial detection of the beat signal between two free-running lasers by the phasemeter and to allow performing a phase-lock between the two lasers with a total power detected of the order of 270 pW.

The requirement is based on previously demonstrated performance.

A.4.2. Prestabilised laser

The requirement for the frequency stability of the prestabilised laser at low frequencies is driven by the expected performance of the TDI scheme and the interspacecraft ranging system of about 1 m.

A.5. Spectral linewidth

The spectral linewidth is not a requirement but is reported for information only.

A.6. Spectral purity

A.6.1. Broadband

The requirement for the spectral purity in the $\pm 5 \text{ nm}$ wavelengthband around the laser line is due to shot-noise and dynmaical range considerations. As this light in this band could be received by the photodetectors, it is important that the contribution is small enough to not introduce unwanted noise or saturate the detectors.

A.6.2. Narrowband

The power excess noise in the frequency band of $\pm 5 \text{ GHz}$ around the laser line needs to be small enough to not affect the ranging codes and the clock transfer. As a guiding principle, the noise on the laser light in the local oscillator should not be bigger than the shotnoise caused by the RF sidebands in the received light.

$$S_{P}(f) \leqslant \sqrt{h\nu P_{sb}} \frac{P_{out}}{P_{LO}}$$

$$= 1.38 \frac{pW}{\sqrt{Hz}}$$
(21)

The excess noise is calulated by taking the rms-noise in a band of 1 MHz (corrsponding to the approximate bandpass in the read-out-electronics of the phasemeter) and multiplying it by 6.

$$\delta_{\rm pp} P = 6S_{\rm P}(f) \sqrt{\Delta f}$$

= 8.3 nW (22)

for the standard values.

A.7. Output Power Stability (RIN)

The four different frequency ranges of the RIN requirement are a result of different physical effects.

Low frequency 3×10^{-5} Hz $\leq f \leq 1$ kHz

The limiting factor is given by the permissable acceleration on the proof mass in the proof mass interferometer due to power fluctuations of the incident laser beam. The total acceleration noise for NGO is given by

$$S_{a}(f) = 3 \times 10^{-15} \frac{m}{s^{2} \sqrt{Hz}} \sqrt{1 + \left(\frac{f}{8 \, mHz}\right)^{4} \sqrt{1 + \frac{0.1 \, mHz}{f}}}$$
 (23)

Assigning a fraction ϵ_a to the contribution from radiation pressure (which is given by 2P/c, neglecting incomplete reflection at the proof mass) yields

$$S_{RIN}(f) < \epsilon_a \frac{mc}{2P} S_a(f)$$
 (24)

With $\epsilon_a = 0.25$, an assumed power level of $P = P_{PM} = 1$ mW, and a mass of the proof mass of m = 2 kg, a requirement of

$$S_{\text{RIN}}(f) < 10^{-4} \, \frac{1}{\sqrt{\text{Hz}}} \qquad \qquad \text{for} \quad 3 \times 10^{-5} \, \text{Hz} \leqslant f \leqslant 1 \, \text{kHz}$$

fulfills the condition from equation (24).



Figure 4: Chosen RIN requirement (dashed line) and permissable RIN ($\epsilon_a = 0.25$) for different power levels for the light in the proof mass interferometer. Nominal power is $P_{PM} = 1 \text{ mW}$.

Mid frequencies, low end, $1 \text{ kHz} \leq f \leq 1 \text{ MHz}$

In this frequency range, the requirement is driven by potential interferences between the RIN and the pseudo-random noise sidebands that are put on the laer for ranging pruposes.

Pending a detailed analysis of the effects, a requirement on the excess noise is

$$S_{\text{RIN}}(f) < 5 \times 10^{-4} / \sqrt{\text{Hz}} \left(\frac{1 \,\text{Hz}}{f}\right)$$
 (25)

$$\delta_{\rm pp} \rm RIN < 10^{-4}$$
 (26)

Mid frequencies, high end, $1 \text{ MHz} \leq f \leq 5 \text{ MHz}$

In this frequency range, the requirement is given by the need to not saturate the photodetectors with out-of-band noise. The peak-to-peak variation of the signal on the photodiode is given by

$$\max s(t) - \min s(t) = 4\sqrt{P_{LO}P_s}$$
(27)

The variation of the noise shall only be a small fraction $\epsilon_p \sim 0.1$ of that value, yielding the requirement for $\delta_{pp}RIN$

$$\delta_{pp}RIN = 4\epsilon_p \sqrt{\frac{P_s}{P_{LO}}}$$
(28)

$$= 1 \times 10^{-4}$$
 (29)

High frequencies $5 \text{ MHz} \leq f \leq 25 \text{ MHz}$

This frequency range coincides with the measurement band of the phasemeter, so the permissable RIN is determined by the susceptibility of the phase measurement system to amplitude modulation.

The signal photocurrent s(t) caused by the interfering light on the photodiode is given by

$$s(t) = 2\eta \sqrt{P_{LO}P_s} \sin(2\pi f(t) + \phi)$$
(30)

where P_{LO} is the power of the local oscillator (typically ~ 1 mW) and P_s) the power of the received light. The efficiency η of the photodiode carries the unit A/W.

The rms of the signal is then approximately²

$$rms s = \eta \sqrt{2P_{LO}P_s} \tag{31}$$

²The relation would be exact if the frequency f(t) was constant. As f(t) is determined by, among other things, the doppler shift, which in turn is determined by orbital mechanics, f(t) is only approximately constant over a time interval long enough to calculate a meaningful rms value.

The noise has three main contributors, the shotnoise $S_{SN}(f)$, the electronical noise $S_{el}(f)$ mainly coming from the photo diodes' pre-amplifiers, and the (technical) amplitude noise of the laser $S_{RIN}(f)$ which has a different power scaling than the shotnoise and is therefore counted seperately. The three different terms are, in order,

$$S_{\rm SN}(f) = \sqrt{2e\eta P_{\rm LO}} \tag{32}$$

$$S_{el}(f) = S_{pa}(f)\sqrt{8}$$
(33)

$$S_{AM}(f) = \eta P_{LO} S_{RIN}(f)$$
(34)

The pre-amplifier noise $S_{pa}(f)$ varies between $2 pA/\sqrt{Hz}$ and $4 pA/\sqrt{Hz}$, depending on the chosen components; the factor $\sqrt{8}$ counts the independent contributions of the different photodiodes.

The total photo current noise $S_{pc}(f)$ is the quadratic sum of the three individual contributions and depends on the power of the local oscillator.

$$S_{pc}(f) = \sqrt{S_{SN}(f)^2 + S_{el}(f)^2 + S_{AM}(f)^2}$$
(35)

$$= \sqrt{2e\eta P_{\rm LO} + 8S_{\rm pa}(f)^2 + \eta^2 P_{\rm LO}^2 S_{\rm RIN}(f)^2}$$
(36)

The corresponding singe-link *displacement* noise is the photocurrent noise scaled by the size of the signal and the inverse wavenumber

$$S_{\delta x}(f) = \frac{\lambda}{2\pi} \frac{S_{pc}(f)}{rms \, s} \leqslant 12 \, pm / \sqrt{Hz} = S_{req}(f) \tag{37}$$

by requirement. The contribution of the shotnoise alone to the displacement noise is independent of the power of the local oscillator

$$S_{\delta x,SN}(f) = \frac{\lambda}{2\pi} \frac{S_{SN}(f)}{rms s}$$
(38)

$$=\frac{\lambda}{2\pi}\sqrt{\frac{e}{\eta P_s}}$$
 $\approx 4.93 \,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ for the standard values (39)

As the different noise sources scale differently with the locals oscillator power, signal-to-noise ratio (SNR) can be optimised with respect to the locals oscillator power

$$SNR = \frac{rms \, s}{S_{pc}(f)} = \sqrt{\frac{2\eta^2 P_{LO} P_s}{2e\eta P_{LO} + 8S_{pa}(f)^2 + \eta^2 P_{LO}^2 S_{RIN}(f)^2}}$$
(40)

which yields an optimal power of the local oscillator which in turn balances the two other noise terms $S_{el}(f)$ and $S_{AM}(f)$.

$$P_{LO,opt} = \frac{\sqrt{8}}{\eta} \frac{S_{pa}(f)}{S_{RIN}(f)}$$
(41)

$$S_{el}(f)|_{P_{LO}=P_{LO,opt}} = S_{AM}(f)|_{P_{LO}=P_{LO,opt}} = \sqrt{8}S_{pa}(f)$$
(42)

Allowing the two noise terms to contribute $\epsilon = 25\%$ to the remaining noise allocation, i.e.

$$S_{pc}(f)^{2} = S_{SN}(f)^{2} + \epsilon \left(S_{req}(f)^{2} - S_{SN}(f)^{2}\right)$$
(43)

$$= (1 - \epsilon) S_{SN}(f)^2 + \epsilon S_{req}(f)^2$$
(44)

$$S_{el}(f)^2 + S_{AM}(f)^2 = \epsilon \left(S_{req}(f)^2 - S_{SN}(f)^2 \right)$$
(45)

or

$$S_{AM}(f) = \sqrt{\frac{\epsilon}{2} \left(S_{req}(f)^2 - S_{SN}(f)^2 \right)}$$
(46)

$$= 3.87 \,\mathrm{pm}/\sqrt{\mathrm{Hz}} \tag{47}$$

gives the power of the local oscillator in terms of the photodiode noise

$$S_{el}(f) = S_{AM}(f) = \sqrt{8} S_{pa}(f) \leqslant \frac{2\pi \operatorname{rms} s}{\lambda} 3.87 \, \mathrm{pm}/\sqrt{\mathrm{Hz}}$$
(48)

$$=\frac{2\pi\eta\sqrt{2P_{\rm LO}P_{\rm s}}}{\lambda}\,3.87\,\rm{pm}/\sqrt{\rm{Hz}}\tag{49}$$

$$P_{LO} = \frac{1}{P_{S}} \left(\frac{\lambda}{\eta \pi}\right)^{2} \left(\frac{S_{pa}(f)}{3.87 \, \text{pm}/\sqrt{\text{Hz}}}\right)^{2}$$
(50)

> 1 mW
$$\left(\frac{0.7 \text{ A/W}}{\eta}\right)^2 \left(\frac{S_{pa}(f)}{4 \text{ pA}/\sqrt{\text{Hz}}}\right)^2$$
 (51)

Using (41) this leads to a RIN of

$$S_{\text{RIN}}(f) \leq 1.6 \times 10^{-8} \,/ \sqrt{\text{Hz}} \left(\frac{\eta}{0.7 \,\text{A/W}}\right)^2 \left(\frac{4 \,\text{pA}/\sqrt{\text{Hz}}}{S_{\text{pa}}(f)}\right)^2 \tag{52}$$

A.8. Polarisation

The polarisation requirement is set so that only a small amount of light is in the "wrong" polarisation. The final polarisation will be set on the optical bench by high-perfomance polarisers

A.9. Spatial mode

The requirement for the spatial mode is set so that the wavefront of the laser light is well defined. Light in higher spatial modes would distort the wavefront of the laser beam. A detailed analysis of this requirement is pending.



Figure 5: Total dose for different thicknesses of Al shield. From data in (LISA-ASU-TN-4013).

A.10. Spatial beam quality

Similarly to section 4.12, this requirement is set to avoid waveform distortion. A detailed analysis of this requirement is pending.

A.11. Radiation

The requirement for the total radiation dose is taken from the radiation analysis for LISA (LISA-ASU-TN-4013), scaled down from 6.5 years to 3.5 years (2 years of mission duration and 1.5 years of transfer), assuming a aluminium shielding of 3 mm for the laser. The fit to the original data yields for the radiation dose γ

$$\gamma = 2.14 \,\mathrm{krad}\left(\frac{1\,\mathrm{mm}}{\mathrm{d}}\right)\left(\frac{\mathrm{T}}{1\,\mathrm{a}}\right) \tag{53}$$

with the thinkness of the aluminium shield d and the total duration T.

To obtain the requirement, a safety factor of 2 has been applied over equation (53). Figure 5 is given as illustration.

References

LISA-ASU-TN-4013. LISA Radiation Analysis. Tech. rep.