# Tales of Hierarchical Three-body 

## Systems

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## HIERARCHICAL THREE-BODY SYSTEMS

- Configuration:



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$$
r_{I} \ll r_{2}
$$

- Hierarchical configurations are COMMON:
- For binaries with period $<3$ days, $\geq 96 \%$ are in systems with multiplicity $\geq 3$. (Tokovinin et al. 2006)
- 282 of the 299 triple systems ( $-94.3 \%$ ) are hierarchical. (Eggleton et al. 2007)
- Hierarchical 3-body dynamics gives insight for hierarchical multiple systems formation/evolution.


## OUTLINE

- Dynamical properties:
- Flips of inner binary
- Eccentricity excitation of the inner binary
- Examples:
- Formation of misaligned hot Jupiters
- Enhancement of tidal disruption rates for stars in galactic nuclei


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- Inner wires $(\mathrm{I})$ : formed by $\mathrm{m}_{\mathrm{I}}$ and $\mathrm{m}_{\mathrm{J}}$.

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Outer wires (2): $\mathrm{m}_{2}$ orbits the center mass of $\mathrm{m}_{\mathrm{I}}$ and $\mathrm{m}_{\mathrm{J}}$.
$a_{1} / a_{2}$ small, expand $H$ in $a_{1} / a_{2}$ and apply perturbative analysis

## LIDOV-KOZAI MECHANISM

## Lidov-Kozai Mechanism

$$
\left(\mathrm{e}_{2}=\mathrm{o}, \mathrm{~m}_{\mathrm{J}} \rightarrow \mathrm{o}\right)
$$

(Kozai 1962; Lidov 1962:
Solar system objects)

- Octupole level $\mathrm{O}\left(\left(\mathrm{a}_{1} / a_{2}\right)^{3}\right)$ is zero.
- Quadrupole level O(( $\left.\left.\mathrm{a}_{1} / a_{2}\right) 2\right)$ :


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\begin{aligned}
\Rightarrow J z= & \sqrt{1-e_{1}^{2}} \cos i_{1} \text { conserved } \\
& \text { (axi-symmetric potential). }
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$e \uparrow, i \downarrow$
$i$ does not cross $90^{\circ}$

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\Rightarrow J z=\sqrt{1-e_{1}^{2}} \cos i_{1} \text { conserved }
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(axi-symmetric potential).
$\Rightarrow>$ when $\mathrm{i}>40^{\circ}, \mathrm{e}_{\mathrm{I}}$ and i oscillate with large amplitude.

Example of Lidov-Kozai Mechanism.

## OCTUPOLE LIDOV-KOZAI MECHANISM

## $e_{2} \neq 0$ (Eccentric Lidov-Kozai

Mechanism) or $\mathrm{m}_{\mathrm{J}} \neq \mathrm{o}$ :
(e.g., Naoz et al. 201I, 2013, test particle case: Katz et al. 201I, Lithwick of Naoz 201I ):

- Jz NOT constant, octupole $\neq 0$.


Cyan: quadrupole only.
Red: quadrupole + octupole. Naoz et al 2013

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- Consequence:
- Tidal disruption rate enhancement ( $e_{1} \rightarrow 1$ )

(e.g., Chen et al. 2009, Bode \& Wegg 2014, Li et al. 2015)

$$
R_{p} \propto 1-e_{1}
$$

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Cyan: quadrupole only.
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$$
40^{\circ}<i<140^{\circ}
$$

## COPLANAR FLIP

- Starting with $i \approx 0$, $e_{1} \geq 0.6, e_{2} \neq \mathrm{O}$ :
$e_{1} \rightarrow \mathrm{I}, i$ flips by $\approx 18 \mathrm{o}^{\circ}$
(Lietal. 2014a).

(Li et al. 20I4a)


## COPLANAR FLIP

- Starting with $i \approx 0$, $e_{1} \geq 0.6, e_{2} \neq 0$ :
$e_{1} \rightarrow \mathrm{I}, i$ flips by $\approx 180^{\circ}$
(Liet al. 2014a).



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(Liet al. 2014a).

=> Increase the parameter space of interesting behaviors.
=> Produces counter orbiting hot Jupiters.
=> Enhance tidal disruption rates.



## DIFFERENCES BETWEEN HIIGH/LOW I FLIP

- Low inclination flip

- For simplicity: take $\mathrm{m}_{\mathrm{j}} \rightarrow \mathrm{O}=>$ outer orbit stationary.
- $z$ direction: angular momentum of the outer orbit.
- $\uparrow$ : direction of $\mathrm{J}_{\mathrm{r}}$.
- $\uparrow: \mathrm{Jz}_{\mathrm{I}}=>$ indicates flip.
- Colored ring: inner orbit. Color: mean anomaly.


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## ANALYTICAL OVERVIEW

- Hamiltonian has two degrees of freedom in test particle limit:

$$
\begin{aligned}
& \left(J=\sqrt{1-e_{1}^{2}}, J z=\sqrt{1-e_{1}^{2}} \cos i_{1}, \omega, \Omega\right) \\
& 2 \text { conjugate pairs: } \mathrm{J} \& \omega, \mathrm{~J} z \& \Omega
\end{aligned}
$$

- The Hamiltonian up to the Octupole order:

$$
H=F_{\text {quad }}(J, J z, \omega)+\epsilon F_{\text {oct }}(J, J z, \omega, \Omega)
$$

Quadrupole order: Independent of $\Omega$
=> Jz constant
$\epsilon$ : hierarchical parameter:
$\epsilon=\frac{a_{1}}{a_{2}} \frac{e_{2}}{1-e_{2}^{2}}$

Octupole order: Depend on both $\Omega \& \omega=>\mathrm{J}$ and Jz not constant

## CO-PLANAR FLIP CRITERION

- Hamiltonian (at $\mathrm{O}(i))$ :
- Evolution of $e_{l}$ only due to octupole terms:
$=>e_{1}$ does not oscillate before flip
- Depend on only $J_{I}$ and $\varpi_{I}=\omega_{\mathrm{I}}+\Omega_{\mathrm{I}}$

> => System is integrable.
$=>e_{I}(\mathrm{t})$ can be solved.
=> The flip timescale can be derived.
=> The flip criterion can be derived.

$$
\varepsilon>\frac{8}{5} \frac{1-e_{1}^{2}}{7-e_{1}\left(4+3 e_{1}^{2}\right) \cos \left(\omega_{1}+\Omega_{1}\right)}
$$

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=> System is integrable.
$\Rightarrow e_{l}(\mathrm{t})$ can be solved.
=> The flip timescale can be derived.
=> The flip criterion can be derived.
Easier to flip:
* $e_{1}$ larger
* $\bar{\varpi}_{1}=\omega_{1}+\Omega_{1} \sim 180^{\circ}$



## ANALYTICAL RESULTS V.S. NUMERICAL RESULTS



- The flip criterion and the flip timescale from secular integration are consistent with the analytical results.


## SURFACE OF SECTIONS

Coplanar Flip:

(Gongjie Li et al. 2014b)

High inclination Flip:


Quadrupole resonances
(e.g., Kozai 1962)

Caused by the octupole resonance, Regular ( $\varpi$ librates around $\pi$ )

Caused by the overlap of quadrupole and octupole resonances, Chaotic: $\mathrm{t}_{\mathrm{L}}-6 \mathrm{t}_{\mathrm{K}}$

Examples --- I. Formation of Misaligned Hot Jupiters via Lidov-Kozai Oscillations

Credit: ESA/C. Carreau

Mass - Period Distribution


## Mass - Period Distribution



## SPIN-ORBIT MISALIGNMENT (ROSSITER-MCLAUGHLIN METHOD)

Stellar Spin


## OBSERVED SPIN-ORBIT MISALIGNMENT

Solar System: misalignment $\Psi \leqslant 70$


## OBSERVED SPIN-ORBIT MISALIGNMENT

Solar System: misalignment $\Psi \leq 7^{\circ}$


## FORMATION OF COUNTER ORBITING HOT

 JUPITERS (LK + TIDE)
## Coplanar Flip

## FORMATION OF COUNTER ORBITING HOT JUPITERS (LK + TIDE)


$e_{I} \rightarrow 1$ during the flip
$\Rightarrow \mathrm{r}_{\mathrm{p}} \downarrow$, tide dominates. $\quad \Rightarrow e_{I} \rightarrow \mathrm{O}, a_{I} \downarrow, i, \psi \approx 180^{\circ}$.

## FORMATION OF COUNTER ORBITING HOT JUPITERS (LKK + TIDE)



May produce tidal disruption events

## DIFFICULTY $\mathbb{N}$ THE FORMATION OF COUNTERORBITING HOT JUPITERS

Including short range forces, a small fraction survive and produce retrograde planets


Xue \& Suto 2016, Xue et al. 2017

## DIFFICULTY $\mathbb{N}$ THE FORMATION OF COUNTERORBITING HOT JUPITERS

Flip condition (with no short range forces) is also a good approximation for migration condition


Xue \& Suto 2016, Xue et al. 2017

## FORMATION OF MISALIGNED HOT JUPITERS

 (LK + TIIDE) BY POPULATION SYNTHESIS

- $15 \%$ of systems produce hot Jupiters
- ELK may account for about $30 \%$ of hot Jupiters (Naoz et al. 20II)


## FORMATION OF MISALIGNED HOT JUPITERS (LK + TIDE) BY POPULATION SYNTHESIS



Population synthesis study of interaction of two giant planets.
=> a different
mechanism is needed
(Petrovich 2015)

## FORMATION OF MISALIGNED HOT JUPITERS (LK + TIDE) BY POPULATION SYNTHESIS



Population synthesis study of interaction of two giant planets.
=> a different mechanism is needed (Petrovich 2015)

LK produces $\sim 20 \%$ of the observed HJs

## FORMATION OF HOT JUPITERS OBSERVATIONAL EVIDENCES



16 Cygni Bb: e $=0.67$

## FORMATION OF HOT JUPITERS




16 Cygni $\mathrm{Bb}: \mathrm{e}=0.67$, can be produced by Lidov-Kozai mechanism

Holman et al. 1997

## FORMATION OF HOT JUPITERS



Each grid square $=0.1 \mathrm{AU} \times 0.1 \mathrm{AU}$
Planet and star not drawn to scale
Naef et al. 2001

## FORMATION OF HOT JUPITERS



Pont et al. 2009

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## FORMATION OF HOT JUPITERS



Pont et al. 2009

HD80606b: $\mathrm{e}=0.93$, can be produced by Lidov-Kozai mechanism

Wu \& Murray 2003
Each grid square $=0.1 \mathrm{AU} \times 0.1 \mathrm{AU}$
Planet and star not drawn to scale
Naef et al. 2001

## FRIENDS OF HOT JUPITERS

Existence an outer companion?

or


LK not dominate
Knutson et al. 2014

## FRIENDS OF HOT JUPITERS

$47 \% \pm 7 \%$ of hot Jupiter have stellar companions with $a$ b.t. 50-200o AU based on 77 transiting hot Jupiters

Ngo et al. 2016

$<16 \% \pm 5 \%$ systems formed via Lidov-Kozai oscillations

## FRIENDS OF HOT JUPITERS



No correlation between misaligned/eccentric hot Jupiter systems and the incidence of stellar companions based on 27 misaligned/eccentric HJs

# EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB 



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- SMBHBs originate from mergers between galaxies.

- SMBHBs with mostly -kpc separation have been observed with direct imagine.
(e.g., Woo et al. 2014; Komossa et al. 2013, Fabbiano et al. 201ı, Green et al. 2010, Civano et al. 20io, Rodriguez et al. 2006, Komossa et al. 2003, Hutchings \& Neff 1989)

Multicolor image of NGC 6240. Red p
 5 keV ), and blue p hard ( $5-8 \mathrm{keV}$ ) X-ray band. (Komossa et al. 2003)

## STARS SURROUNDING SMBHB

- At -Ipc separation it is more difficult to identify SMBHBs. SMBHBs can be observed with photometric and spectral features.
(e.g., Shen et al. 2013, Boroson \& Lauer 2009, Valtonen et al. 2008, Loeb 2007)

Example of multi-epoch spectroscopy (Shen et al. 2013):


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(e.g., Shen et al. 2013, Boroson \& Lauer 2009, Valtonen et al. 2008, Loeb 2007)
- Identify SMBHB at -I pe separation by stellar features due to $^{\text {p }}$ interactions with SMBHB.
(e.g., Chen et al. 2009, 20II, Wegg \& Bode 201ı, Li et al. 2015)


## PERTURBATIONS ON STARS SURROUNDING SMBHB

- Identify SMBHB at -I pc separation by stellar features due to interactions with SMBHB.
(e.g., Chen et al. 2009, 2011, Wegg \& Bode 201r, Li et al. 2015)



RATES

$e_{I}$ max determines the closest distance:
$\mathrm{r}_{\mathrm{p}} \propto\left(\mathrm{I}-\mathrm{e}_{\mathrm{I}}\right)$
$t_{K}=\frac{8}{3} P_{i n} \frac{m_{1}}{m_{2}}\left(\frac{a_{2}}{a_{1}}\right)^{3}\left(1-e_{2}^{2}\right)^{3 / 2}$
$e_{\text {max }}$ reaches $\mathrm{I}^{-1 \mathrm{IO}^{-6}}$ over $-30 \mathrm{t}_{\mathrm{K}}$ ( -Myrs )

Starting at $a-\mathrm{IO}^{6} \mathrm{R}_{\mathrm{t}}$, it's still possible to be disrupted in $\sim 30 \mathrm{t}_{\mathrm{K}}$ !

## SUPPRESSION OF ELK

- Eccentricity excitation suppressed when precession timescale < Kozai timescale.


$$
\mathrm{m}_{\circ}=107 \mathrm{M}_{\odot}, \mathrm{m}_{2}=109 \mathrm{M}_{\odot}, \mathrm{e}_{\mathrm{I}}=2 / 3, \mathrm{a}_{2}=0.3 \mathrm{pc}, \mathrm{~m}_{\mathrm{I}}=\mathrm{I} \mathrm{M}_{\odot}, \mathrm{e}_{2}=0.7
$$

## SUPPRESSION OF ELK

- Eccentricity excitation suppressed when precession timescale < Kozai timescale. $\mathrm{m}_{0}=107 \mathrm{M}_{\odot}, \mathrm{m}_{2}=10{ }^{9} \mathrm{M} \odot$

$e_{1}=2 / 3, a_{2}=0.3 \mathrm{pc}, \mathrm{m}_{\mathrm{I}}=\mathrm{I} \mathrm{M}_{\odot}, e_{2}=0.7$.
(Liet al. 2015)


## EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

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- Kozai affects more stars when perturbing more massive SMBH.


## SUPPRESSION OF ELK



## EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

- 57/1000 disrupted; 726/1000 scattered.

- Example: $m_{1}=10^{7} \mathrm{M}_{\odot}, m_{2}=10^{8} \mathrm{M} \odot, a_{2}$ $=0.5 \mathrm{pc}, e_{2}=0.5$, Run time: 1 Gyr .


## EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

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=> Disruption rate can reach $\sim 10^{-3} / \mathrm{yr}$.
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## EFFECTS ON STARS SURROUNDING $A \mathbb{N} \mathbb{N} M B H I \mathbb{N}$ GC

- Example: $m_{1}=10^{4} \mathrm{M}_{\bullet}, m_{2}=4 \times 10^{6} \mathrm{M}_{\odot}, a_{2}=0.1 \mathrm{pc}, e_{2}=0.7$ (Run time: 100 Myr)


## EFFECTS ON STARS SURROUNDING AN IMBH IN GC

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- 40/1000 disrupted; 500/1000 $=>\sim 50 \%$ stars survived. scattered.
$=>$ Disruption rate can reach $\sim 10^{-4} / \mathrm{yr}$.


## EFFECTS ON STARS SURROUNDING AN IMBH IN GC

- Example: $m_{l}=10^{4} \mathrm{M}_{\odot}, m_{2}=4 \times 10^{6} \mathrm{M}_{\odot}, a_{2}=0.1 \mathrm{pc}, e_{2}=0.7, \alpha=1.75$ (Run time: 100 Myr )



## CONCLUSION

- Perturbation of the outer object can produce flips of the inner orbit and excite inner orbit eccentricity

O Under tidal dissipation, the perturbation of a farther companion can produce misaligned hot Jupiters

O Perturbation of a SMBH may enhance the tidal disruption rate of stars.

## THANK YOU!

## Systematic Study of the Parameter Space

- Identify the resonances and the chaotic region.
- Characterize the parameter space that give rise to the interesting behaviors --- eccentricity excitation and orbital flips.


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## EFFECTS OF EKM ON STARS SURROUNDING BBH

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## SUPPRESSION OF ELK



## ROSSITER-MCLAUGHLIN METHOD (SPIN-ORBITT MISALIGNMENT)



## ROSSITER-MCLAUGHLIN METHOD (SPIN-ORBITT MISALIGNMENT)



## DIFFERENCES BETWEEN HIGH/LOW I FLIP

Low inclination flip


High inclination flip


Low inclination flips:
$e_{\mathrm{I}} \uparrow$ monotonically, inclination stays low before flip.
Flip occurs faster.

## Resonances and Chaotic Regions

- The Hamiltonian $\mathrm{H}_{\text {res }}$ takes form of a pendulum.
- Two dynamical regions: libration region and circulation region.




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Overlap of resonances can cause chaos


Separatrix
Circulation

## Surface of Section

Example of a 2-degree freedom $\mathrm{H}(\mathrm{J}, \omega, \mathrm{Jz}, \Omega)$

(Li et al. 20I4b)

- Resonant zones: points fill I -D lines. trajectories are quasi-periodic.
- Chaotic zones: points fill a higher dimension.


## Surface of Section

- Surface of section of hierarchical three-body problem in the test particle limit in the $J-\omega$ Plane.
- $J=\sqrt{1-e_{1}^{2}}$ (specific angular momentum);
$\omega$ : argument of periapsis


Li et al. 20I4b

## Surface of Section

Resonances exist for all surfaces:
Low i
High i ( $40-60^{\circ}$ )


Quadrupole order dominates

Octupole order stronger




Quadrupole resonances:
centers at low $\mathrm{e}_{\mathrm{I}}, \omega=\pi / 2$ and $3 \pi / 2$ (e.g. Kozai 1962)
Octupole resonances:
centers at high $e_{1}, \omega=\pi$ or $\pi / 2$ and $3 \pi / 2$

## Surface of Section



High i ( $40^{-60^{\circ}}$ )


- $e_{1}$ excitation $(J \rightarrow 0)$ are caused by octupole resonances.
- Near coplanar flip due to octupole resonances alone.
- High inclination flip due to both quadrupole and octupole order resonances.


## Summary

- Hierarchical Three Body Dynamics:
- Starting with near coplanar configuration, the inner orbit of a hierarchical $3^{-b o d y}$ system can flip by $\sim 180^{\circ}$, and $\mathrm{e}_{\mathrm{I}} \rightarrow \mathrm{I}$.
- This mechanism is regular, and the flip criterion and timescale can be expressed analytically.
- This mechanism can produce counter orbiting hot exoplanets, and can enhance collision/tidal disruption rate.
- Underlying resonances:
- Flips and $e_{1}$ excitations are caused by octupole resonances.
- High inclination flips are chaotic, with Lyapunov timescale - $6 \mathrm{t}_{\mathrm{K}}$.


## Summary

- Coplanar flip:
- Starting with near coplanar configuration, the inner orbit of a hierarchical $3^{-b o d y}$ system can flip by $\sim 180^{\circ}$, and $\mathrm{e}_{\mathrm{I}} \rightarrow \mathrm{I}$.
- This mechanism is regular, and the flip criterion and timescale can be expressed analytically.
- This mechanism can produce counter orbiting hot exoplanets, and can enhance collision/tidal disruption rate.
- Characterization of parameter space:
- Near coplanar flip and $e_{1}$ excitations are caused by octupole resonances.
- High inclination flips are chaotic, with Lyapunov timescale $-6 t_{\text {K }}$.


## Potential Applications

- Captured stars in BBH systems may affect stellar distribution around the BHs (e.g., Ann-Marie Madigan, Smadar Naoz, Ryan O'Leary).
- Tidal disruption and collision events for planetary systems (e.g., Eugene Chiang, Bekki Dawson, Smadar Naoz).
- Production of supernova (e.g., Rodrigo Fernandez, Boaz Katz, Todd Thompson).
- Other aspects:
- Involving more bodies (e.g., Smadar Naoz, Todd Thompson).
- Obliquity variation of planets.


## COHJ Contradict with popular Planets' Formation Theory

- Formation Theory:

- Planet systems form from cloud contraction.
- Spin of the star ends up aligned with the orbit of the planets


## Analytical Overview --- Test Particle Limit

- Hamiltonian has two degrees of freedom:
isolated 3-body: 6 dof $\xrightarrow{\text { secular }} 4$ dof $\xrightarrow{\text { test-particle }} 2$ dof
2 conjugate pairs: J \& $\omega, \mathrm{Jz} \& \Omega$

$$
\left(J=\sqrt{1-e_{1}^{2}}, J z=\sqrt{1-e_{1}^{2}} \cos i_{1}\right)
$$

Pericenter
$\omega$ : orientation in orbital plane.
$\Omega$ : orientation in reference plane.


## Analytical Overview

- Hamiltonian (Harrington 1968, 1969; Ford et al., 2000):
- In the octupole order: $\mathrm{H}=-\mathrm{F}_{\text {quad }}-\varepsilon \mathrm{F}_{\text {oct }}, \varepsilon=\left(\mathrm{a}_{\mathrm{I}} / \mathrm{a}_{2}\right) \mathrm{e}_{2} /\left(\mathrm{I}-\mathrm{e}_{2}{ }^{2}\right)$

$$
\begin{aligned}
F_{\text {quad }} & =-\left(e_{1}^{2} / 2\right)+\theta^{2}+3 / 2 e_{1}^{2} \theta^{2} \\
& +5 / 2 e_{1}^{2}\left(1-\theta^{2}\right) \cos \left(2 \omega_{1}\right), \\
F_{\text {oct }} & =\frac{5}{16}\left(e_{1}+\left(3 e_{1}^{3}\right) / 4\right) \\
& \times\left(\left(1-11 \theta-5 \theta^{2}+15 \theta^{3}\right) \cos \left(\omega_{1}-\Omega_{1}\right)\right. \\
& \left.+\left(1+11 \theta-5 \theta^{2}-15 \theta^{3}\right) \cos \left(\omega_{1}+\Omega_{1}\right)\right) \\
& -\frac{175}{64} e_{1}^{3}\left(( 1 - \theta - \theta ^ { 2 } + \theta ^ { 3 } ) \operatorname { c o s } \left(3 \omega_{1}-\Omega_{1}\right.\right. \\
& \left.+\left(1+\theta-\theta^{2}-\theta^{3}\right) \cos \left(3 \omega_{1}+\Omega_{1}\right)\right),
\end{aligned}
$$

- Independent of $\Omega_{\mathrm{t}}, \mathrm{J}_{\mathrm{z}}$ const.
- Depend on both $\omega_{\mathrm{I}}$ and $\Omega_{\mathrm{I}}$ $\rightarrow$ both J and $\mathrm{J}_{z}$ are not const.

$$
t_{K}=\frac{8}{3} P_{i n} \frac{m_{1}}{m_{2}}\left(\frac{a_{2}}{a_{1}}\right)^{3}\left(1-e_{2}^{2}\right)^{3 / 2}
$$

## Analytical Derivation for Flip Criterion and Timescale

- Hamiltonian (at O(i)):
- Evolution of $\mathrm{e}_{\mathrm{r}}$ only due to octupole terms:
$\Rightarrow e_{I}$ does not oscillate before flip.
- Depend on only $J_{\mathrm{I}}$ and $\varpi_{\mathrm{I}}=\omega_{\mathrm{I}}+\Omega_{\mathrm{I}}$
=> System is integrable.
$\Rightarrow e_{I}(t)$ can be solved.
- Flip at $\mathrm{e}_{\mathrm{I}, \max }-\mathrm{I}$
$\Rightarrow$ The flip timescale can be derived.
- Flip when $\varpi_{\mathrm{r}}=180^{\circ}$
=> The flip criterion can be derived.

$$
\varepsilon>\frac{8}{5} \frac{1-e_{1}^{2}}{7-e_{1}\left(4+3 e_{1}^{2}\right) \cos \left(\omega_{1}+\Omega_{1}\right)}
$$

## Analytical Overview

- Hamiltonian has two degrees of freedom:

$$
\left(J=\sqrt{1-e_{1}^{2}}, J z=\sqrt{1-e_{1}^{2}} \cos i_{1}, \omega, \Omega\right)
$$

2 conjugate pairs: J \& $\omega, J z \& \Omega$

- Hamiltonian (Harrington 1968, 1969; Ford et al. 2000): In the octupole order:

Interaction Energy (H) of two orbital wires:

$$
H=F_{q u a d}(J, J z, \omega)+\epsilon F_{o c t}(J, J z, \omega, \Omega)
$$

Quadrupole order: Independent of $\Omega$
$\Rightarrow J z$ constant
$\epsilon$ : hierarchical parameter:
$\epsilon=\frac{a_{1}}{a_{2}} \frac{e_{2}}{1-e_{2}^{2}}$

Octupole order: Depend on both $\Omega \& \omega=>\mathrm{J}$ and Jz not constant

## Analytical Der ar

- Hamiltonian (at $O(i))$ depend on only $e_{1}$ and $\varpi_{1}=\omega_{1}+\Omega_{1}$ :
- Evolution of $\mathrm{e}_{\mathrm{r}}$ only due to octupole terms:

$$
\dot{e}_{1}=\frac{5}{8} J_{1}\left(3 J_{1}^{2}-7\right) \varepsilon \sin \left(\varpi_{1}\right) \quad \dot{\varpi}_{1}=J_{1}\left(2+\frac{5\left(9 J_{1}^{2}-13\right) \varepsilon \cos \left(\varpi_{1}\right)}{\sqrt{1-J_{1}^{2}}}\right)
$$

- $\mathrm{e}_{\mathrm{I}}(\mathrm{t})$ can be solved $=>$

The flip criterion and the flip timescale can be derived:

$$
\varepsilon>\frac{8}{5} \frac{1-e_{1}^{2}}{7-e_{1}\left(4+3 e_{1}^{2}\right) \cos \left(\omega_{1}+\Omega_{1}\right)}
$$

## FLIIP CRITERION

- Averaging the quadrupole oscillations in limit $j_{z} \sim 0$, Katz et al. 2011 obtain the constant:

$$
f\left(C_{K L}\right)+\epsilon \frac{\cos i_{\text {tot }} \sin \Omega_{1} \sin \omega_{1}-\cos \omega_{1} \cos \Omega_{1}}{\sqrt{1-\sin ^{2}{ }_{1 \text { tot }} \sin ^{2} \omega_{1}}}
$$



Requiring $j_{z}=0$, during the flip:


## Analytical Results v.s. Numerical Results



## Why do analytical results with low inclination approximation work?

$I C: m_{I}={ }_{I} M_{\odot}, m_{2}=0 . I M_{\odot}, a_{I}=I A U, a_{2}=$ $45.7 A U, \omega_{I}=0^{\circ}, \Omega_{I}=180^{\circ}, i_{I}=5^{\circ}$.

## Analytical Results v.s. Numerical Results

## Why do analytical results with low inclination approximation work?



## Small inclination assumption holds for most of the evolution.



Li, et al., 2013

## Examples --- I. Produce Counter Orbiting Hot Jupiters (+ tide)

## Question:

Does this
mechanism produce a peak at $\psi \approx 180^{\circ}$ ?

## No.



## Examples --- ı. Produce Counter

 Orbiting Hot Jupiters (+ tide)Question:
Will planet be tidally disrupted?


Li et al., 20I4a

Applications --- r. Produce Counter Orbiting Hot Jupiters (+ tide)

- Hot Jupiters:
- massive exoplanets ( $m \geq m_{J}$ ) with close-in orbits (period: $\mathrm{I}^{-4}$ day).
- Counter Orbiting Hot Jupiters:
- Hot Jupiters that orbit in exactly the opposite direction to the spin of their host star.
- Disagree with the classical planet formation theory: the orbit aligns with the stellar spin.


## Rossiter-McLaughlin Method


http://www.subarutelescope.org/

# FORMATION OF MISALIGNED HOT JUPITERS 

## (LK + STELLAR OBLATENESS + TIDE)

Anderson et al. 2016:
$\mathrm{Mp}<3 \mathrm{M}_{\mathrm{J}}$ => bimodal
$\mathrm{Mp} \sim 5 \mathrm{M}_{\mathrm{J}}$
=> low misalignment (solar-type stars)
=> higher misalignment (more massive
 stars)

## FORMATION OF MISALIGNED HOT JUPITERS (LK + STELLAR OBLATENESS + TIDE)

If the host star is spinning and oblate, gravity from the planet makes stellar spin precess around L , and can cause chaos under Lidov-Kozai oscillations (Storch \& Lai 2015).


Storch \& Lai 2015
Chaos: precession period $\sim$ Lidov-Kozai oscillation period

## Take Home Message

- Eccentric Coplanar Kozai Mechanism can flip an eccentric coplanar inner orbit to produce counter orbiting exoplanets


Eccentric inner orbit flips due to eccentric coplanar outer companion


## Observational Links to Counter Orbiting Hot Jupiters

- Distribution of sky projected spin-orbit angle ( $\lambda$ ) of Hot Jupiters



There are retrograde hot jupiters $\left(\lambda>90^{\circ}\right)$

It is possible to have counter orbiting planets.

## Applications --- 2. Effects of EKM of Stars Surrounding BBH

- Tidal disruption rate is highly uncertain:
- It is observed to be $10-5-4 / \mathrm{galaxy} / \mathrm{yr}$ from a very small sample by Gezari et al. 2008.
- It roughly agrees with theoretical estimates. (e.g. Wang \& Merritt 2004)
- The disruption rate may be greatly enhanced:
- due to non-axial symmetric stellar potential. (Merritt \& Poon 2004)
- due to SMBHB (Ivanov et al. 2005, Wegg \& Bode 201r, Chen et al. 20II)
- due to recoiled SMBHB (Stone \& Loeb 201I)


## Examples --- 3. Effects of EKM of Stars Surrounding BBH

- Example: $m_{1}=10^{7} \mathrm{M} \odot, m_{2}=10^{8} \mathrm{M} \odot, a_{2}=0.5 \mathrm{pc}, e_{2}=0.5, \alpha=1.75$ (stellar distribution), normalized by $\mathrm{M}-\sigma$ relation. Run time: 1 Gyr .





(Li, et al.


## Examples --- 3. Effects of EKM of Stars Surrounding BBH

- Example: $m_{1}=10^{4} \mathrm{M} \odot, m_{2}=4 \times 10^{6} \mathrm{M}_{\odot}, a_{2}=0.1 \mathrm{pc}, e_{2}=0.7, \alpha=1.75$ (stellar distribution), normalized by M- $\sigma$ relation. Run time: 100Myr.






(Li, et al.


## COMPARISON OF TIMESCALES



## STARS SURROUNDING SMBHB

- At -Ipc separation it is more difficult to identify SMBHBs. SMBHBs can be observed with spectral features.
(e.g., Shen et al. 2013, Boroson \& Lauer 2009, Valtonen et al. 2008, Loeb 2007)

Example of multi-epoch spectroscopy (Shen et al. 2013):


active BH dominates the BL features, multi-epoch BL features => binary orbital parameters

## COPLANAR HIGH ECCENTRICITY MIGRATION


Population synthesis study. $\mathrm{tv}=\mathrm{O} . \mathrm{ryr}$

## Initial v.s. Final Distribution

- Example: $m_{1}=10^{6} \mathrm{M}_{\bullet}, m_{2}=10^{10} \mathrm{M}_{\bullet}, a_{2}=1 \mathrm{pc}, e_{2}=0.7, \alpha=1.75$ (stellar distribution), normalized by M- $\sigma$ relation. Run time: 1 Gyr .



## Initial Condition in i



## Maximum $\mathrm{e}_{\mathrm{I}}$ for different H

## and $\epsilon$



Maximum $e_{1}$ for low $i$, high $e_{1}$ case, and high i cases

## Surface of Section



- Trajectories chaotic only for $\mathrm{H}=-0.5,-0.1$ at high $\epsilon$.
- High inclination flips are chaotic.
- Overall evolution of the trajectories: evolution sensitive on the initial angles.


## Surface of Section

- Surface of section in the $\mathrm{Jz}-\Omega$ plane
$J z=\sqrt{1-e_{1}^{2}} \cos i_{1} \Omega$ : longitude of node
Low i, high $e_{1}$
High i , low $\mathrm{e}_{\mathrm{I}}$

Quadrupol e order dominates

Octupole order dominates


- All features are due to octupole effects.
- Trajectories are chaotic only possible when $\mathrm{H}=-0.5,-0.3,-0.1$, for high $\epsilon$.


## Characterization of Chaos

- Lyapunov exponents $(\lambda): \lambda \uparrow$, more chaotic.

- Chaotic when $\mathrm{H} \leq \mathrm{o}$ (correspond to high i cases).
- In chaotic region, Lyapunov timescale $\mathrm{t}_{\mathrm{L}}=(\mathrm{I} / \lambda) \approx 6 \mathrm{t}_{\mathrm{K}}$. ( $\mathrm{t}_{\mathrm{K}}$ corresponds to the oscillation timescale of $\mathrm{e}_{\mathrm{r}}$ and i )

$$
t_{K}=\frac{8}{3} P_{\text {in }} \frac{m_{1}}{m_{2}}\left(\frac{a_{2}}{a_{1}}\right)^{3}\left(1-e_{2}^{2}\right)^{3 / 2}
$$

## Surface of Section

Low i, high $\mathrm{e}_{\mathrm{I}}$
High i, low $\mathrm{e}_{\mathrm{I}}$
Quadrupol e order dominates

Octupole order dominates



- All features are due to octupole effects.
- Trajectories are chaotic only when $\mathrm{H} \leq 0$.
- Flips are due to octupole resonances.
(Li, et al., 2014 in prep)


## Applications --- 2. Tidal Disruption of Stars Surrounding BBH

- SMBHBs originate from mergers between galaxies. Following the merger, the distance of the SMBHB decreases.
(Complete numerical simulations: e.g. Khan et al. 2012)
- SMBHBs with -kpc separation have been observed with direct imagine.
(e.g. Fabbiano et al. 201ı, Green et al. 2010, Civano et al. 20ır, Komossa et al. 2003, Hutchings \& Neff 1989)
- At - Ipc separation it is more difficult to identify SMBHBs. SMBHBs have been observed with optical spectra, light variability and radio lines.
(e.g. Boroson \& Lauer 2009, Valtonen et al. 2008, Rodriguez et al. 2006)
- Motivation of tidal disruption of stars by - Ipc SMBHB:

Identify SMBHB at -I pe separation with tidal disruption rate

## Effects on Stars Surrounding BBH

- Dynamics of stars around BH or BBH:
- Secular dynamics introduce instability in eccentric stellar disks around a single BH (e.g. Madigan, Levin \& Hopman 2009)
- Tidal disruption event rate can be enhanced due to BBH and the recoil of BBH (Ivanov et al. 2005, Wegg to Bode 201I, Chen et al. 20II, Stone o Loeb 201I)
- Relic stellar clusters of recoiled BH may uncover MW formation history (e.g. O'Leary \& Loeb 2009).
- Here we study the effect of EKM to stars surrounding BBH


## Effects of EKM on Stars Surrounding BBH

- Study the role of eccentric $\left(\mathbf{e}_{2} \neq 0\right)$ Kozai mechanism in the presence of general relativistic (GR) precession and Newtonian (NT) precession for stars surrounding SMBHB.
- Set the separation of the BBH at $a_{2}=1 p c, e_{2}=0.7$ and assuming $\varrho * \propto a^{-1.75}$, normalized by $\mathrm{M}-\sigma$ relation.
- $\mathrm{N} *$ is the number of stars affected by the eccentric Kozai Mechanism. (Requirement: $\mathrm{t}_{\mathrm{GR}}<\mathrm{t}_{\text {Kozai }}$, $\left.\mathrm{t}_{\mathrm{NT}}<\mathrm{t}_{\text {Kozai }}, \varepsilon<0.1, a_{1}<\mathrm{r}_{\mathrm{RL}}\right)$.




## Effects of EKM on Stars Surrounding BBH

- Example: $m_{l}=10^{6} \mathrm{M}_{\circ}, m_{2}$
$=10^{10} \mathrm{M}_{\circ}, a_{2}=1 \mathrm{pc}, e_{2}=0.7$, Run time: 1 Gyr.
- 14/1000 disrupted; 535/1000 captured. Disruption/capture timescales are short.
$\Rightarrow$ Captured stars may change stellar density profile of the other BH
=> With rapid diffusion, disruption rate $\sim 10^{-3} / \mathrm{yr}$.

(Li, et al., in prep)


## SURFACE OF SECTION




- Resonant zones: points fill $\mathrm{I}^{-} \mathrm{D}$ lines. trajectories are quasi-periodic.
- Chaotic zones: points fill a higher dimension. trajectories are chaotic.


## SURFACE OF SECTION

Quadrupole order dominates Octupole order stronger

resonances resonances
Quadrupole resonances:

centers at low $\mathrm{e}_{\mathrm{I}}, \omega=\pi / 2$ and $3 \pi / 2$ (e.g., Kozai 1962)
Octupole resonances:
centers at high $\mathrm{e}_{\mathrm{I}}, \omega=\pi$ or $\pi / 2$ and $3 \pi / 2$

## SURFACE OF SECTION

Quadrupole order dominates

Octupole order stronger


chaos
quadru1 resonances



Octupole resonances: responsible for $\mathrm{e} \rightarrow \mathrm{I}$
Chaos: overlap of quadrupole and octupole resonances high inclination flips

## CHARACTERIZATION OF CHAOS

OChaotic when $\mathrm{H} \leq \mathrm{O}$ (correspond to high i cases).


- In chaotic region, Lyapunov timescale $\mathrm{t}_{\mathrm{L}}=(\mathrm{I} / \lambda) \approx 6 \mathrm{t}_{\mathrm{K}}$.
( $\mathrm{t}_{\mathrm{K}}$ corresponds to the oscillation timescale of $\mathrm{e}_{\mathrm{I}}$ and i )

$$
t_{K}=\frac{8}{3} P_{i n} \frac{m_{1}}{m_{2}}\left(\frac{a_{2}}{a_{1}}\right)^{3}\left(1-e_{2}^{2}\right)^{3 / 2}
$$

## DIFFERENCES BETWEEN HIGH/LOW I FLIP

Low inclination flip


High inclination flip


Low inclination flips:
$e_{I} \uparrow$ monotonically, inclination stays low before flip.
$i$ stays low before flip.

## HIERARCHICAL THREE-BODY SYSTEMS

- Configuration:

$$
r_{I} \ll r_{2}
$$

- Hierarchical configurations are COMMON:
- For binaries with periods shorter than io days, $>40 \%$ of them are in systems with multiplicity $\geq 3$. (Tokovinin 1997)
- For binaries with period $<3$ days, $\geq 96 \%$ are in systems with multiplicity $\geq 3$. (Tokovinin et al. 2006)
- 282 of the 299 triple systems ( $-94.3 \%$ ) are hierarchical. (Eggleton et al. 2007)
- Hierarchical 3-body dynamics gives insight for hierarchical multiple systems.


## EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

- For stellar systems:

Short Period Binaries


Image credit: NASA/Tod Strohmayer/Dana Berry e.g., Harrington 1969; Mazeh \&b Shaham 1979; Ford et al. 2000; Eggleton \&o Kiseleva-Eggleton 2001; Fabrycky \& Tremaine 2007; Shappee 也 Thompson 2013

Type Ia Supernova

e.g., Katz \&゚ Dong 2012; Kushnir et al. 2013

## EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

- Exoplanetary systems:

Eccentric Orbits

e.g., Holman et al. 1997; Ford et al. 2000; Wu \&゙ Murray 2003;

Exoplanets with large spinorbit misalignment


Image credit: ESO/A. C. Cameron
e.g., Fabrycky \& Tremaine 2007; Naoz et al. 2011, 2012; Petrovich 2015; Storch et al. 2014; Anderson et al. 2016

## EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

- Black hole systems: Merger of short period black hole binaries


## Tidal disruption events



## Spin-orbit Misalignment



* No correlation between misaligned/eccentric hot Jupiter systems and the incidence of stellar companions


## Eccentric Proto-Hot Jupiters

Existence of eccentric portoHot 7upiters?


High e migration

## Proto-Hot Jupiters

* A paucity of proto-hot Jupiters on super-eccentric orbits

* <44\% formed via LK mechanism


## Closer Companions of Hot Jupiters

Existence a closer companion?

High e migration<br>=> No close companions

LK not dominate
LK dominate

## Closer Companions of Hot Jupiters




Hot Jupiters (< 10 days) are no more or less likely to have exterior companions than giant planets (>10 days) => high e migration does not dominate

