Understanding the Universe

Summary Lecture

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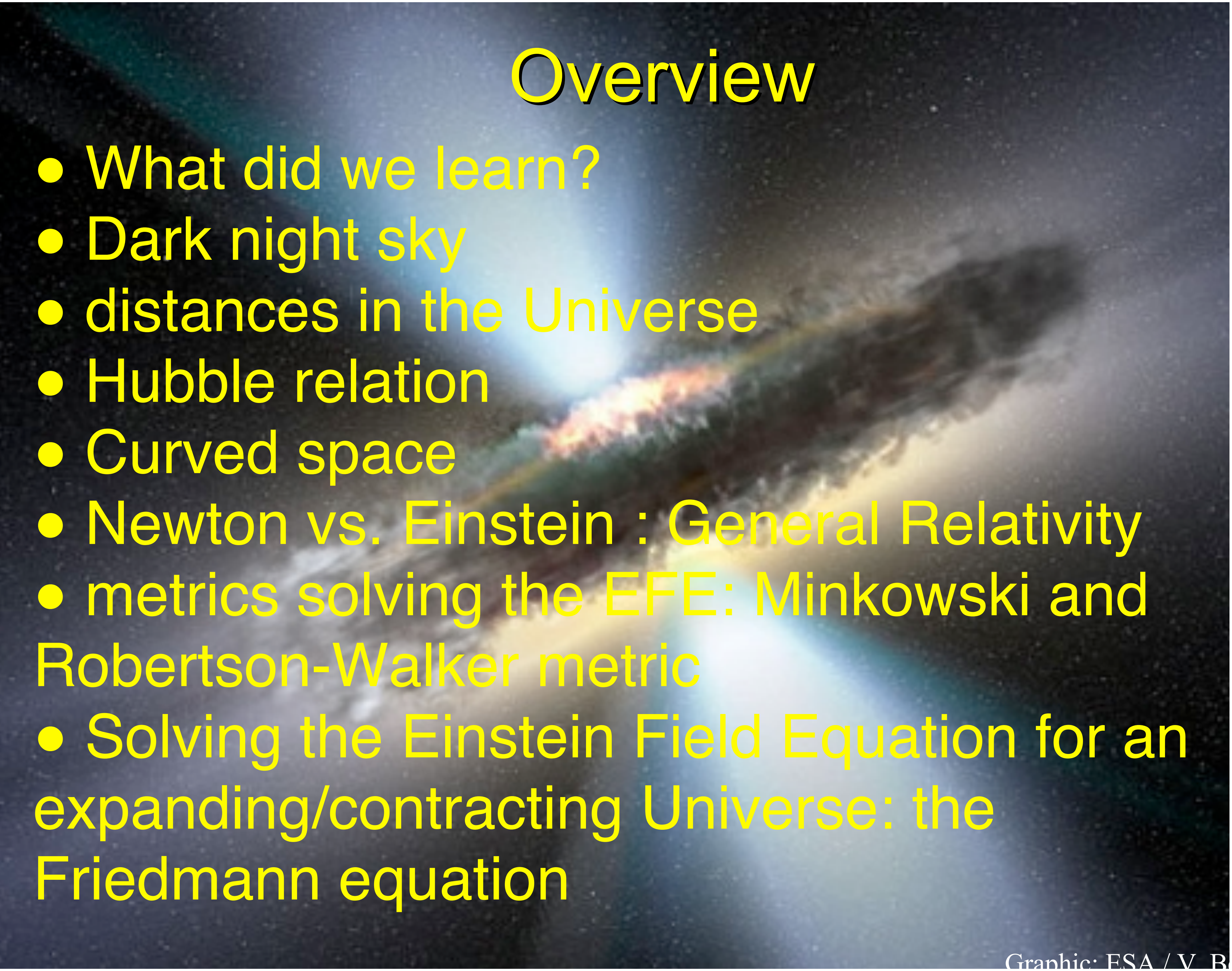
UMBC, December 12, 2006

Overview

- What did we learn?
- Dark night sky
- distances in the Universe
- Hubble relation
- Curved space
- Newton vs. Einstein : General Relativity
- metrics solving the EFE: Minkowski and Robertson-Walker metric
- Solving the Einstein Field Equation for an expanding/contracting Universe: the Friedmann equation



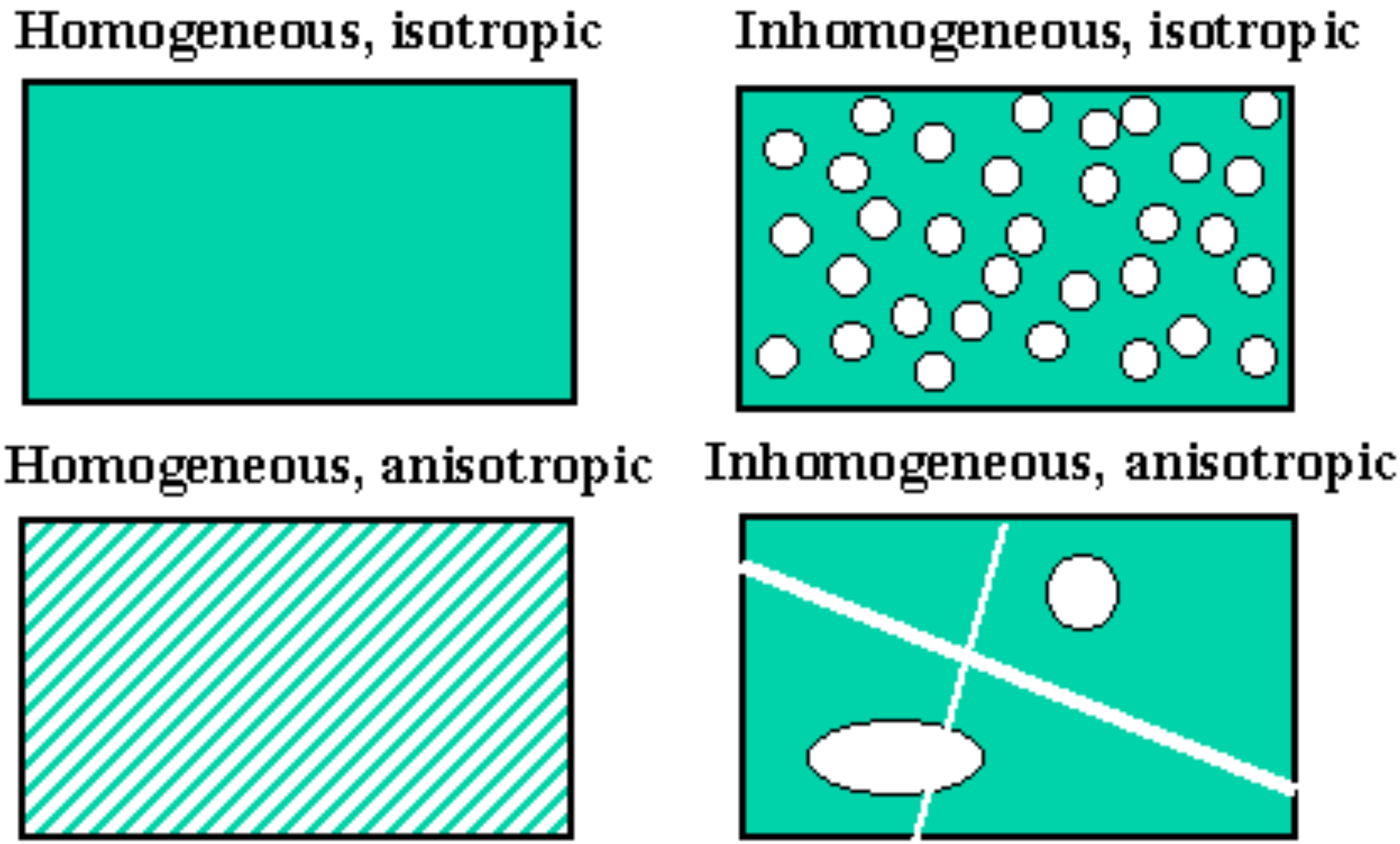
Graphic: ESA / V. Be

Overview

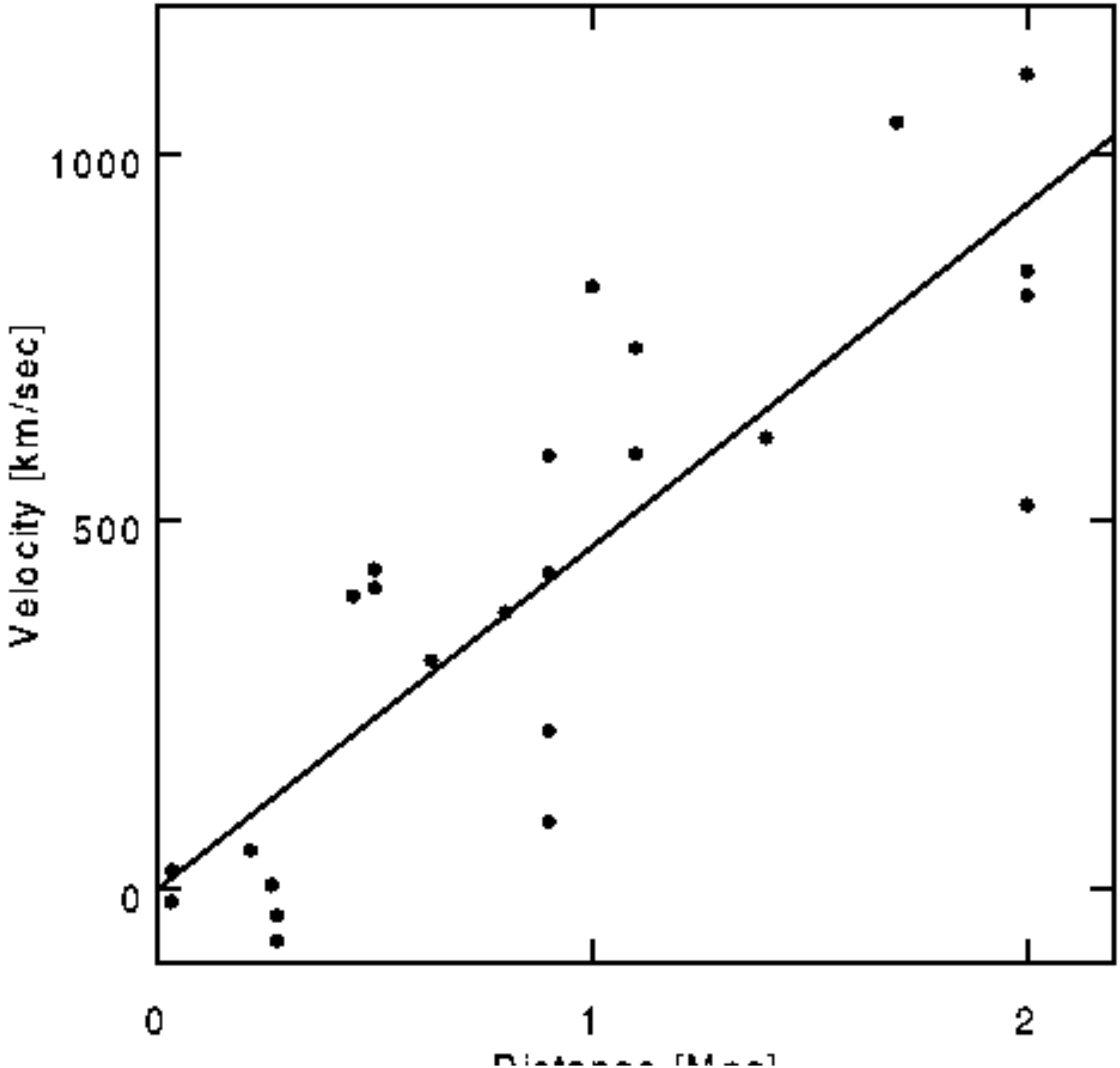
- Friedmann equation
- Fluid equation
- Acceleration equation
- Equation of state
- Evolution in a single/multiple component Universe
- Cosmic microwave background
- Nucleosynthesis
- Inflation

Graphic: ESA / V. B

Homogeneity vs Isotropy



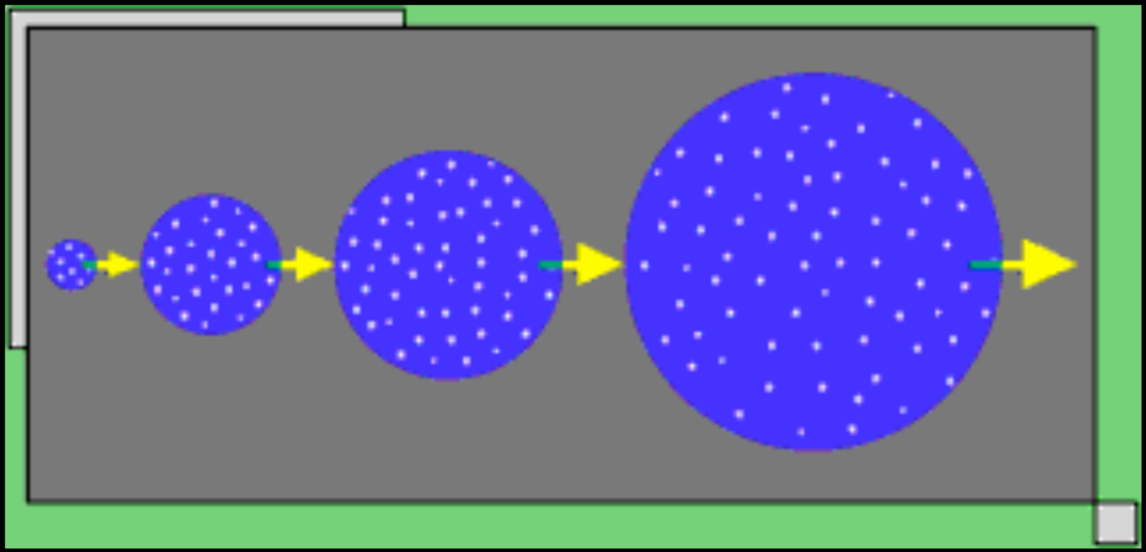
Graphic by Michael C. Wang (U



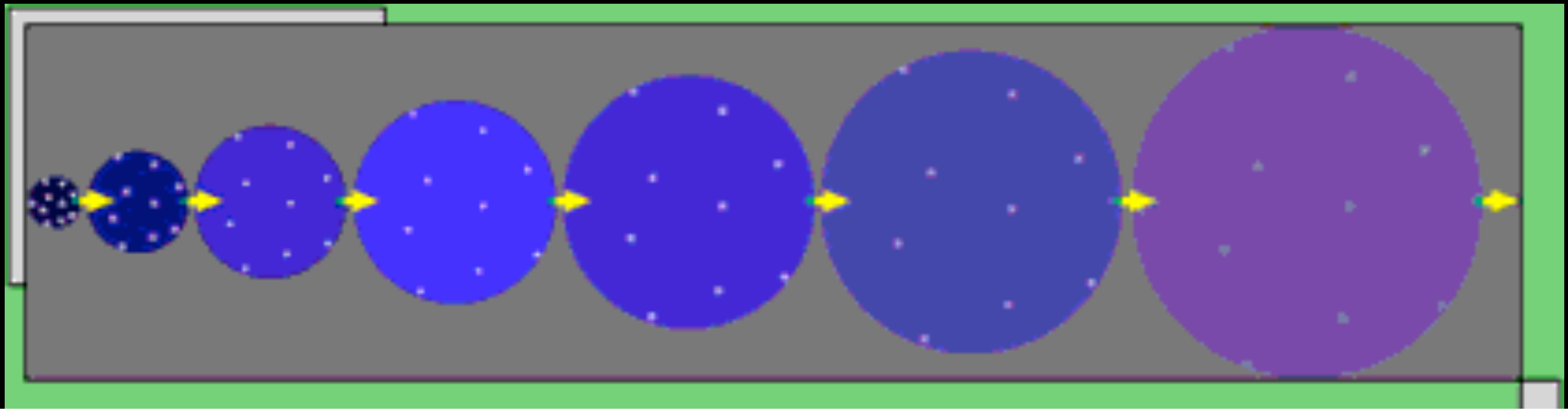
The veloci
distance
relation fo
galaxies f
by Edwin
Hubble.

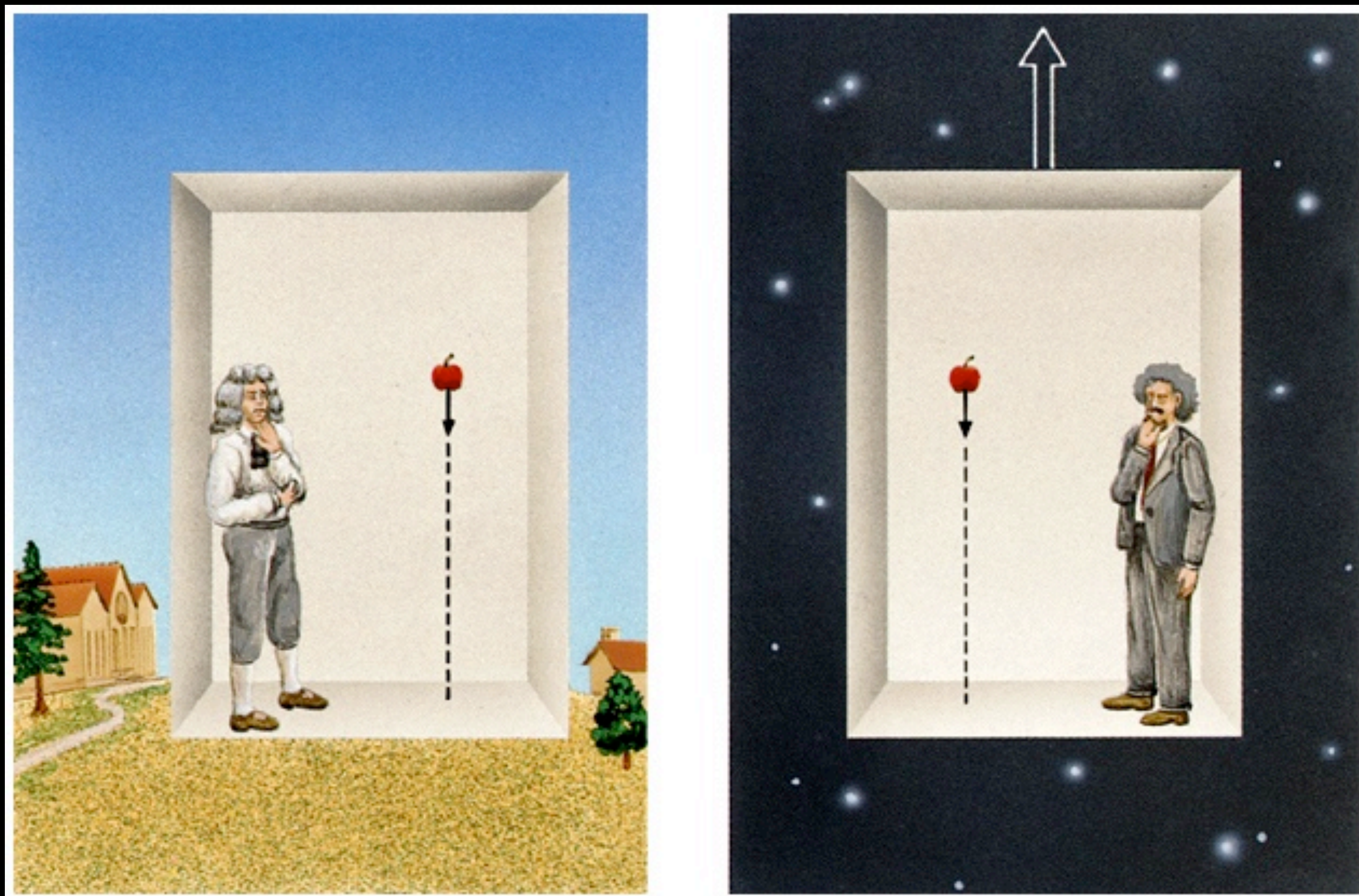
Graphic: Edw

Expansion in a steady state Universe



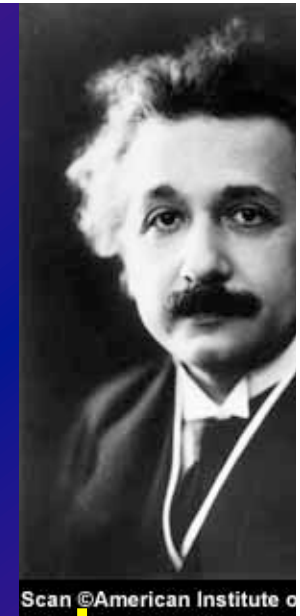
Expansion in a non-steady-state Universe





The equivalent principle: You cannot distinguish whether you are in an accelerated system or in a gravitational field

Newton vs. Einstein



Newton:

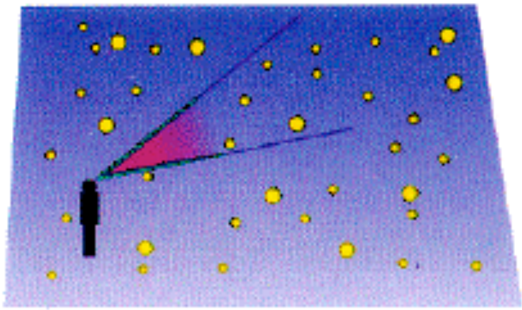
- mass tells gravity how to exert a force, force tells mass how to accelerate ($F = m\ a$)

Einstein:

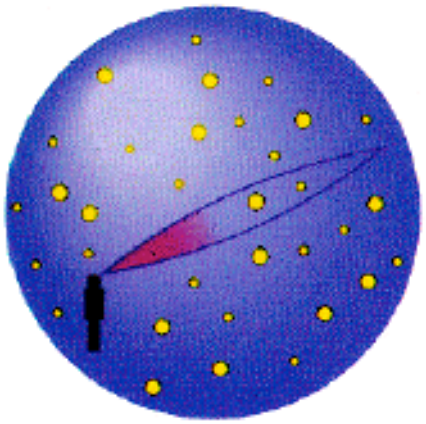
- mass-energy ($E=mc^2$) tells space time how to curve, curved space-tim tells mass-energy how to move

(John Wheeler)

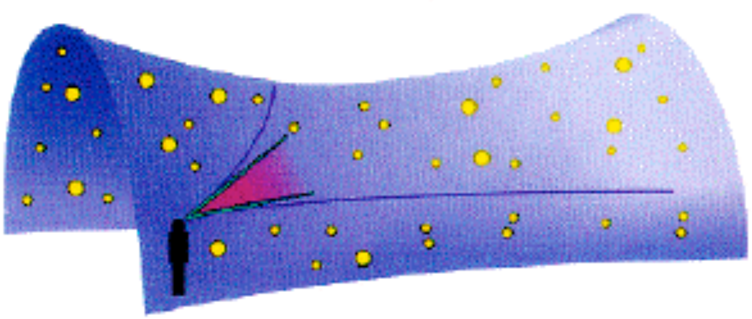
The effect of curvature



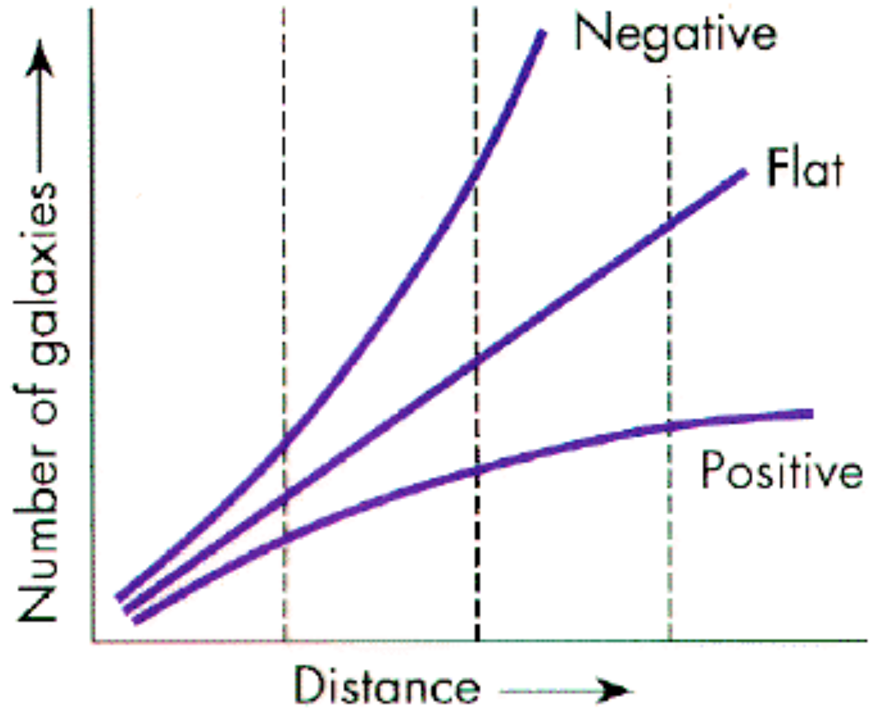
Flat universe



Positively curved universe



Negatively curved universe



A glimpse at Einstein's field equation

$$R_{ab} - \frac{1}{2} R \, g_{ab} = \frac{8\pi G}{c^4} T_{ab}.$$

Left side (describes the action of gravity through the curvature of space tim

R_{ab} = Ricci curvature tensor (4x4, 10 independent)

R = Ricci scalar (each point on a Riemannian manifold assigned a real num describing intrinsic curvature)

g_{ab} = metric tensor (~ gravitational field)

Solutions to the EFE are called "metrics of spacetime" ("metrics")

In flat space time for example we get the Minkowski metric:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

and the metric tensor becomes:

$$\eta = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The Minkowski metric

(a solution to the Einstein field equation for flat space time)

$$ds^2 = -c^2\, dt^2 + dL^2 = -c^2\, dt^2 + dr^2 + r^2\, d\Omega^2$$

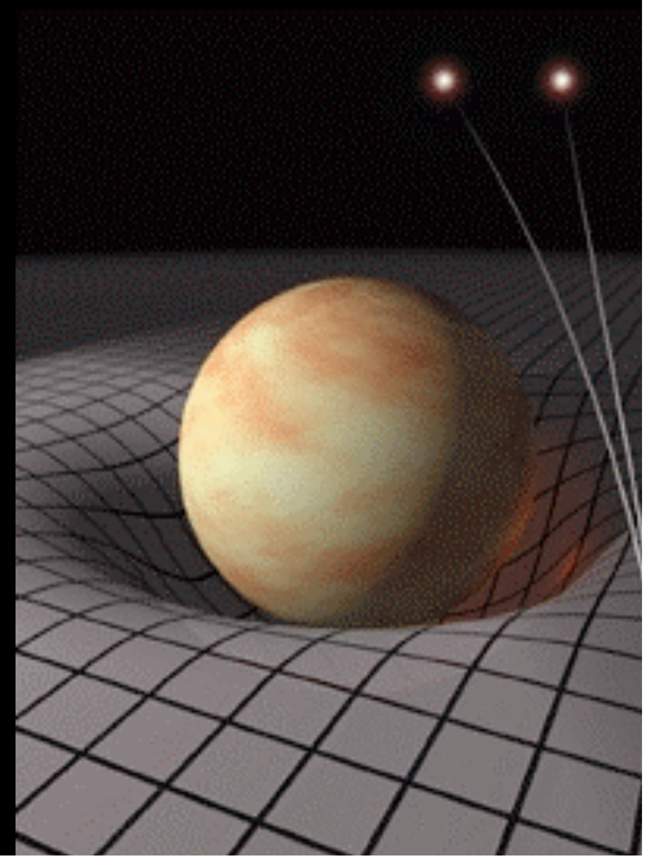
$ds^2 = 0$ represents the path of light (*null geodes*

assume movement along r only :

$c^2\, dt^2 = dr^2$

or

$dr / dt = +- c$



Robertson-Walker metric

Howard Percy Robertson (1903-1961) &
Arthur Geoffrey Walker (1909-2001)

$$R_{ab} - \frac{1}{2} R\, g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

Which metric solves the Einstein equation if there is curvature?
Robertson-Walker metric is an exact solution for General Relativity

$$ds^2 = c^2 dt^2 - a(t)^2 [dr^2 + \bar{r}^2 d\Omega^2]$$

$$\bar{r} = \begin{cases} R_C \sinh(r/R_C), & \text{for negative curvature} \\ r, & \text{for zero curvature} \\ R_C \sin(r/R_C), & \text{for positive curvature} \end{cases}$$

r : comoving distance from observer

$\underline{r}$: proper motion distance

RC : absolute value of the radius of curvature

homogeneous, isotropic expanding universe

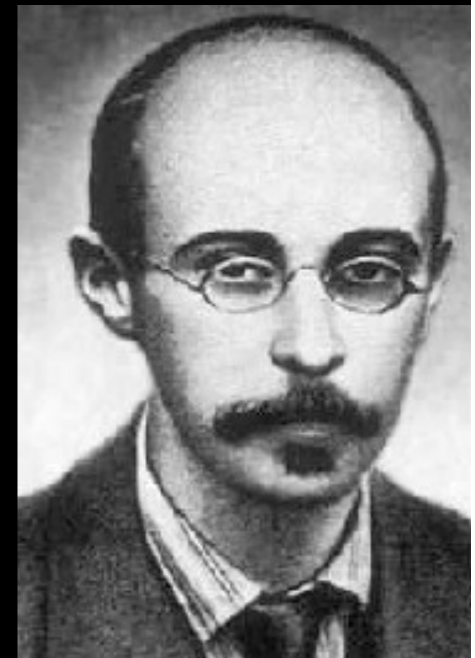
Fundamental Principles of GR

- general principle of relativity
- principle of general covariance
- inertial motion is geodesic motion
- local Lorentz invariance
- spacetime is curved
- spacetime curvature is created by stress-energy within the spacetime

(1 and 3 result in the equivalent principle)

Friedmann Equation

Aleksander Friedmann (1888-1925)



Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{\kappa c^2}{R_0^2 a^2}$$

Friedmann Equation

Critical density:

$$\epsilon_c(t) \equiv \frac{3c^2}{8\pi G} \cdot H(t)^2$$

Critical density (today):

$$\epsilon_{c,0} = \frac{3c^2}{8\pi G} \cdot H_0^2 = (8.3 \pm 1.7) \times 10^{-10}\,\mathrm{J\,m^{-3}}$$
$$= 5200 \pm 1000\,\mathrm{MeV\,m^{-3}}$$

$$\rho_{c,0} \equiv \frac{\epsilon_{c,0}}{c^2} = (0.2 \pm 1.8) \times 10^{-27}\,\mathrm{kg\,m^{-3}}$$

$$= (1.4 \pm 0.3) \times 10^{11}\,\mathrm{M_\odot\,Mpc^{-3}}$$

Friedmann equation with cosmological constant

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3}$$

Fluid equation (stays the same) :

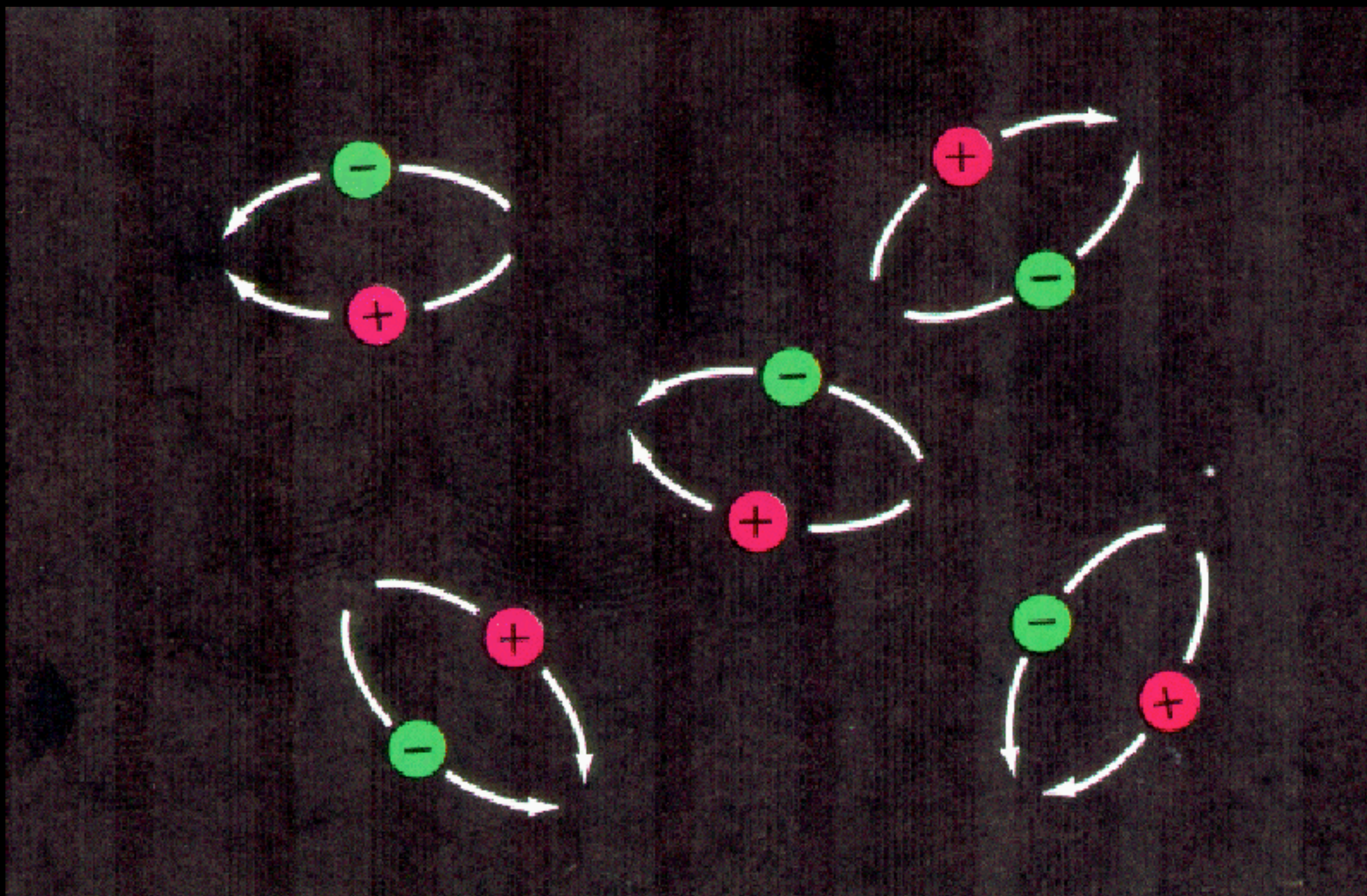
$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

Equation of state

$$P = \omega \epsilon$$

Acceleration equation:

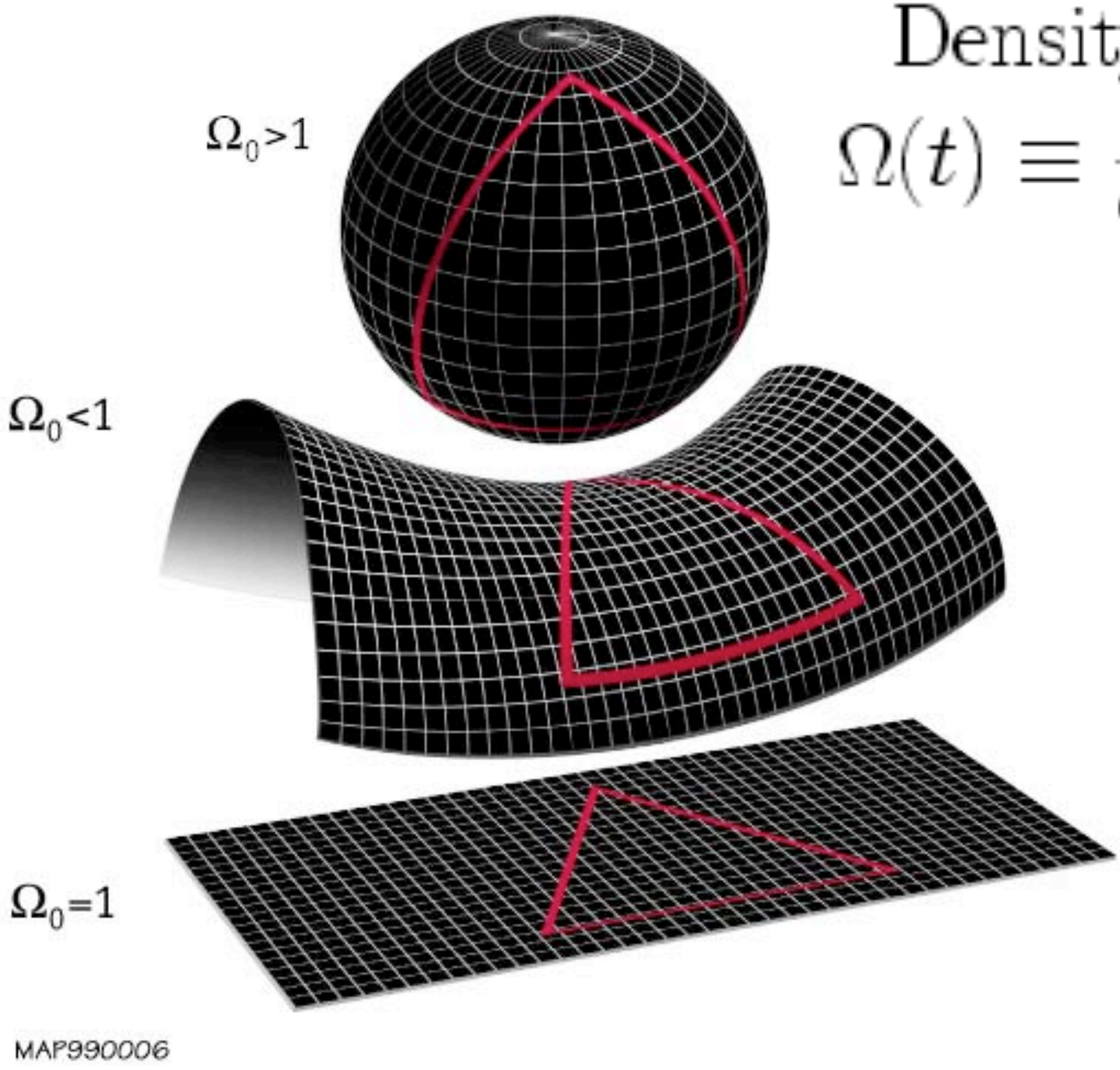
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P) + \frac{\Lambda}{3}$$



Key question: what is the average energy density of virtual particles in the universe?

Look for the lecture about dark energy on the course webpage (under “Literature”). Fast connection required (170 MByte; 45 minutes).

Density parameter Ω and curvature



Density parameter:

$$\Omega(t) \equiv \frac{\epsilon(t)}{\epsilon_c(t)}$$

Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_{\omega} \epsilon_{w,0}\, a^{-1-3\omega} \; - \; \frac{\kappa\, c^2}{R_0^2}$$

Friedmann equation (empty universe):

$$\dot{a}^2 = -\frac{\kappa\, c^2}{R_0^2}$$

Friedmann equation (flat, single-component):

$$\dot{a}^2 = \frac{8\pi G\, \epsilon_0}{3\, c^2}\, a^{-1-3\omega}$$

Flat, single compo

$$H_0 \equiv \left(\frac{\dot{a}}{a}\right)_{t=t_0} = \frac{2}{3(1+\omega)}\, t_0^{-1} \qquad t_0 = \frac{2}{3\,(1+\omega)}\, H_0^{-1}$$

The first part of the lecture

- Universe is expanding (Hubble relation)
- Newton's not enough: Einstein's idea about space-time
- General relativity for curved space-time
- Four equations to describe the expanding/contracting universe

The second part of the lecture

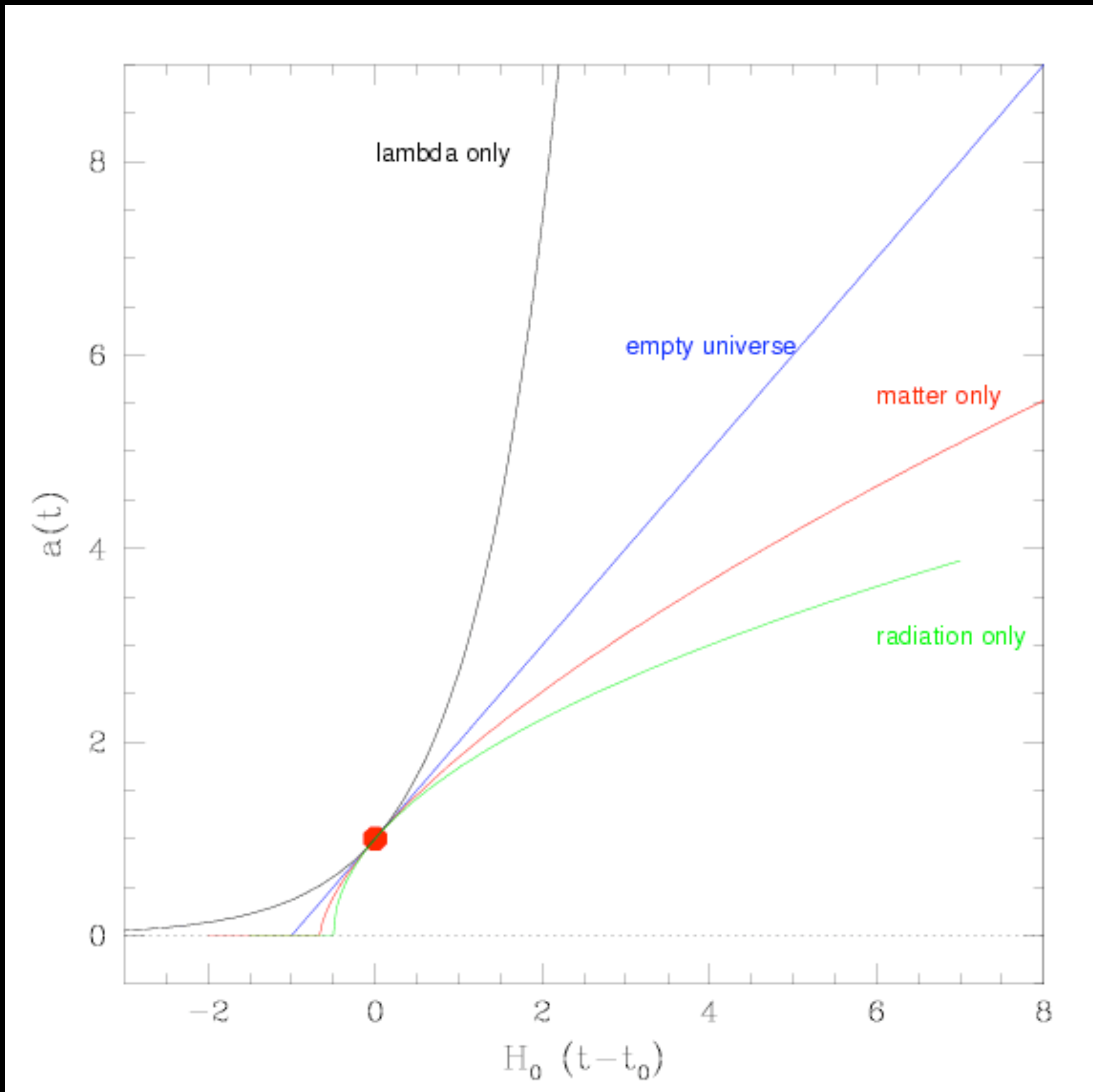
- How to model the Universe
- the Friedmann equation as a function of density parameters
- Matter-, radiation-, lambda-, curvature- only universe
- mixed-component universes
- the important times in history: $a_{r,m}$ and a $_{m,\Lambda}$

The second part of the lecture

- How to measure the Universe
- the Friedmann equation expressed in a Taylor series: H_0 and q_0 (deceleration parameter)
- luminosity distance, angular size distance
- distance ladder: parallax, Cepheids, SuperNova Type Ia
- results from the SuperNova measurements

The second part of the lecture

- What is the matter contents of the Universe?
- matter in stars
- matter between stars
- matter in galaxy clusters
- dark matter



Scale factor a(t) in a flat, single component universe

Universe with matter and curvature only

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$$P = \omega\,\epsilon$$

Density parameter:

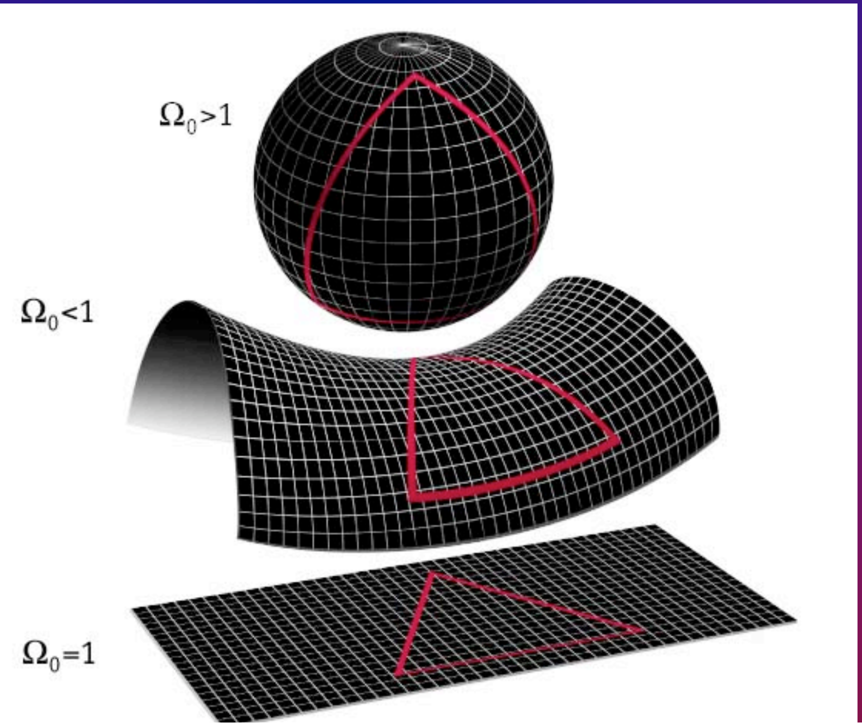
$$\Omega(t) \equiv \frac{\epsilon(t)}{\epsilon_c(t)}$$

Curved, matter dominated Universe

$\Omega_0 < 1$ $\kappa = -1$ Big Chill $(a \propto t)$

$\Omega_0 = 1$ $\kappa = 0$ Big Chill $(a \propto t^{2/3})$

$\Omega_0 > 1$ $\kappa = -1$ Big Crunch



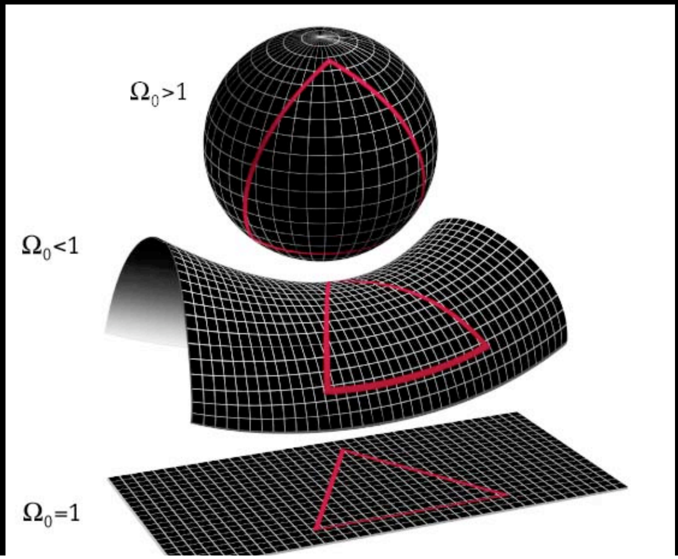
Universe with matter, Λ, and curvature

(matter + cosmological constant + curvature)

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{1-\Omega_{m,0}-\Omega_{\Lambda,0}}{a^2} + \Omega_{\Lambda,0}$$

$$\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0}$$

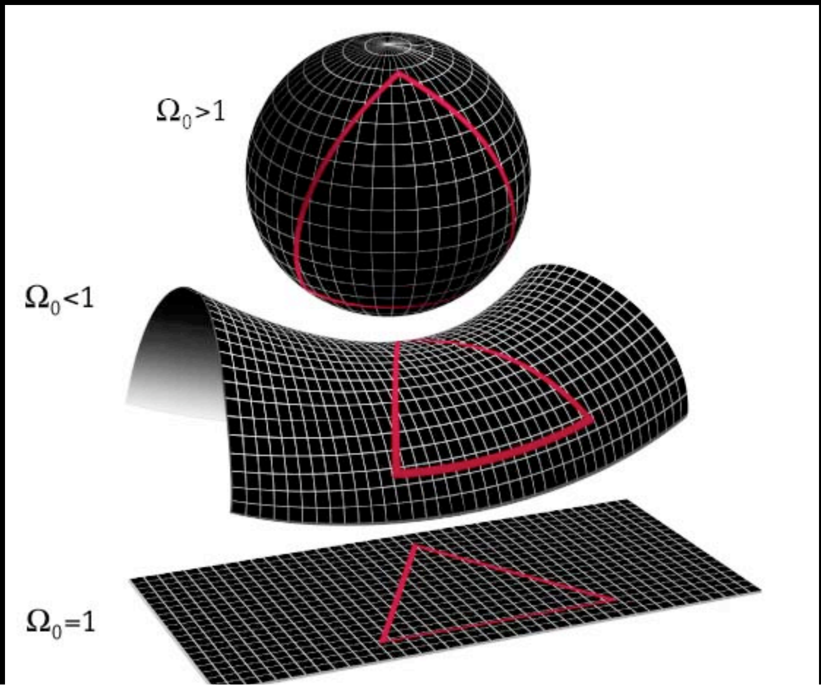


Flat Universe with matter and radiation

(e.g. at $a \sim a_{rm}$)

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

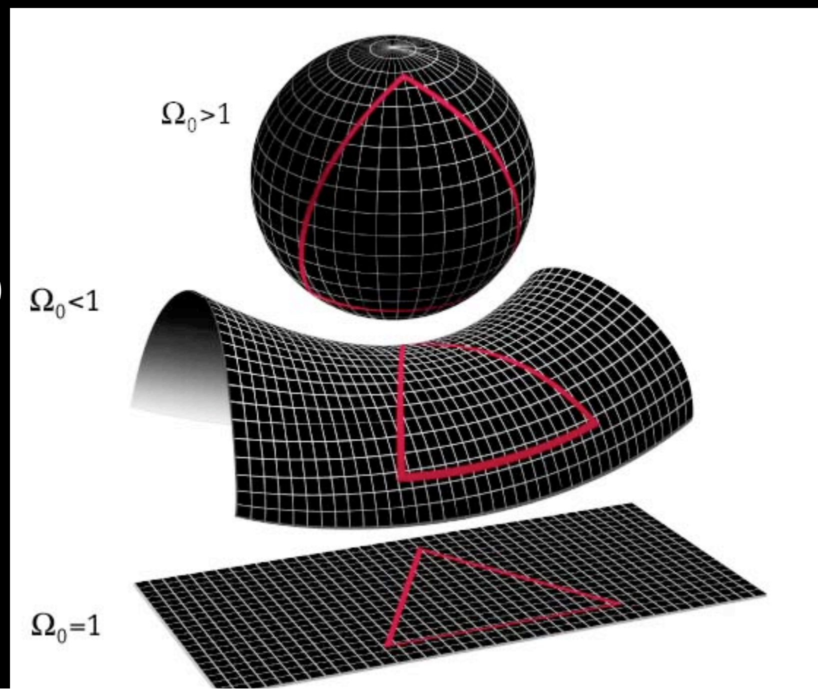
$$\Omega_0 = \Omega_{m,0} + \Omega_{r,0}$$



Describing the real Universe - the "benchmark" model

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$\Omega_0 = \Omega_L + \Omega_{m,0} + \Omega_{r,0}$
$= 1.02 \pm 0.02$
(Spergel et al. 2003, ApJS, 148, 175)

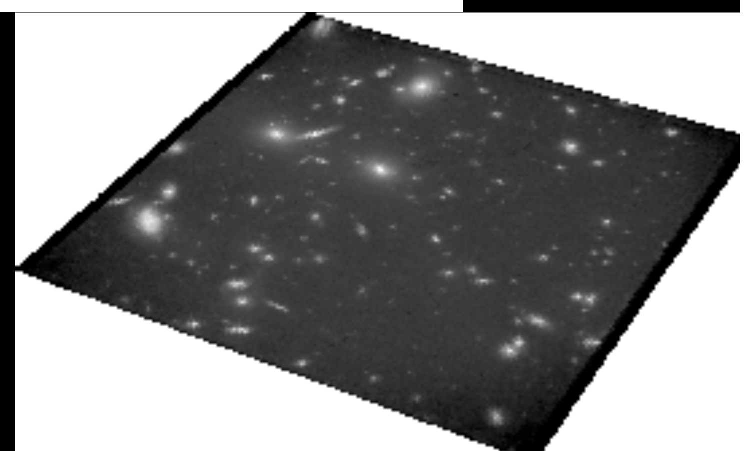


The "benchmark" model

photons	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic	$\Omega_{bary,0} = 0.04$
dark matter	$\Omega_{dm,0} = 0.26$
total matter	$\Omega_{m,0} = 0.30$
dark energy	$\Omega_{\Lambda,0} \simeq 0.70$

$$\Omega_0 = \Omega_L + \Omega_{m,0} + \Omega_{r,0}$$
$$= 1.02 \pm 0.02$$

(Spergel et al. 2003, ApJS, 148, 175)

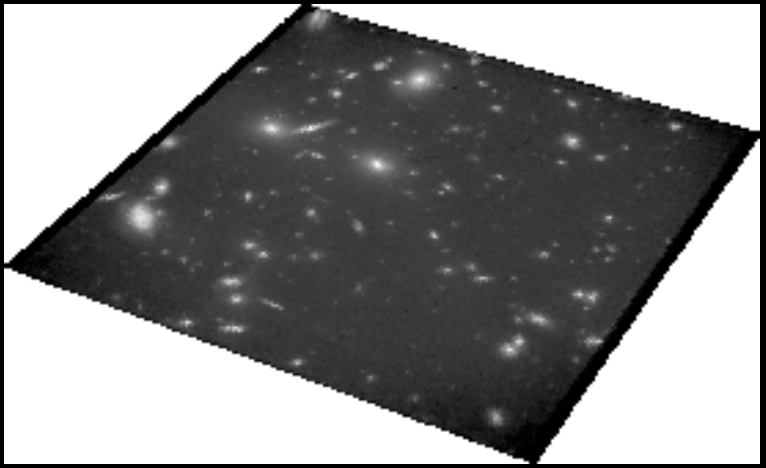


"The Universe is flat and full of stuff we cannot see"

The "benchmark" model

Important Epochs in our Universe:

Epoch	scale factor	time
radiation-matter	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 47{,}000\,\mathrm{yr}$
matter-lambda	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8\,\mathrm{Gyr}$
Now	$a_0 = 1$	$t_0 = 13.5\,\mathrm{Gyr}$



"The Universe is flat and full of stuff we cannot see - and we are even dominated by dark energy right now"

The “benchmark” model

Some key questions:

- Why, out of all possible combinations, we have $\Omega_0 = \Omega_\Lambda + \Omega_{m,0} + \Omega_{r,0} = 1.0$?
- Why is $\Omega_\Lambda \sim 1$?
- What is the dark matter?
- What is the dark energy?
- What is the evidence from observations for the benchmark model?

“The Universe is flat and full of stuff we cannot see”

How do we verify our models with observations?

$$H_0 \cdot t = \int_0^a \frac{da}{\sqrt{\Omega_{r,0}\, a^{-2} + \Omega_{m,0}\, a^{-1} + \Omega_{\Lambda,0}\, a^2 + (1 - \Omega_0)}}$$

Hubble Space Telescope Credit: NASA/STS-82

Scale factor as Taylor Series

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots$$

$$a(t) \simeq 1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0^2 (t-t_0)^2$$

q_0 = deceleration parameter

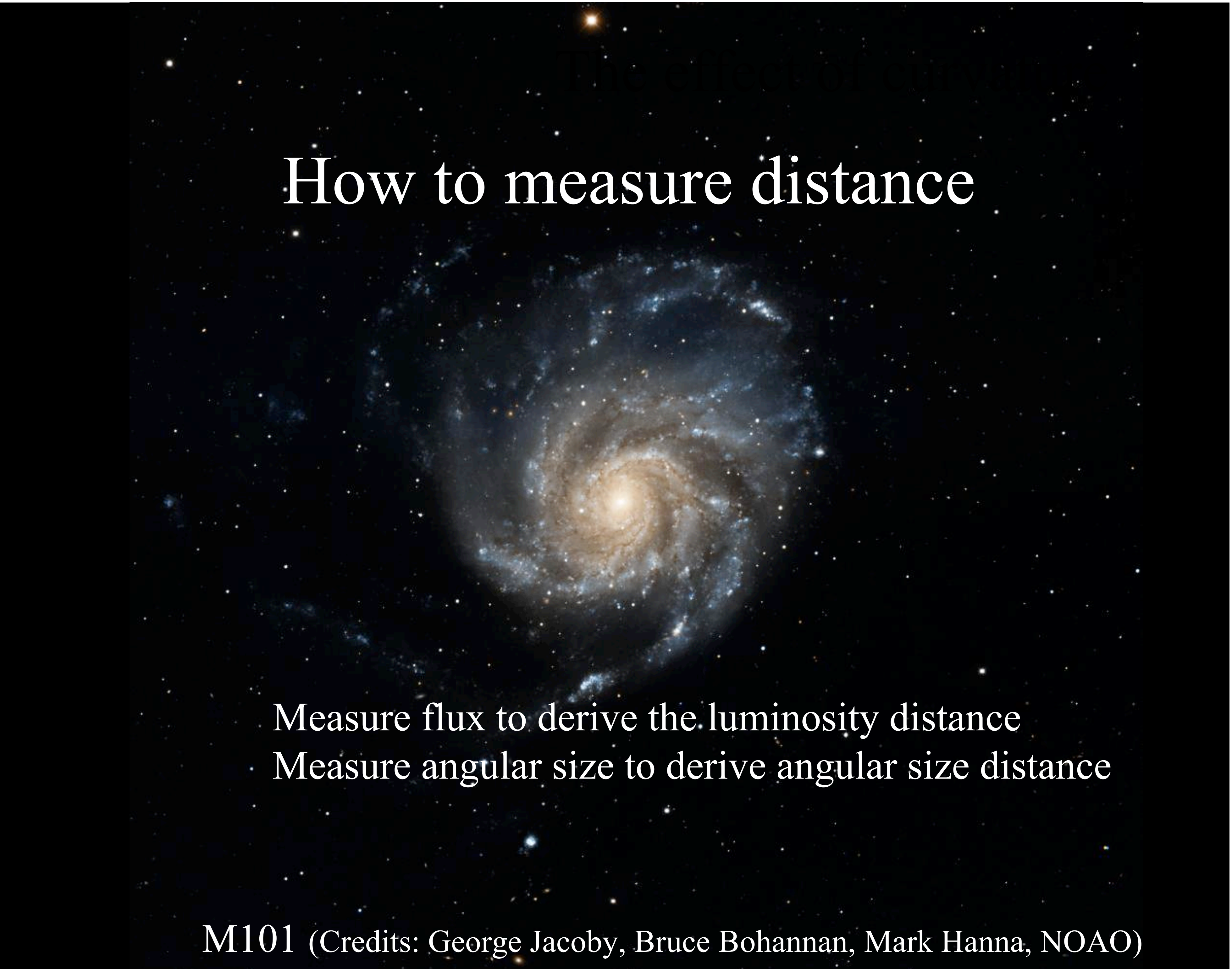
Deceleration parameter q_0

$$a(t) \simeq 1 + H_0(t - t_0) - \tfrac{1}{2} q_0 H_0^2 (t - t_0)^2$$

Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)$$

$$-\frac{\ddot{a}_0}{a_0 H_0^2} = q_0 = \frac{1}{2} \sum_{\omega} \Omega_\omega (1 + 3\omega)$$


How to measure distance
Measure flux to derive the luminosity distance
Measure angular size to derive angular size distance
M101 (Credits: George Jacoby, Bruce Bohannan, Mark Hanna, NOAO)

Luminosity Distance

In a nearly flat universe:

$$d_L \simeq \frac{c}{H_0} z \left[1 + \frac{1-q_0}{2} z\right]$$

How to determine a(t) :

- determine the flux of objects with known luminosity to get luminosity distance

- for nearly flat: $d_L = d_p(t_0)\,(1+z)$

- measure the redshift

- determine H_0 in the local Universe

$\rightarrow$ q_0

Angular Diameter Distance

$$d_A = \text{length} / \delta\Theta = d_L / (1+z)^2$$

For nearly flat universe:

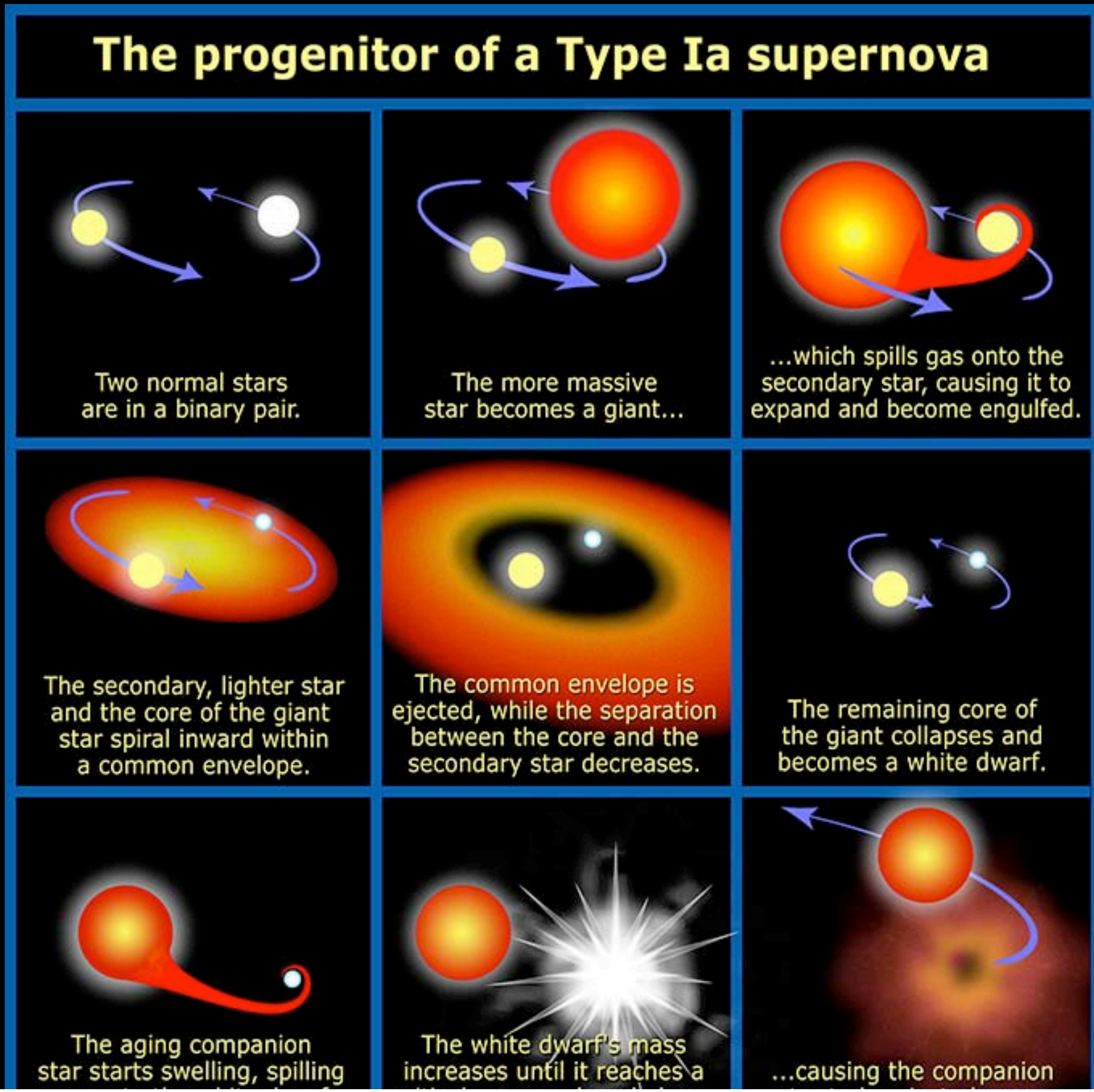
$d_A = dp(t_0) / (1+z)$

Distance measurements

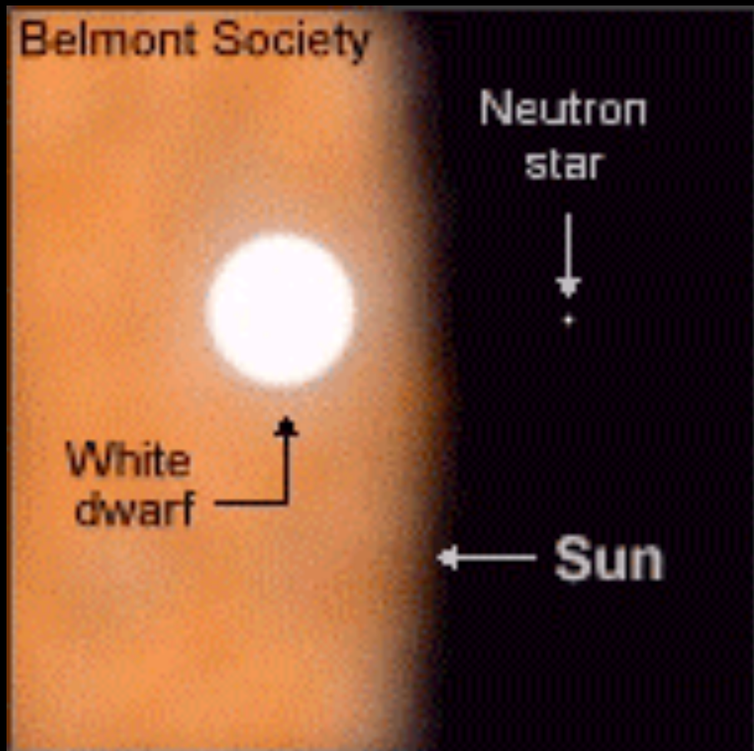
- Distance ladder:
- measure directly distances in solar system
- nearby stars with parallax (up to 1kpc)
- nearby galaxies with variable stars (Cepheids)
- distant galaxies with Supernovae Type Ia ("standard candles") up to redhsift z~2
- see also "literature" on the course webpage

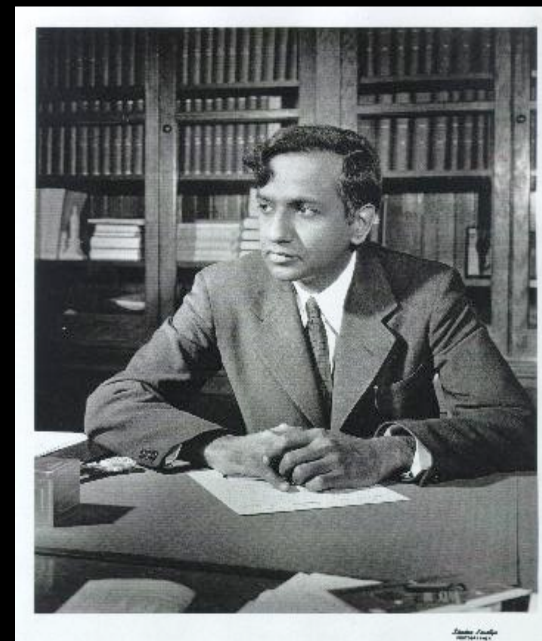
For nearly flat Universe:

$$d_L \simeq \frac{c}{H_0}\, z \left[1 + \frac{1-q_0}{2}\, z\right]$$


The progenitor of a Type Ia supernova
Two normal stars
are in a binary pair.
The more massive
star becomes a giant...
...which spills gas onto the
secondary star, causing it to
expand and become engulfed.
The secondary, lighter star
and the core of the giant
star spiral inward within
a common envelope.
The common envelope is
ejected, while the separation
between the core and the
secondary star decreases.
The remaining core of
the giant collapses and
becomes a white dwarf.
The aging companion
star starts swelling, spilling
The white dwarf's mass
increases until it reaches a
...causing the companion

Chandrasekhar limit





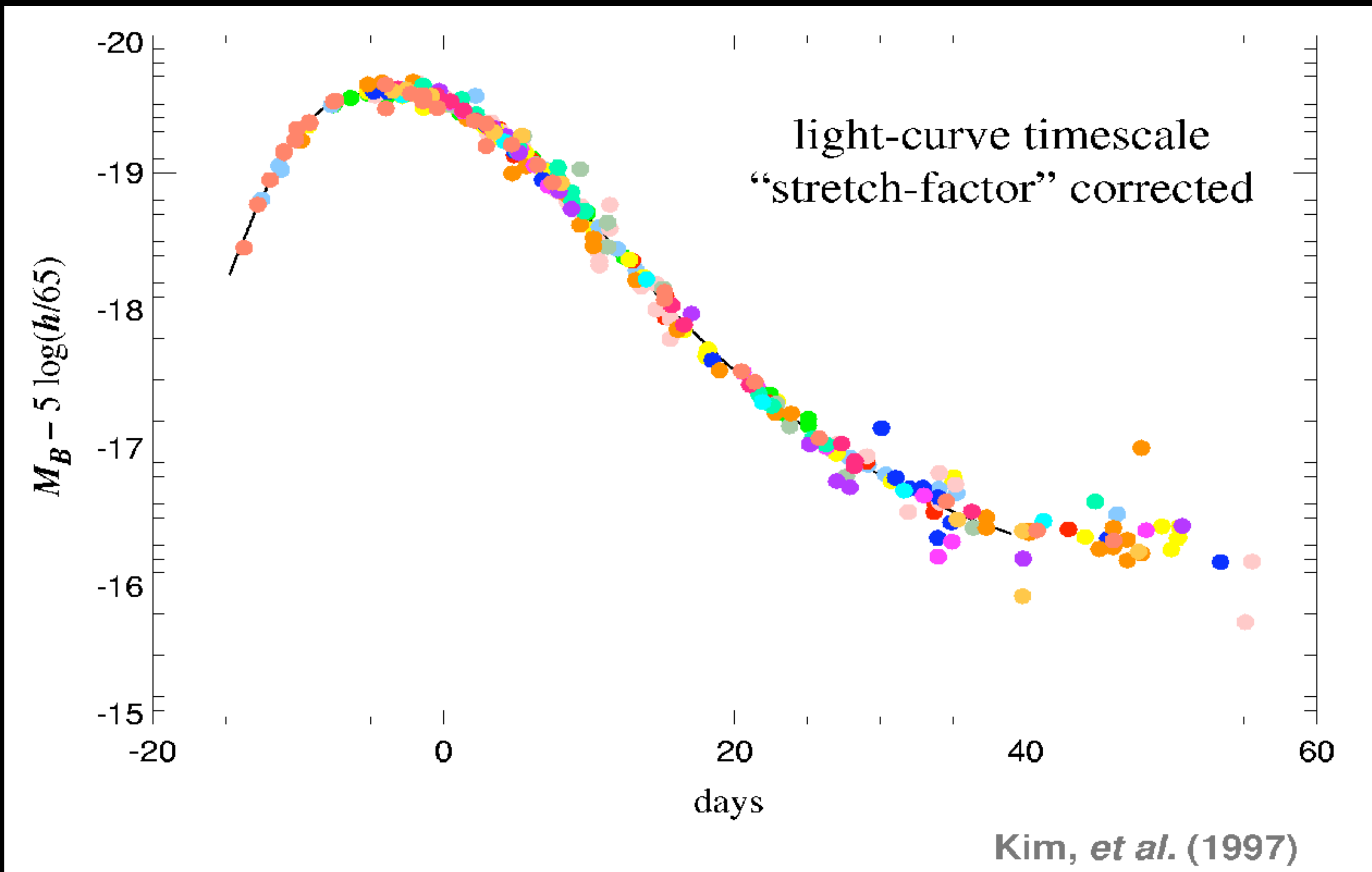
Subrahmanyan Chandrasekhar (1910-1995)

Nobel prize winner

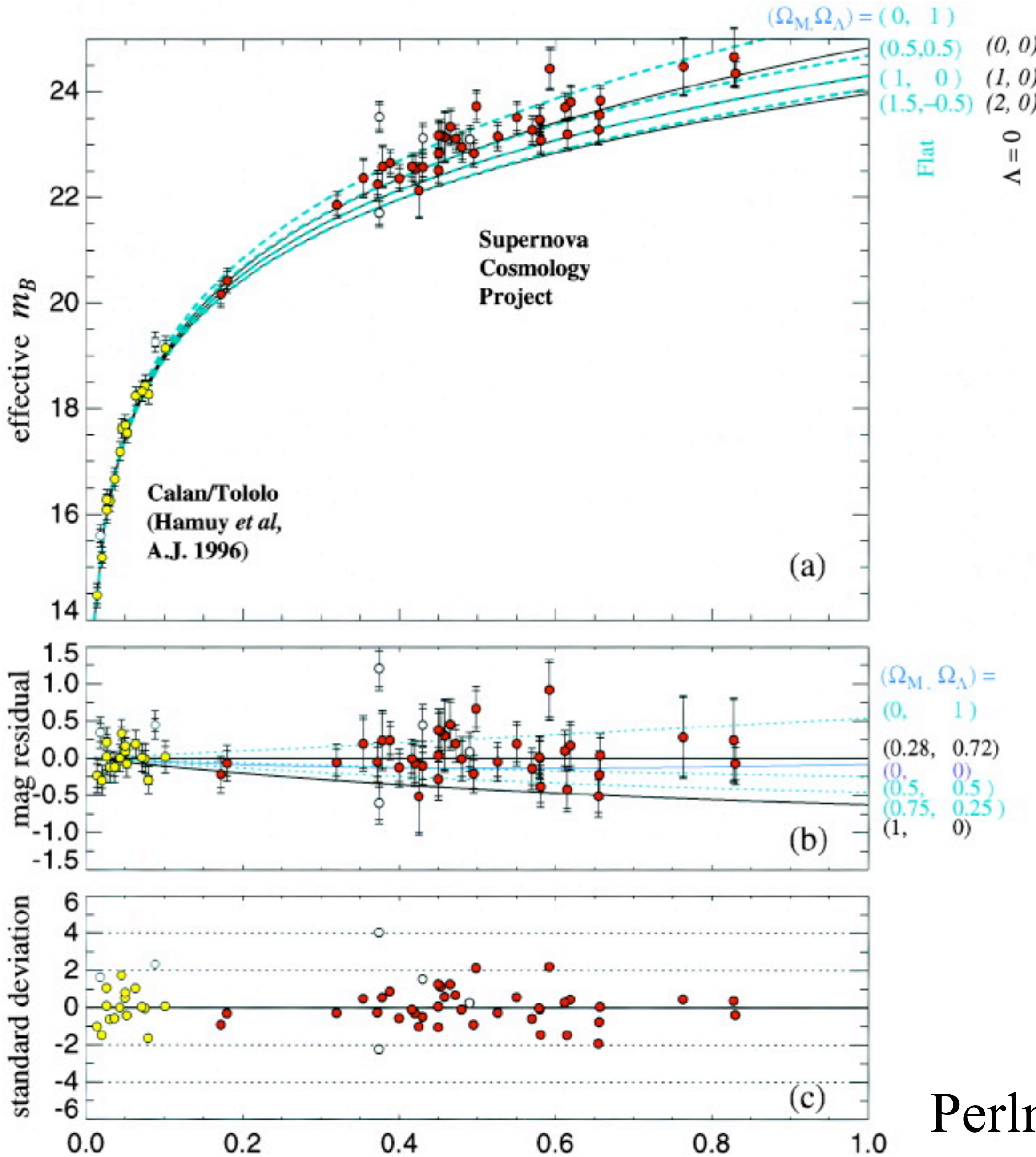
Electron degeneracy pressure can support an electron star (White Dwarf) up to a size of ~1.4 solar masses

$$M_{Ch} \simeq \frac{3\sqrt{2}\pi}{8}\left(\frac{hc}{2\pi G}\right)^{3/2}\left[\frac{Z}{A}\frac{1}{m_H}\right]^2$$

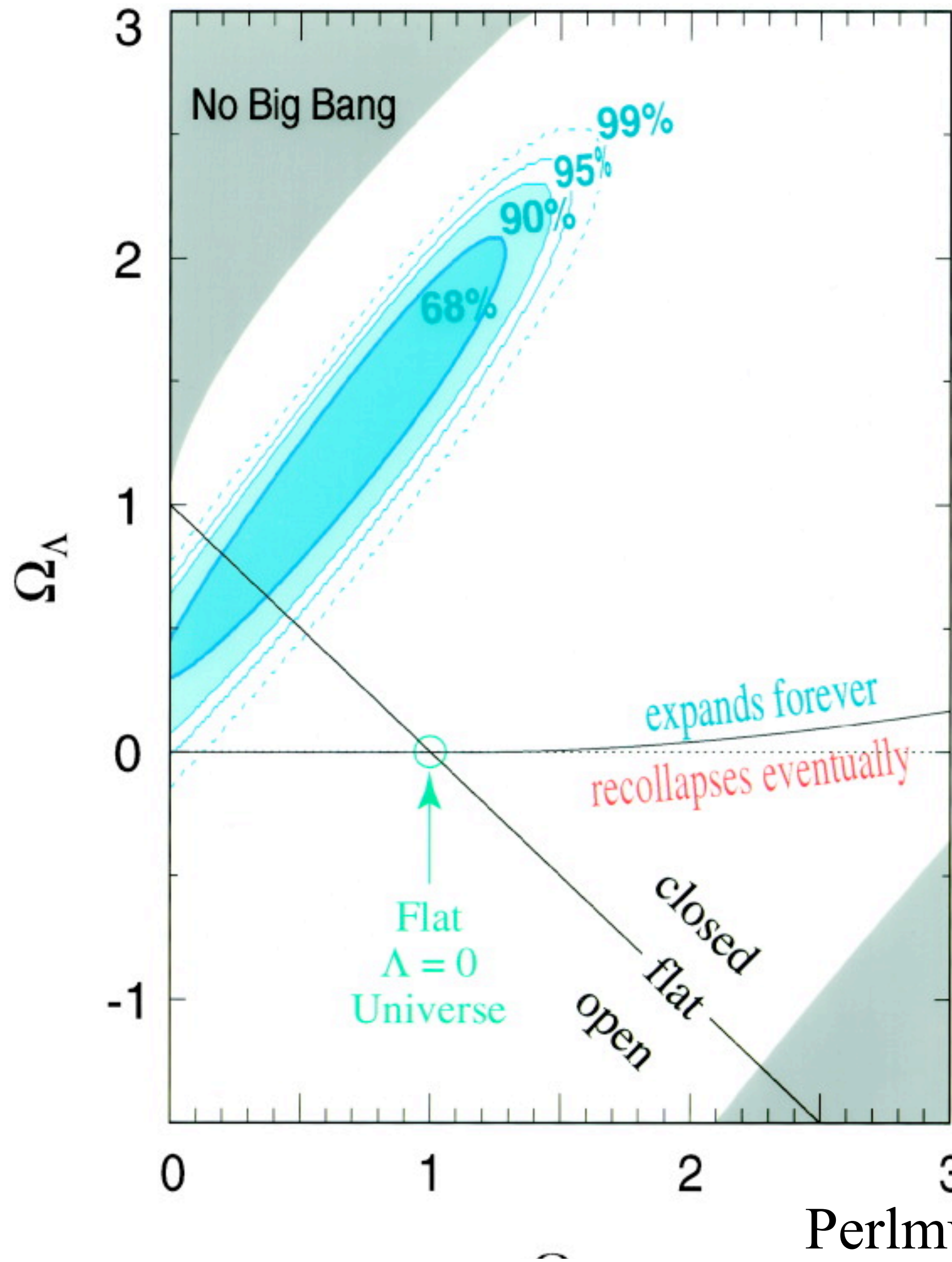
Super Nova Type Ia lightcurves



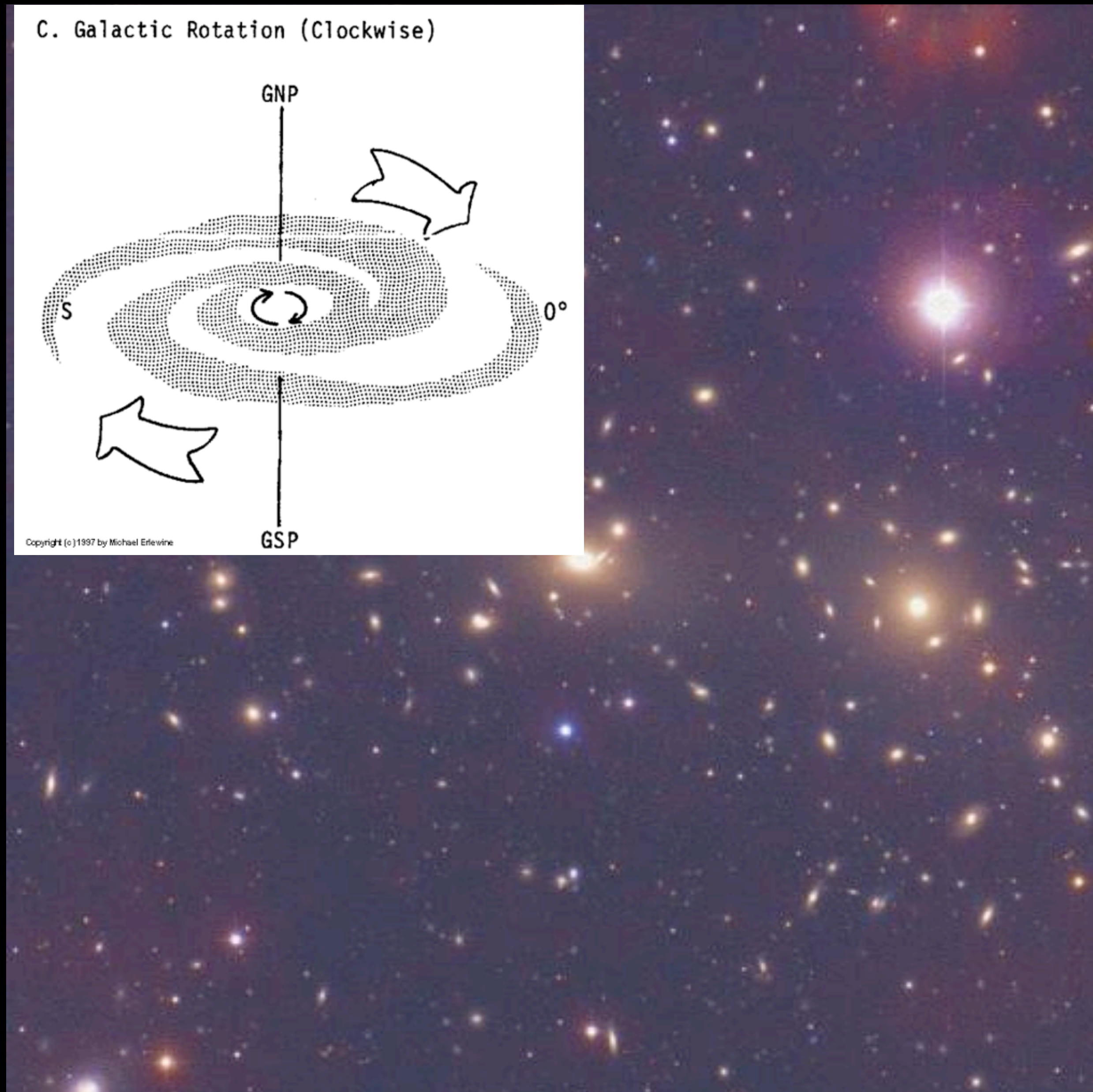
Corrected lightcurves



Perlmutter et al. 1999



Perlmutter et al. 1999



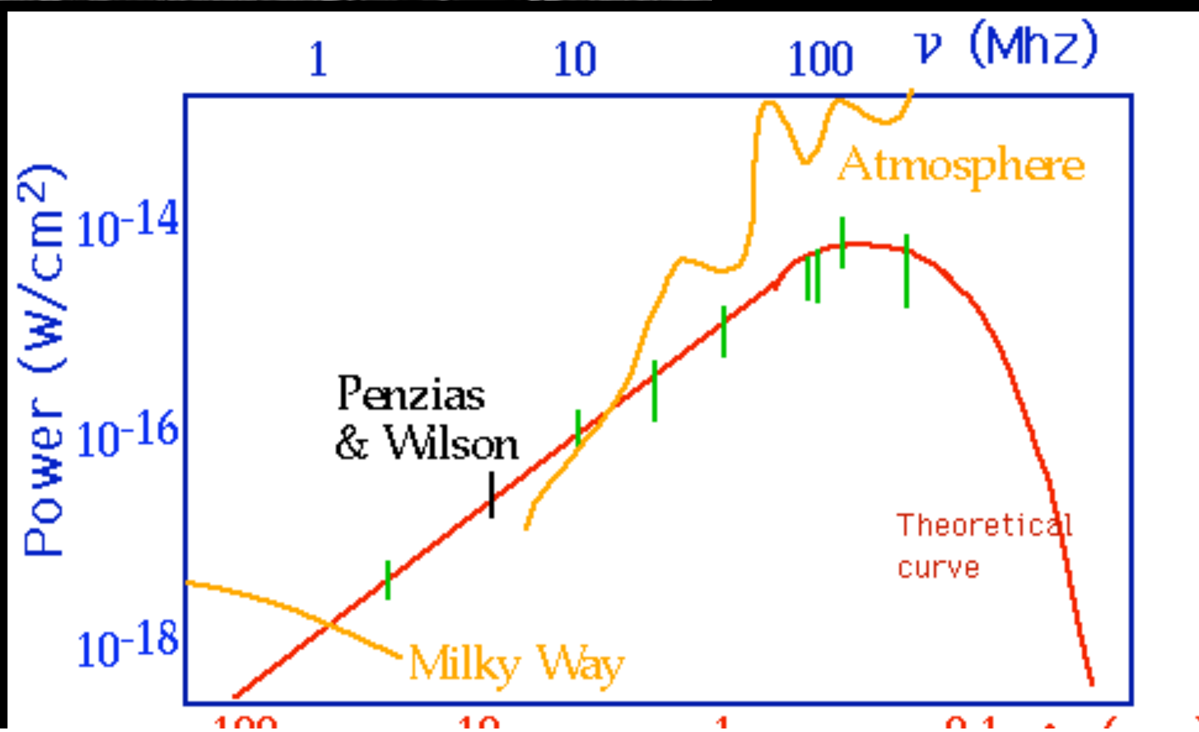
Dark matter can “seen” in rotation curves of galaxies
and in velocity dispersion of galaxies in clusters

The third part of the lecture

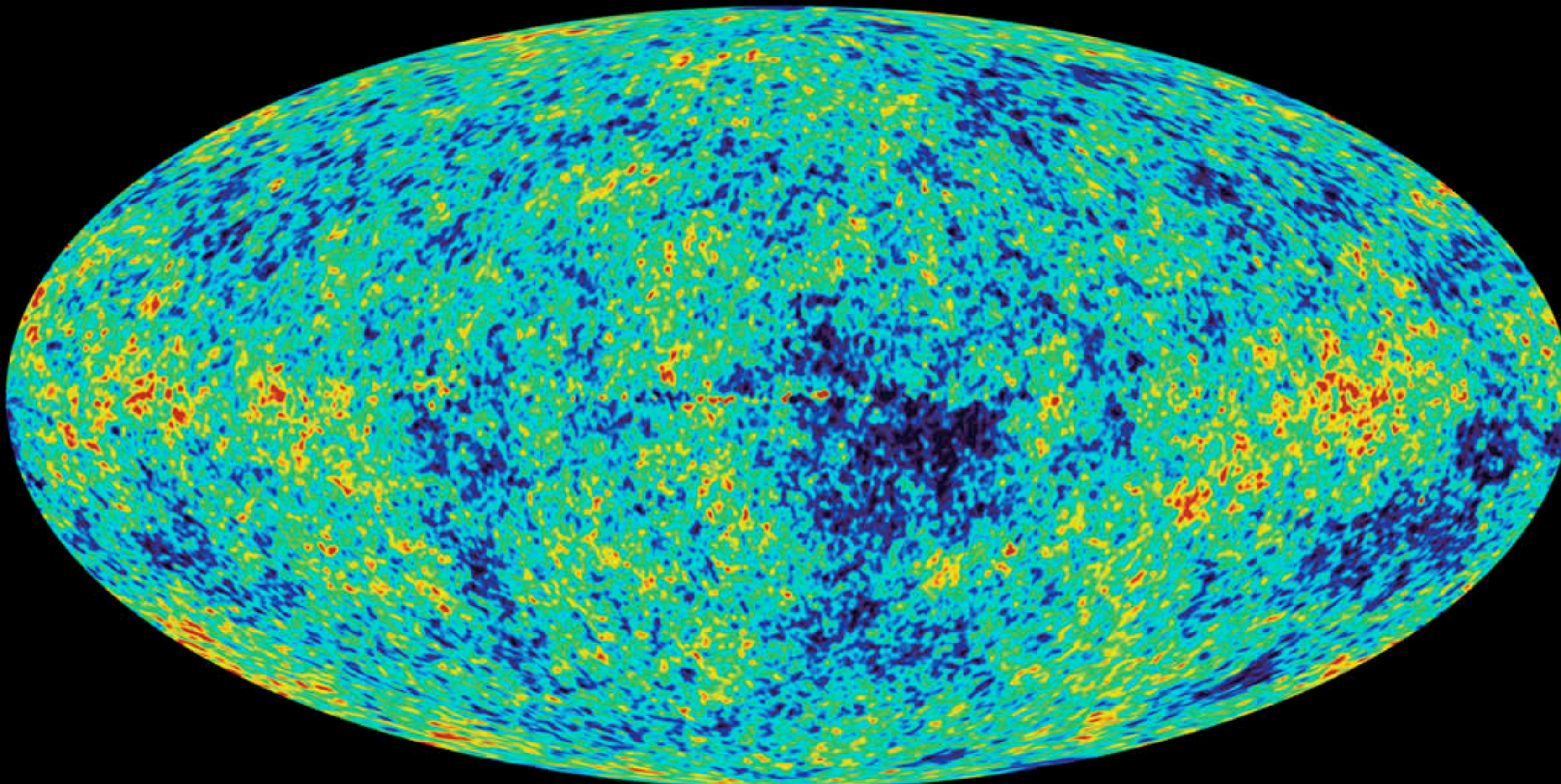
- Cosmic microwave background
- Nucleosynthesis; the first 3 minutes
- inflation: solving the flatness, horizon, and magnetic monopol problem by inflating the universe exponentially for a very short time starting at $t_i = 10^{-35}$ s

Penzias & Wilson (1965): 3˚ K background radiation

Bell Telephone Lab in Murray Hill (New Jersey)



The 2.75 ˚K background as seen by WMAP

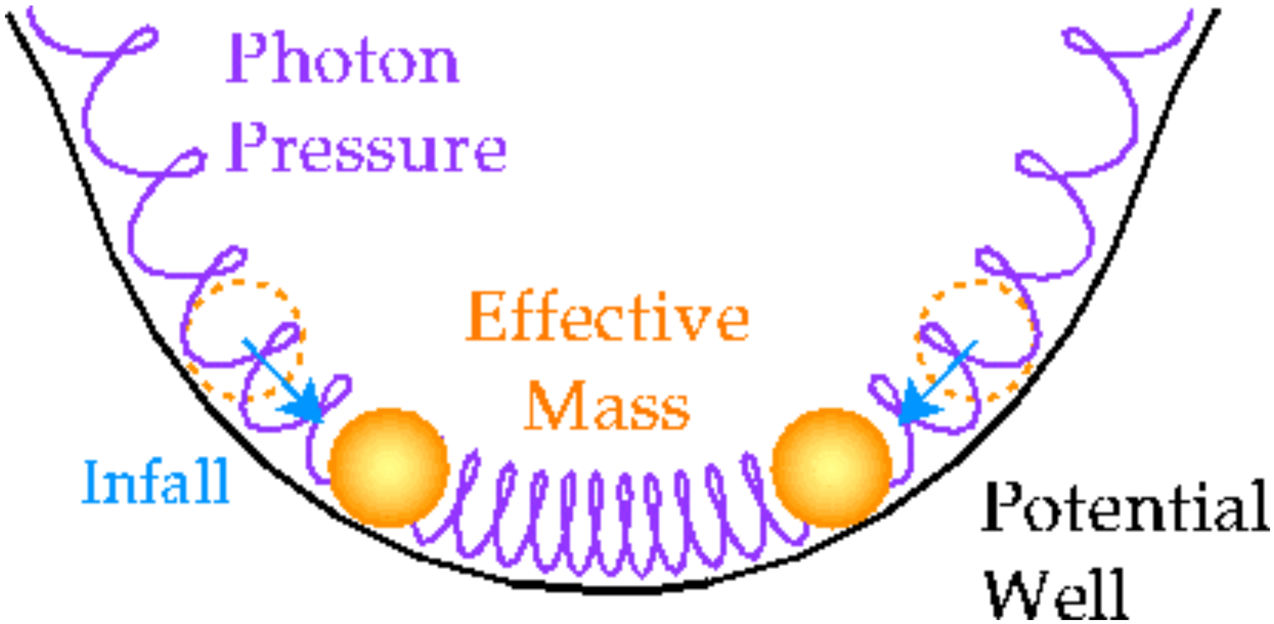


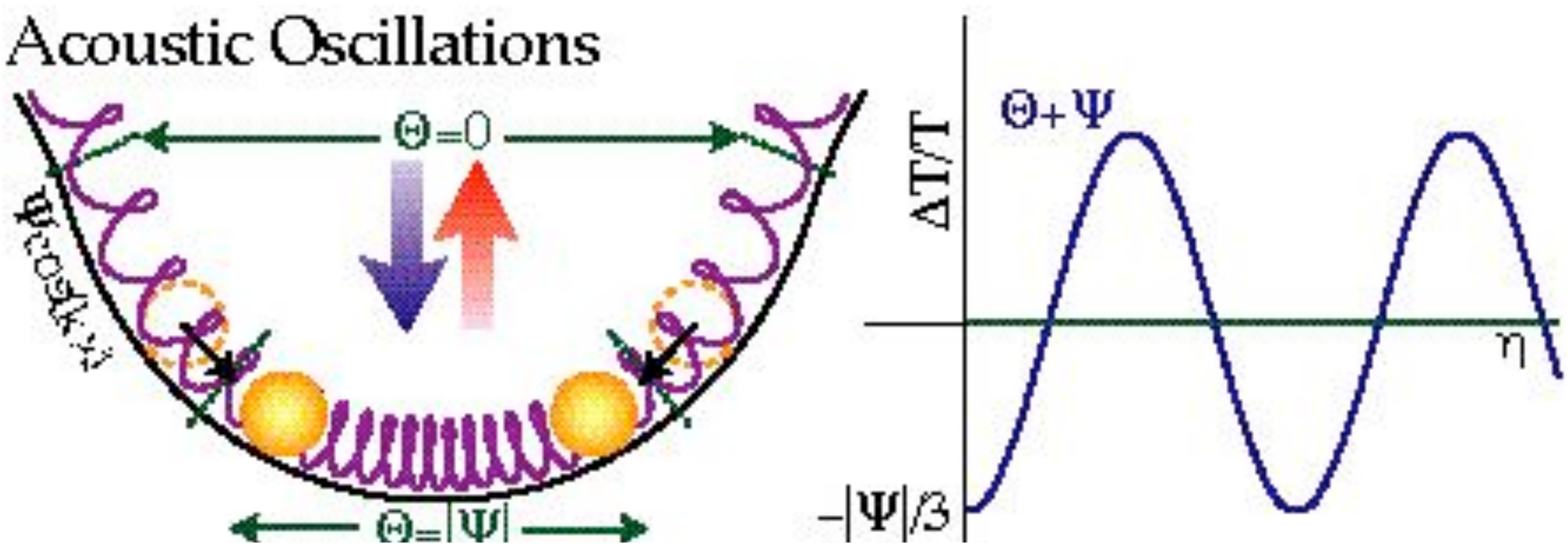
Credit: NASA/WMAP Science Team

Poisson's equation:

$$\nabla^2\phi = 4\pi G\rho$$

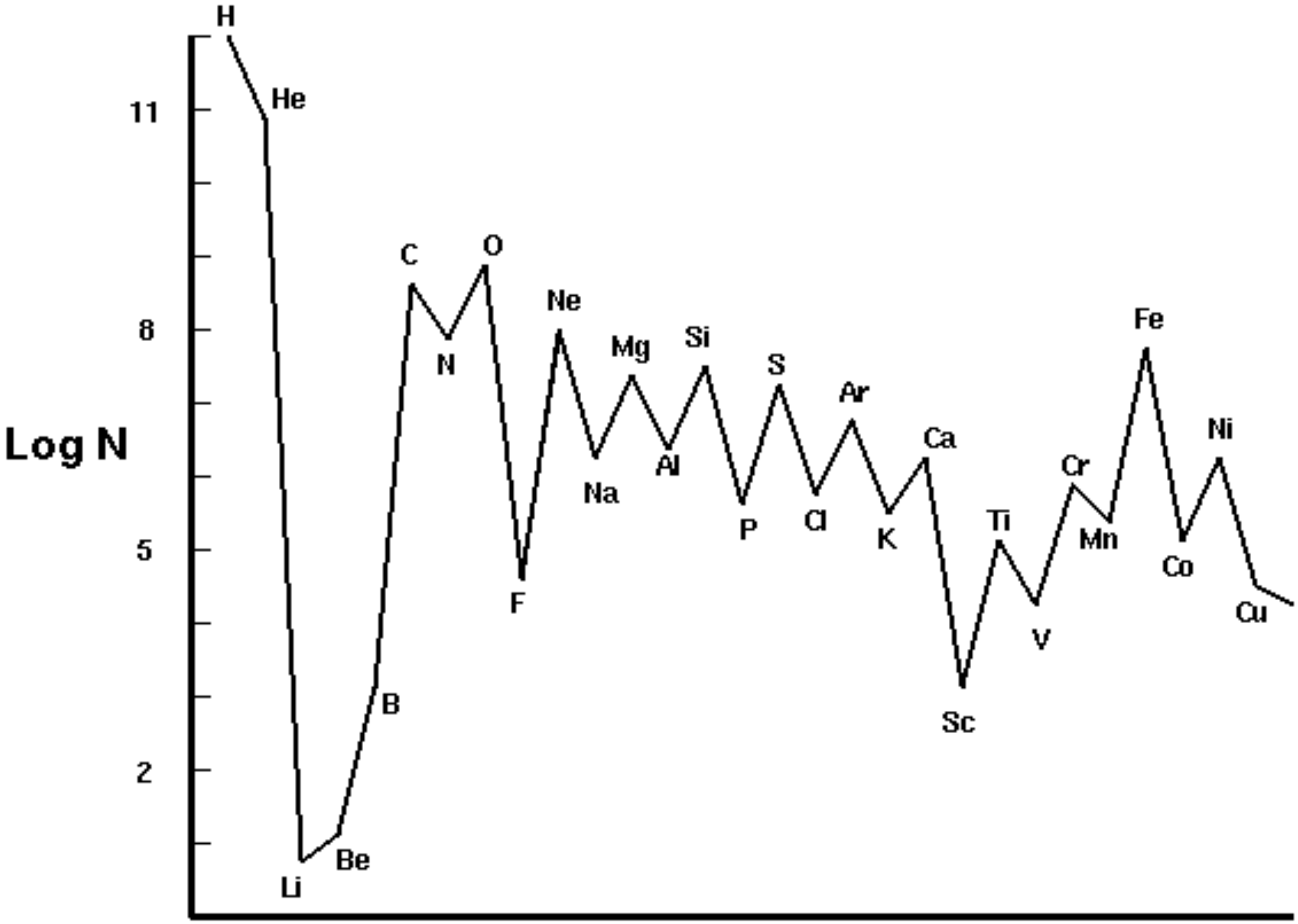
Acoustic Oscillations

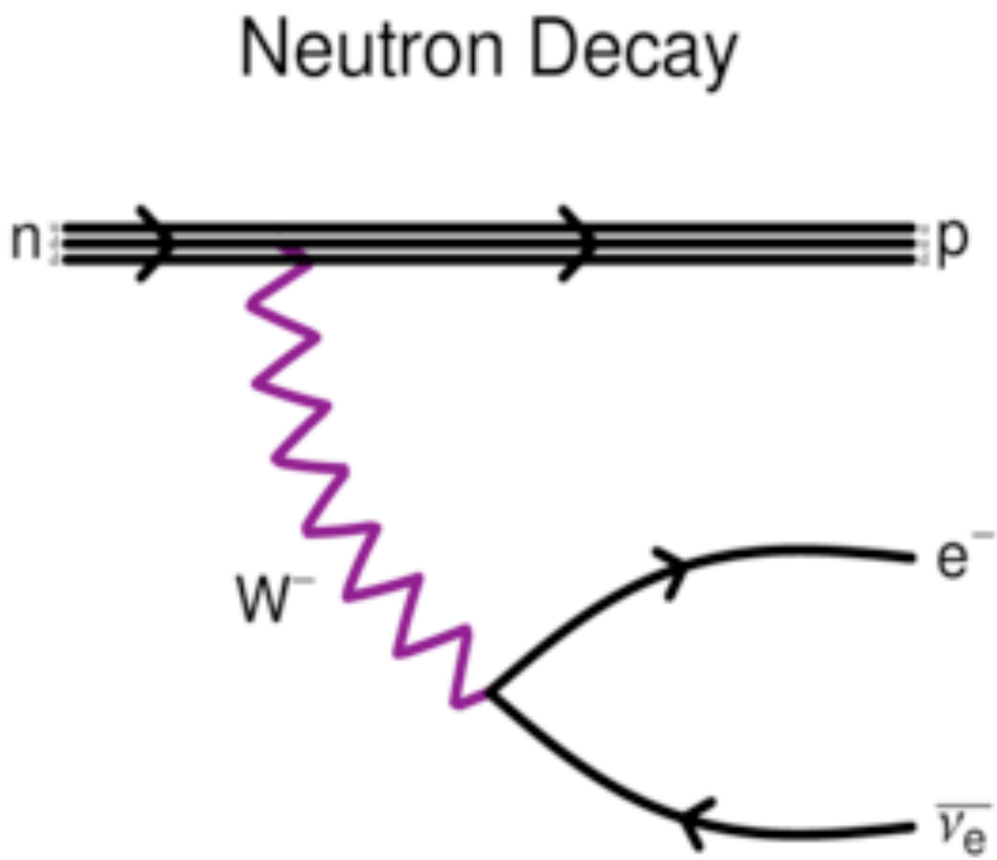




Where do the elements come from???

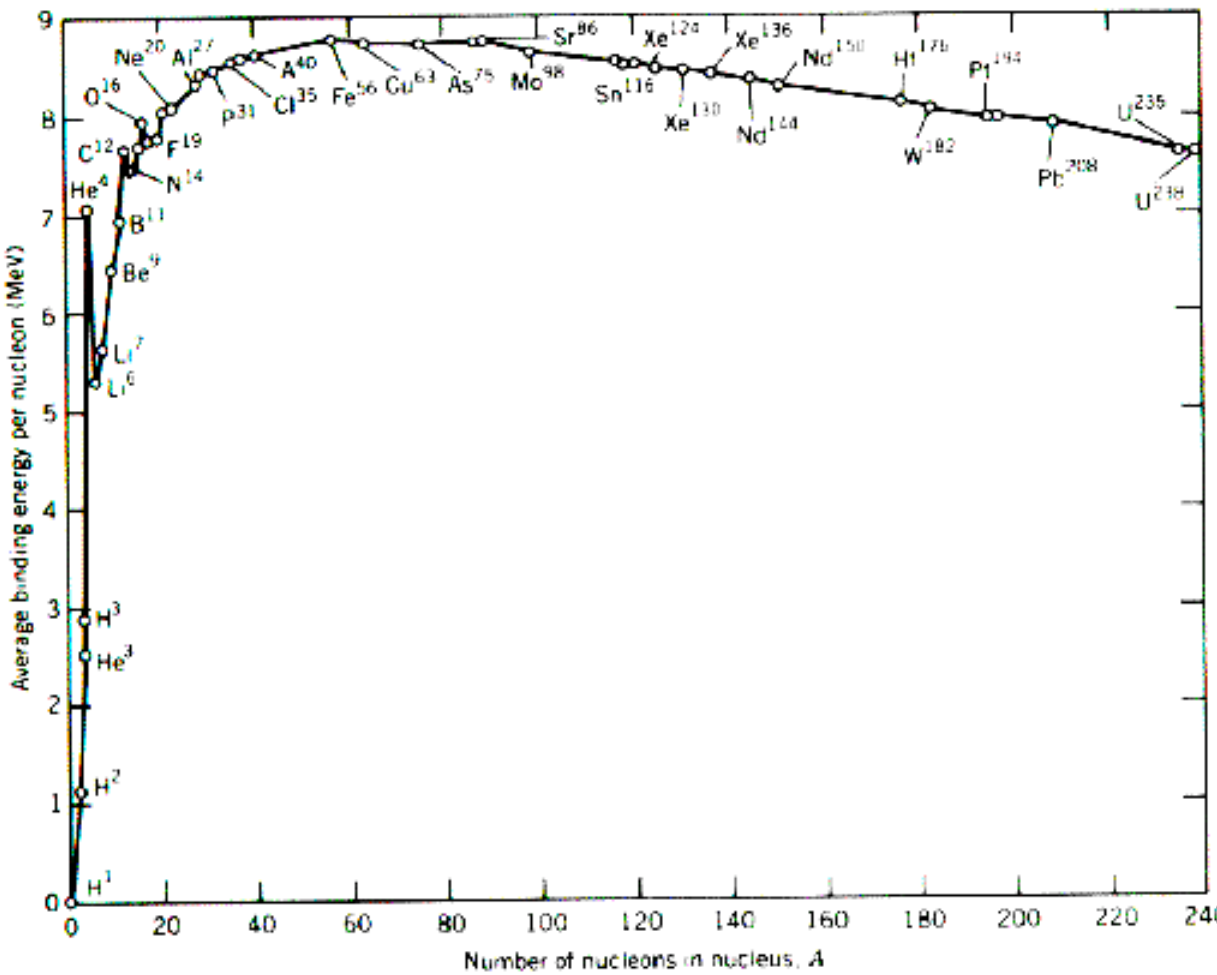
(beats every chemist or biologist!!!)



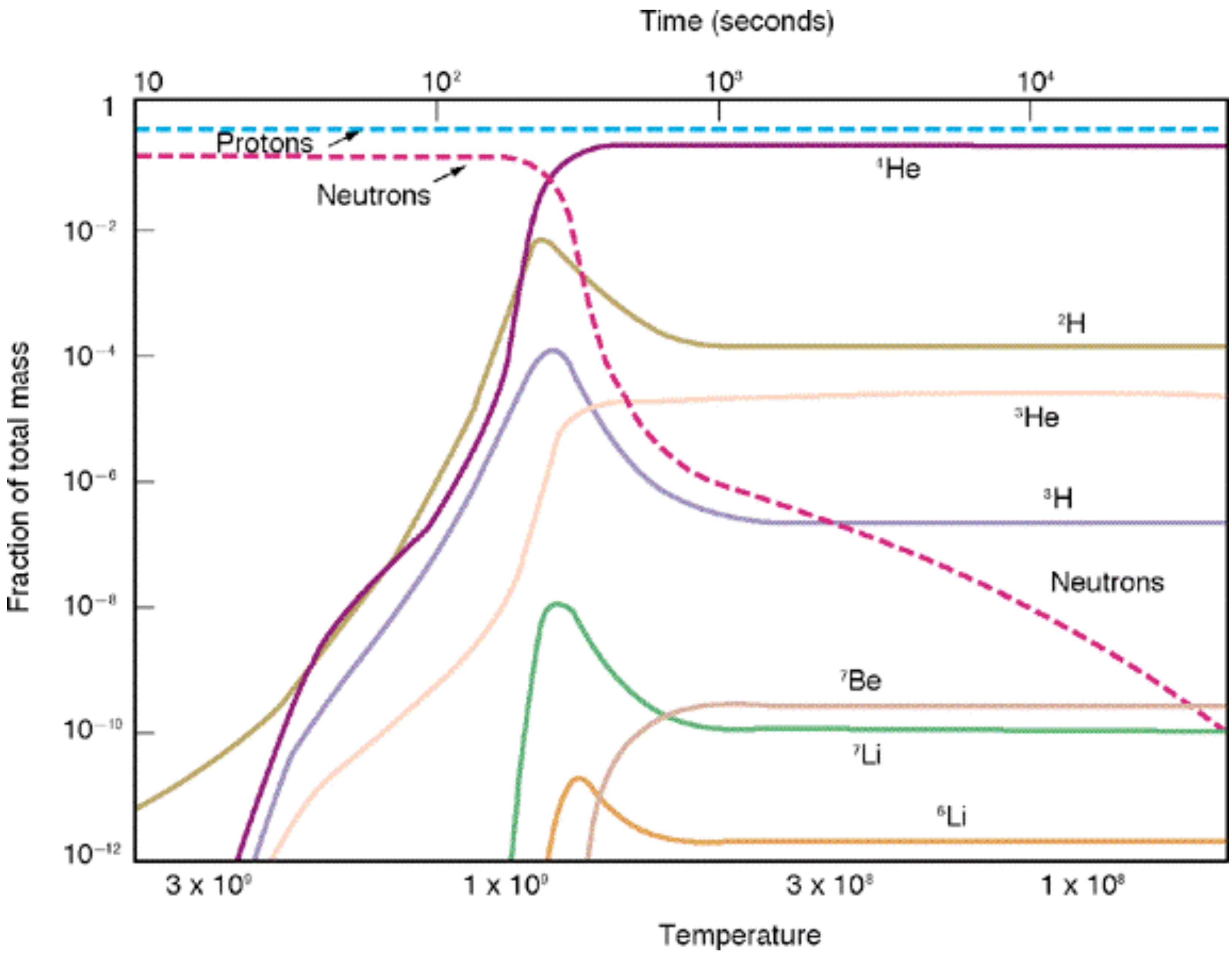


Neutron decay involves the weak force: small cross section which decreases with time (proportional to 1/t). Thus the neutron fraction freezes out after the universe is 1 second old. The neutron fraction ends up to be ~ 0.15

Illustration: Christine M. Griffin

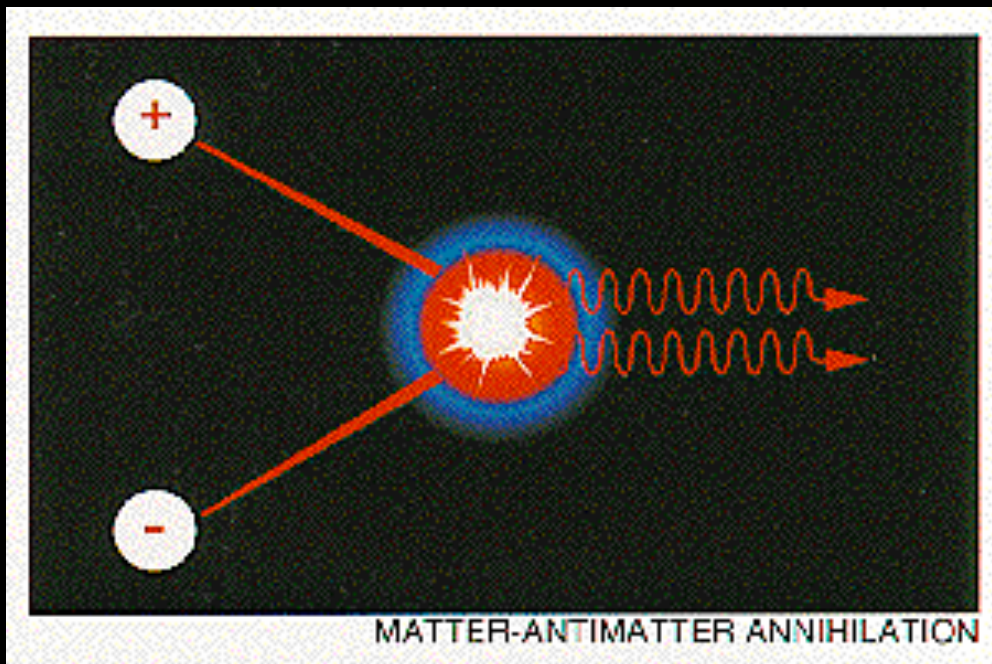


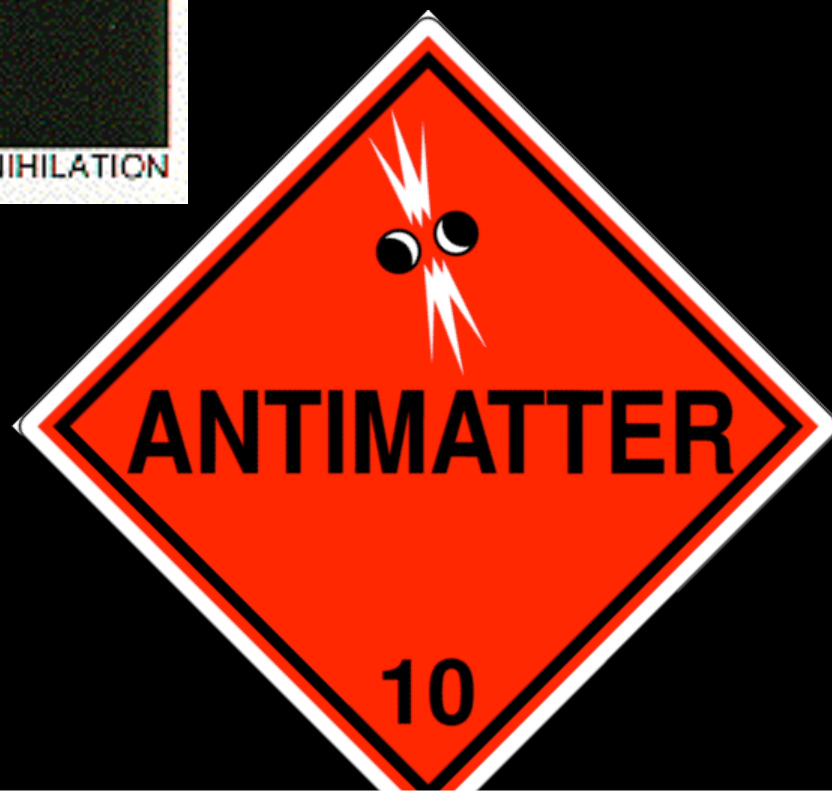
^{4}He is a "dead end" for primordial nucleosynthesis. Helium amounts



Primordial nucleosynthesis is finished after 1000 seconds

Why is the baryon-to-photon fraction $\eta = 5.5\ 10^{-10}$?





Our Universe model works fine so far, but…

- flatness problem (nearly flat but not totally flat)
- horizon problem (isotropic CMB)
- monopole problem (there are no magnetic monopoles)

Inflation of the Universe

- phase of the Universe with accelerated expansion

Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)$$

Alan Guth, MIT
(*1947)

Final Exam

- Thursday, 8:00 a.m. - 10 a.m. (here)
- show up 5 minutes early
- bring calculator, pen, paper
- no books or other material
- formulas provided on page 4 of exam
- questions are ordered in increasing difficulty

Final exam preparation

- Ryden Chapter 2 - 11 (incl.)
- look at homework (solutions)
- look at midterm #1 and #2
- check out the resources on the course web page (e.g. paper about benchmark model, videos of lectures)