Modelling and measuring the Universe

Summary

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The first part of the lecture

- Universe is expanding (Hubble relation)
- Newton's not enough: Einstein's idea abou
space-time
- General relativity for curved space-time
- Four equations to describe the
expanding/contracting universe

The second part of the lecture

- How to model the Universe
- the Friedmann equation as a function of density parameters
- Matter-, radiation-, lambda-, curvature- only universe
- mixed-component universes
- the important times in history: $a_{r,m}$ and $a_{m,\Lambda}$

The second part of the lecture

- How to measure the Universe
- the Friedmann equation expressed in a Taylor series: H_0 and q_0 (deceleration parameter)
- luminosity distance, angular size distance
-distance ladder: parallax, Cepheids, SuperNova Type Ia
- results from the SuperNova measurements

The second part of the lecture

- What is the matter contents of the Universe?
- matter in stars
- matter between stars
- matter in galaxy clusters
- dark matter

Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{\kappa c^2}{R_0^2 a^2}$$

Fluid equation

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

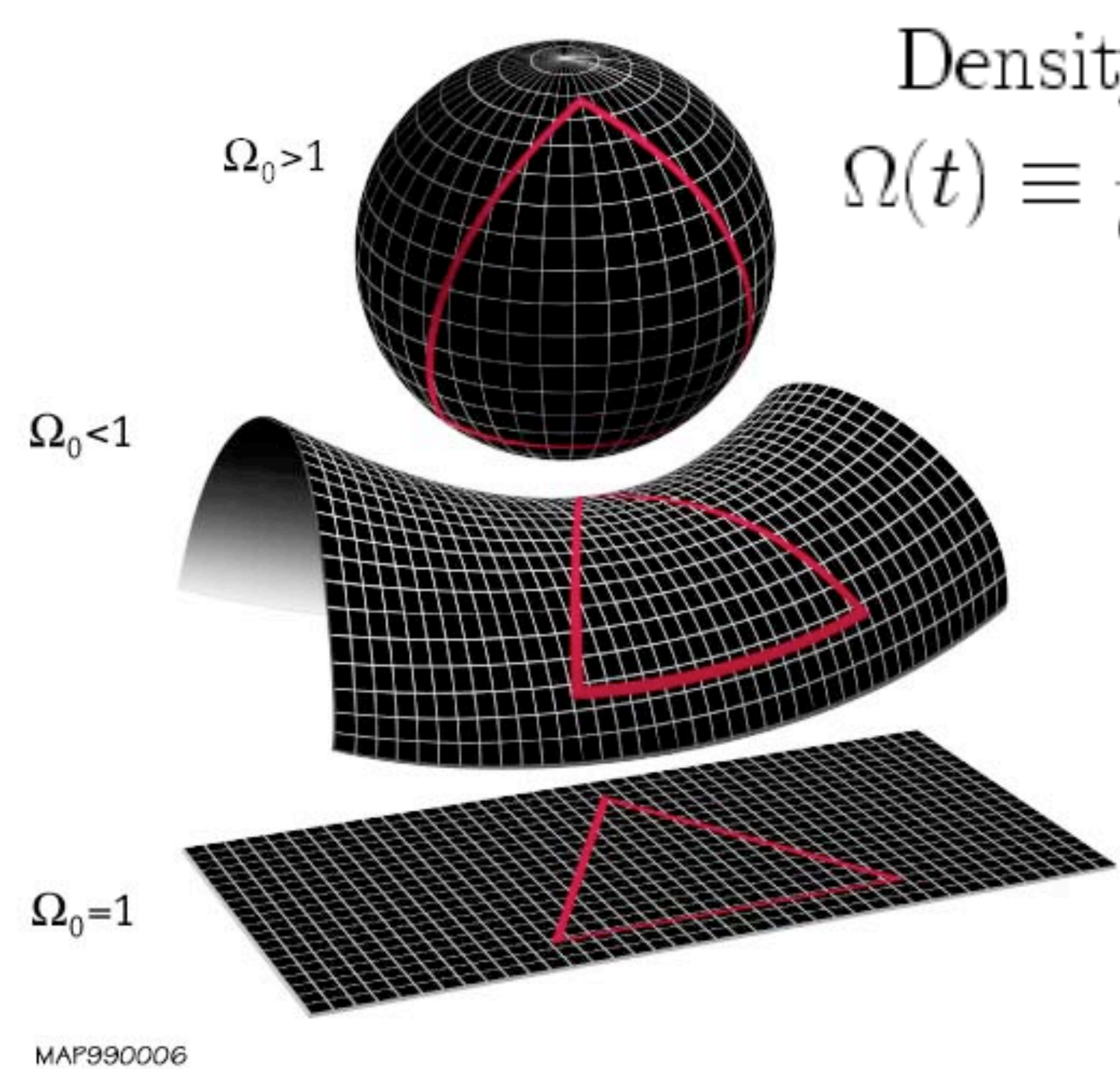
Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)$$

Equation of state

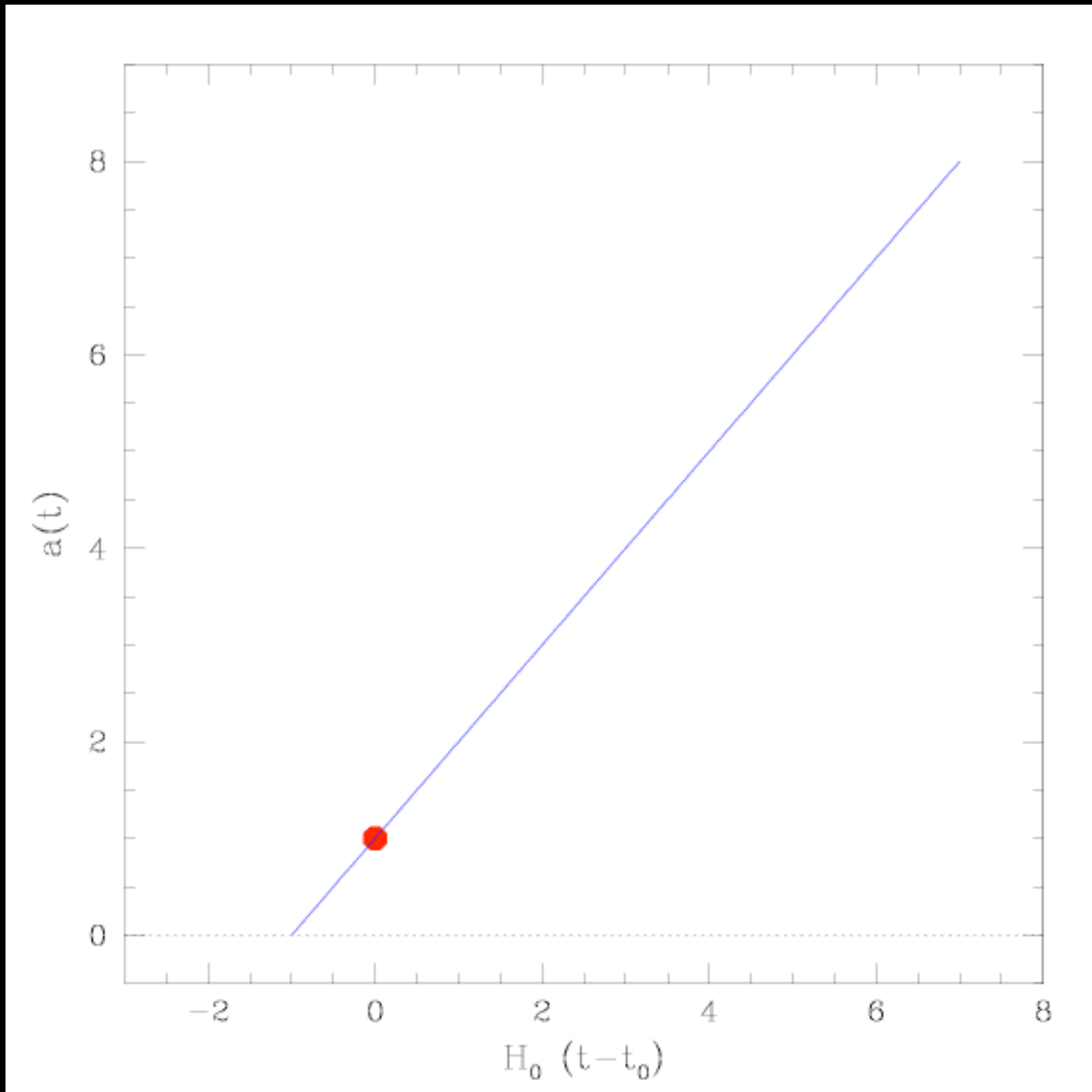
$$P = \omega\,\epsilon$$

Density parameter Ω and curvature



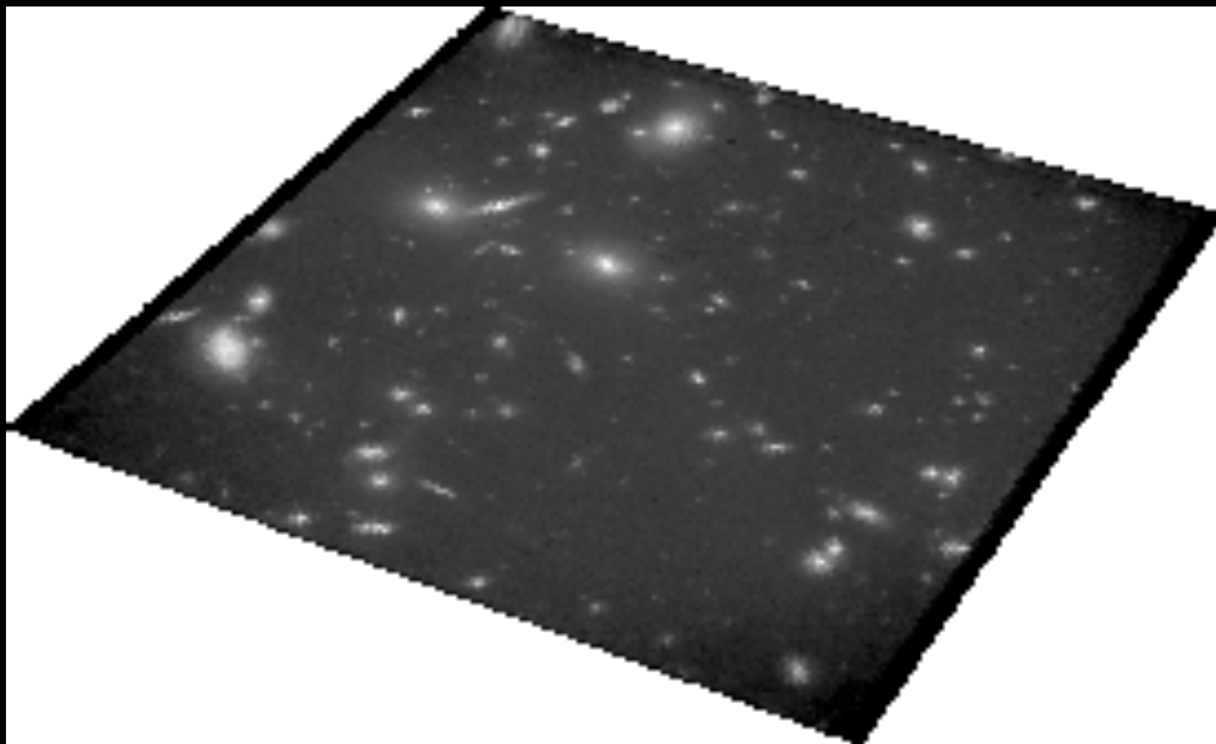
Density parameter:

$$\Omega(t) \equiv \frac{\epsilon(t)}{\epsilon_c(t)}$$



Scale factor a(t) in an empty universe

Spatially flat Universe



Our key questions for any type of Universe:
Scale factor a(t) ? What is the age of the Universe t_0 ?
Energy density $\varepsilon(t)$? Distance of an object with redshift z ?

Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_{\omega} \epsilon_{w,0}\, a^{-1-3\omega} - \frac{\kappa\, c^2}{R_0^2}$$

What happens in a flat universe ?
One component only ?

Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_{\omega} \epsilon_{w,0}\, a^{-1-3\omega} - \frac{\kappa\, c^2}{R_0^2}$$

Friedmann equation (flat, single-component):

$$\dot{a}^2 = \frac{8\pi G\, \epsilon_0}{3\, c^2}\, a^{-1-3\omega}$$

Flat, single component universe:

$$H_0 \equiv \left(\frac{\dot{a}}{a}\right)_{t=t_0} = \frac{2}{3(1+\omega)}\, t_0^{-1}$$

Friedmann equation (flat, single-component):
$$\dot{a}^2 = \frac{8\pi G\,\epsilon_0}{3\,c^2}\,a^{-1-3\omega}$$

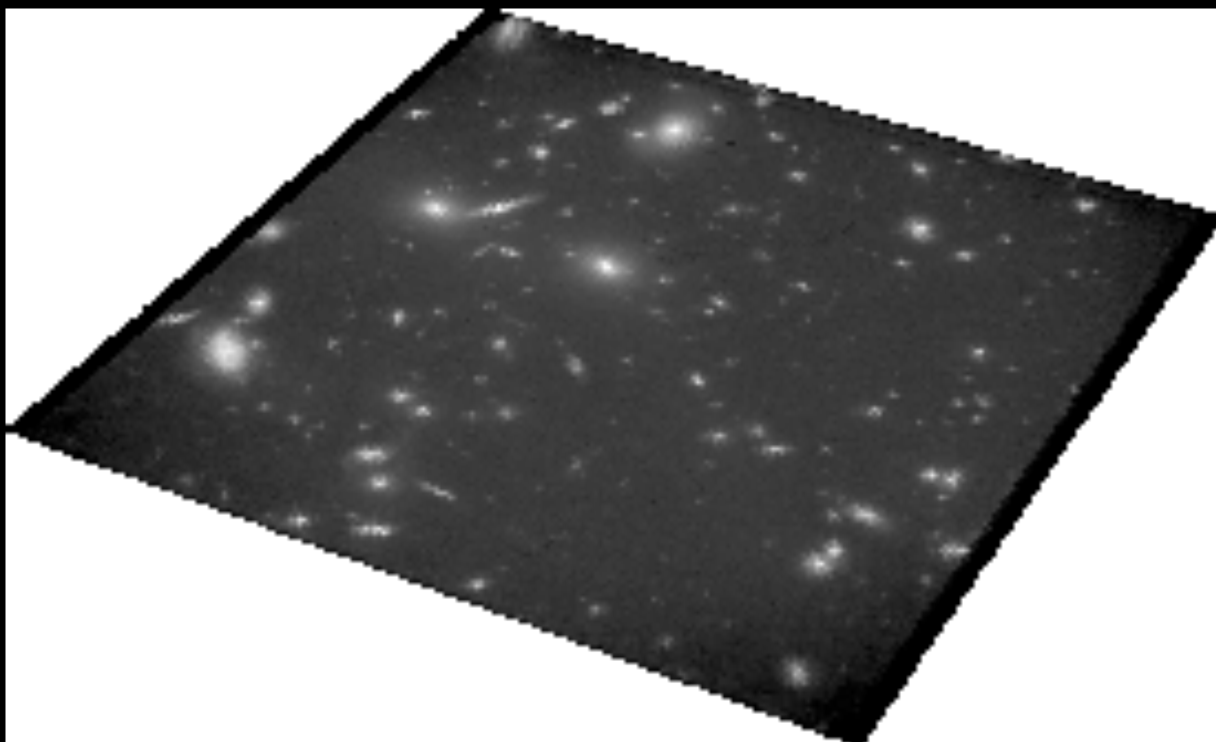
Flat, single component universe:
$$H_0 \equiv \left(\frac{\dot{a}}{a}\right)_{t=t_0} = \frac{2}{3(1+\omega)}\,t_0^{-1}$$

Flat, single component universe:
$$t_0 = \frac{2}{3\,(1+\omega)}\,H_0^{-1}$$

Proper distance:
$$d_P(t_0) = \frac{c}{H_0}\,\frac{2}{1+3\omega}\,\left[1-(1+z)^{-(1+3\omega)/2}\right]$$

Spatially flat Universe with matter only

aka Einstein-de Sitter Universe



Willem de Sitter
(1872 - 1934)

Our key questions for any type of Universe:
Scale factor $a(t)$? What is the age of the Universe t_0 ?
Energy density $\varepsilon(t)$? Distance of an object with redshift z ?

Friedmann equation (flat, single-component):

$$\dot{a}^2 = \frac{8\pi G\,\epsilon_0}{3\,c^2}\,a^{-1-3\omega}$$

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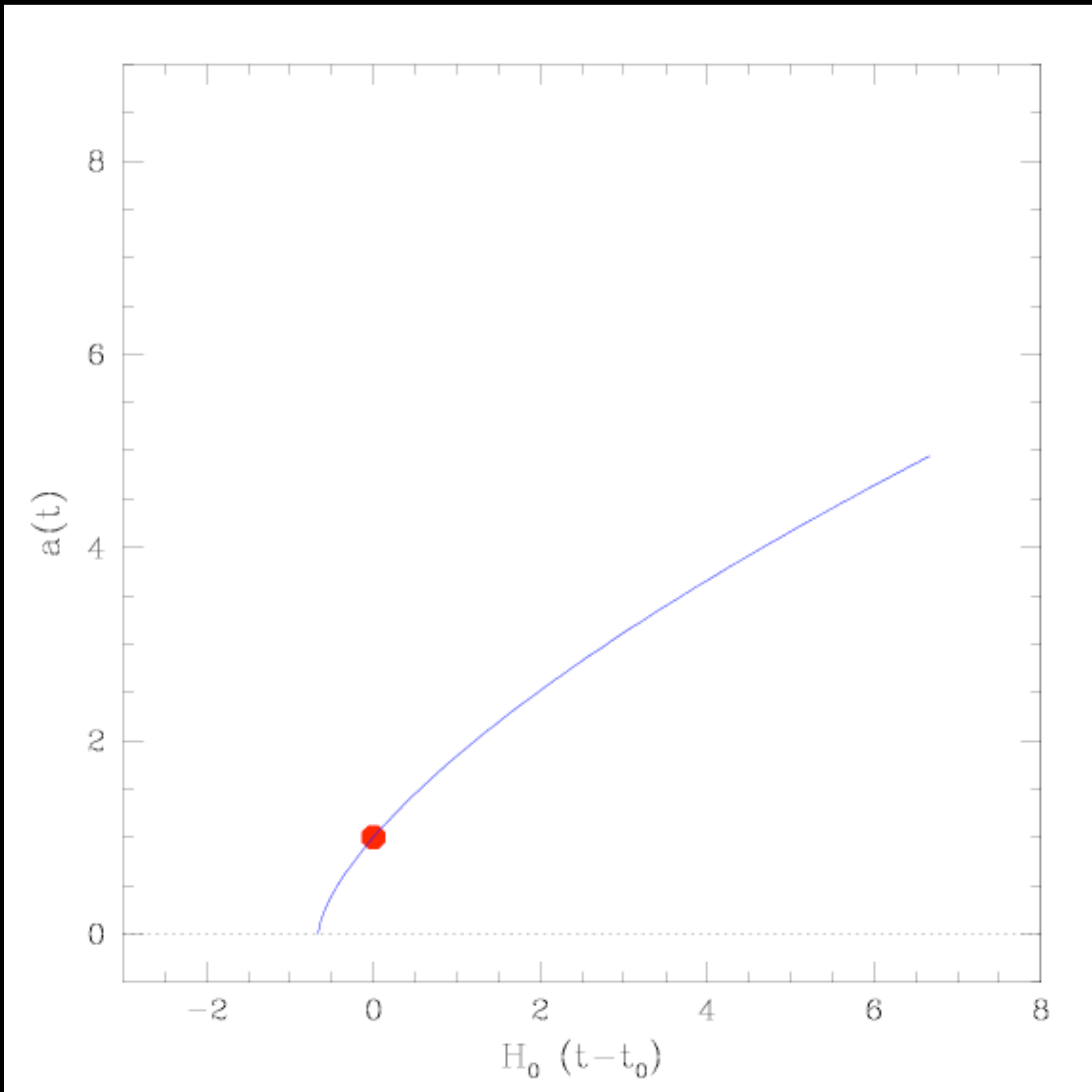
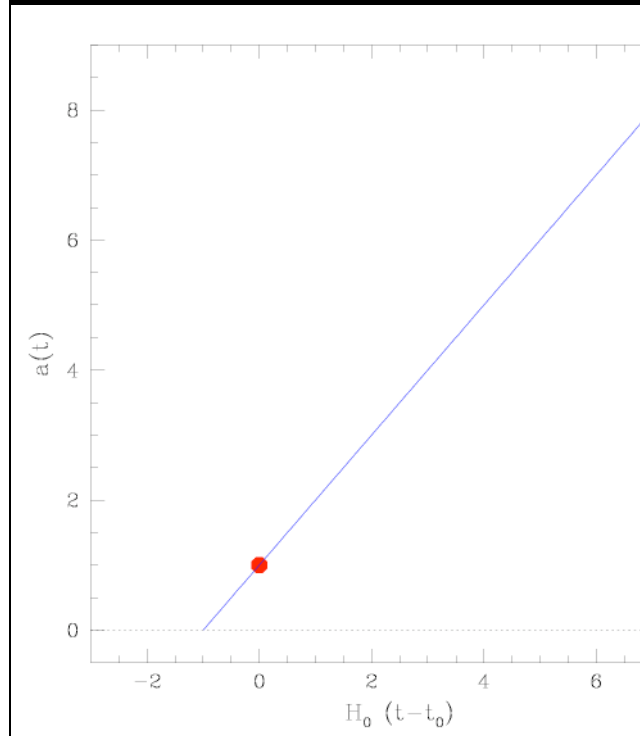
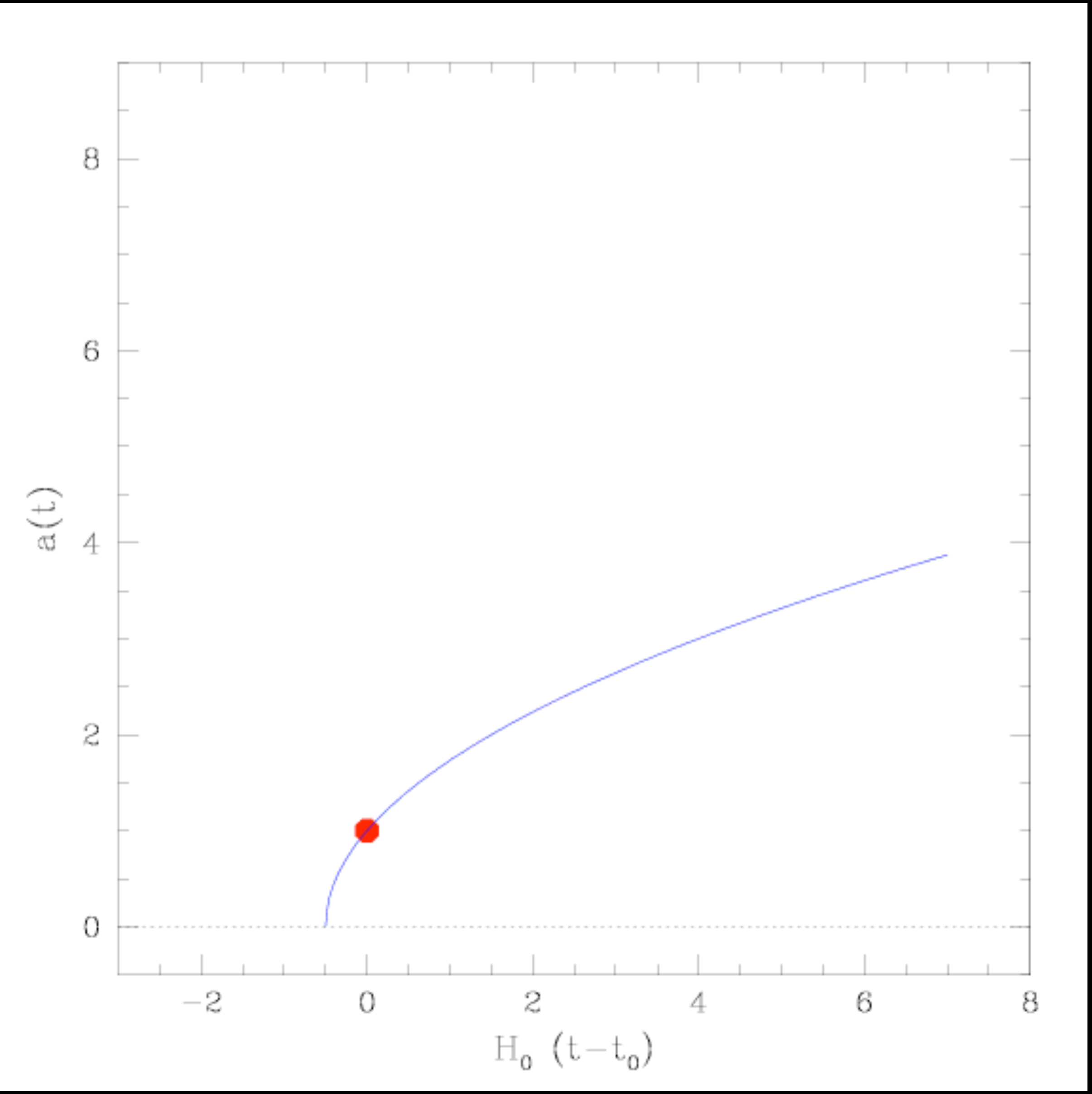
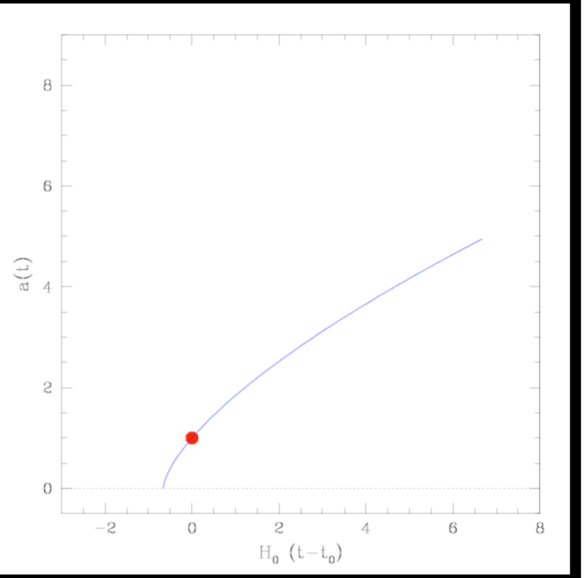


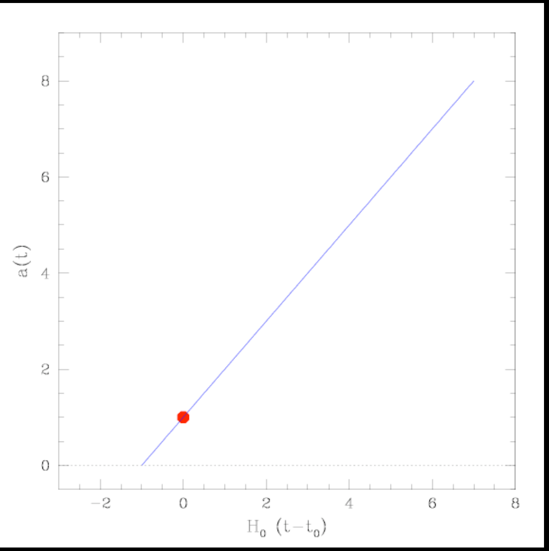


empty universe

Scale factor a(t) in a matter-only (nonrelativistic) universe



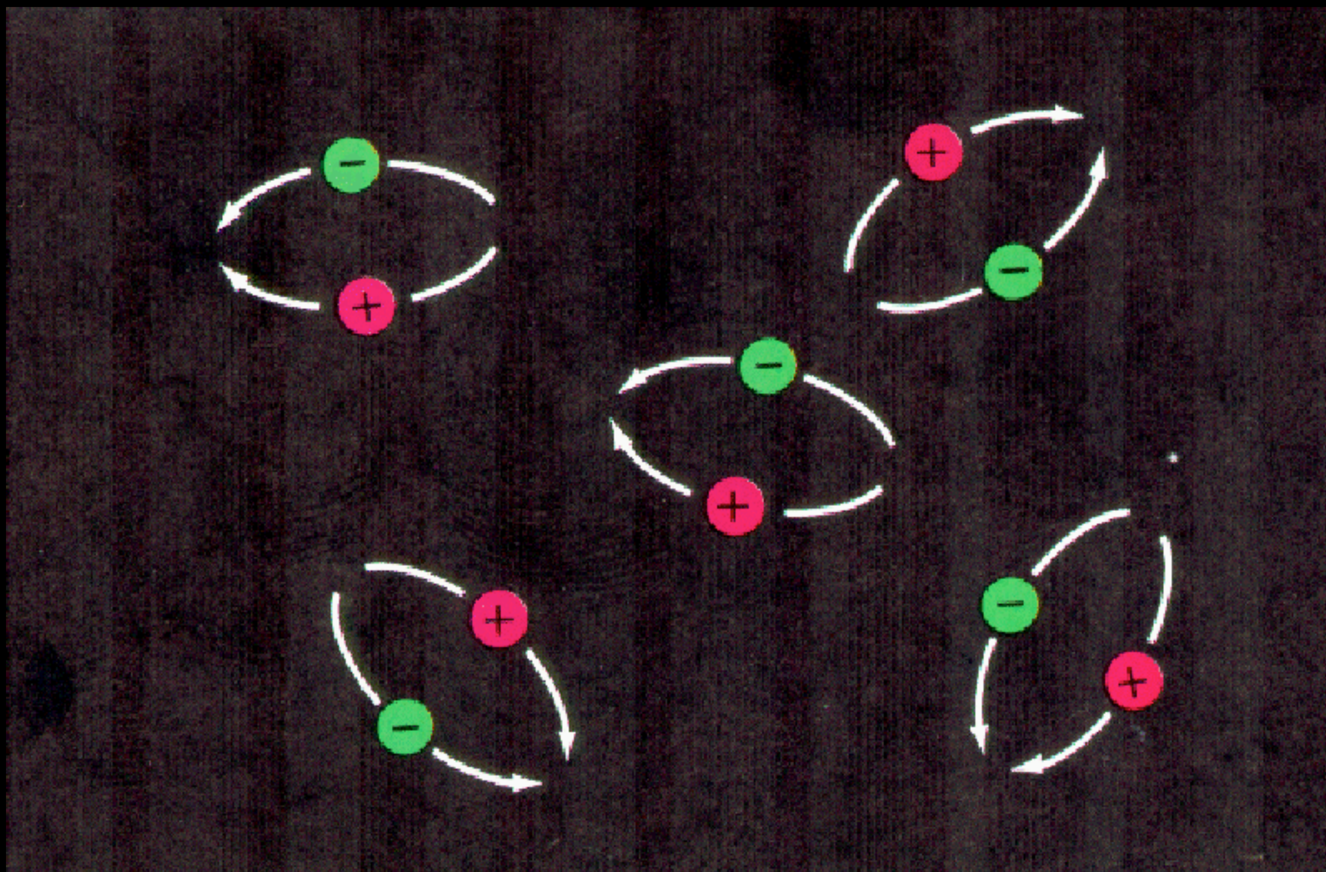




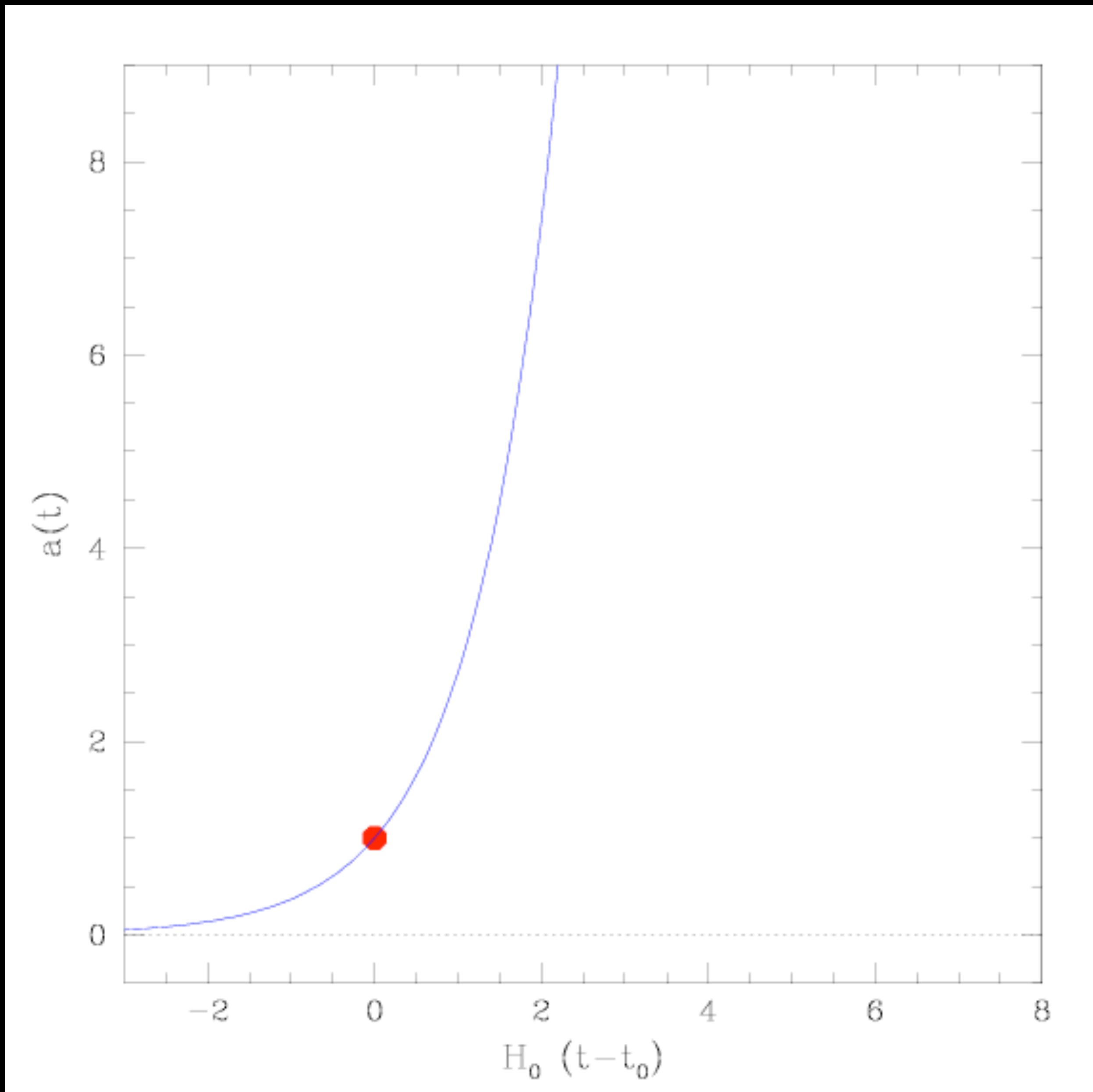
Scale factor a(t) in a radiation-only universe

Matter (top) & empty (bottom) universe

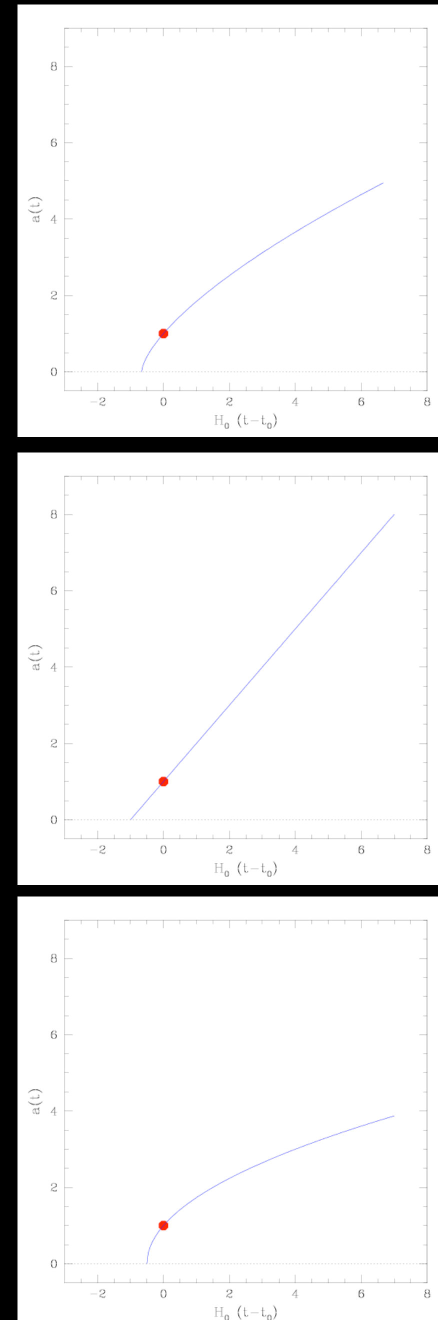
Spatially flat Universe with dark energy only



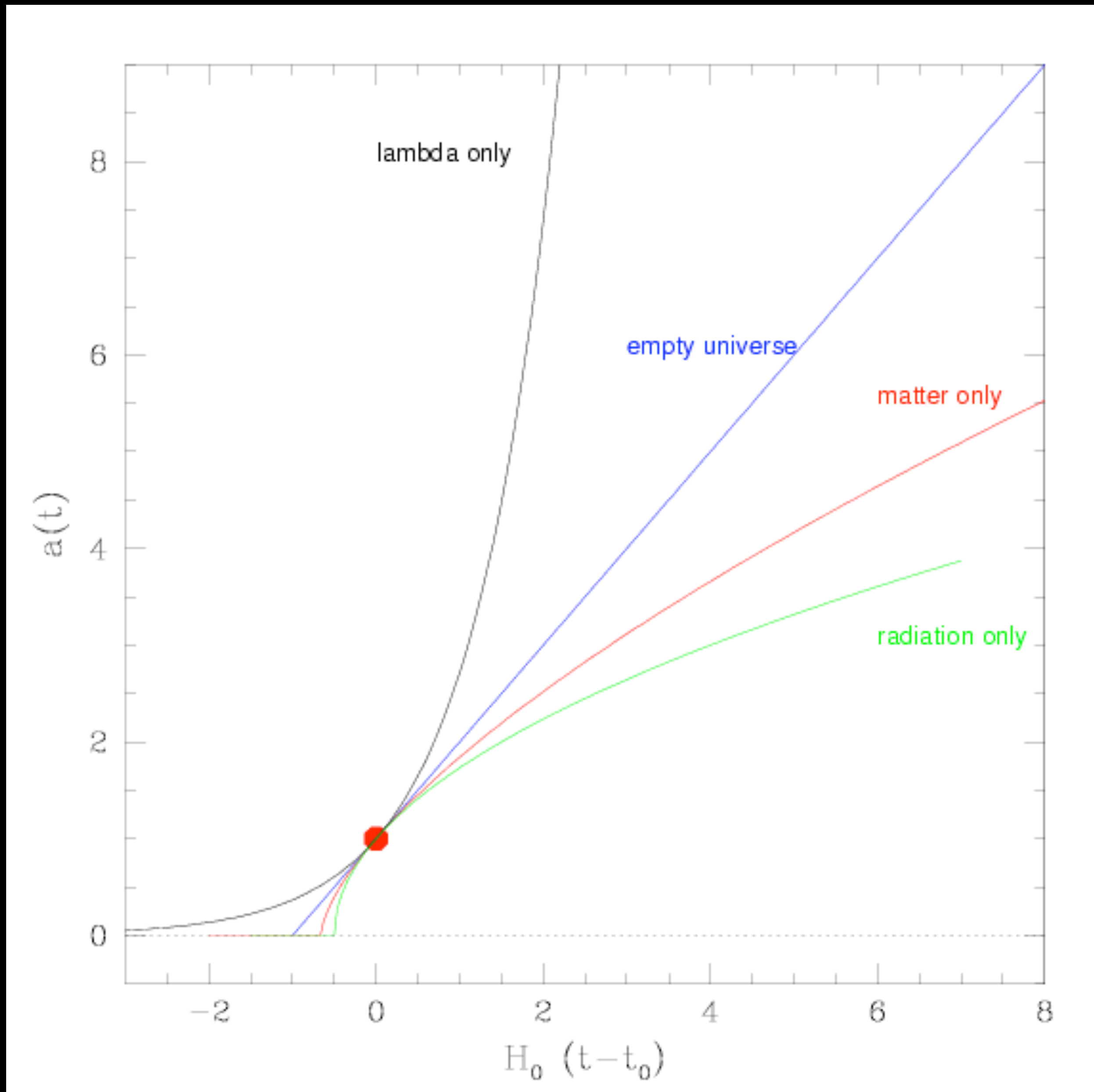
Our key questions for any type of Universe:
Scale factor a(t) ? What is the age of the Universe t_0 ?
Energy density $\varepsilon(t)$? Distance of an object with redshift z ?



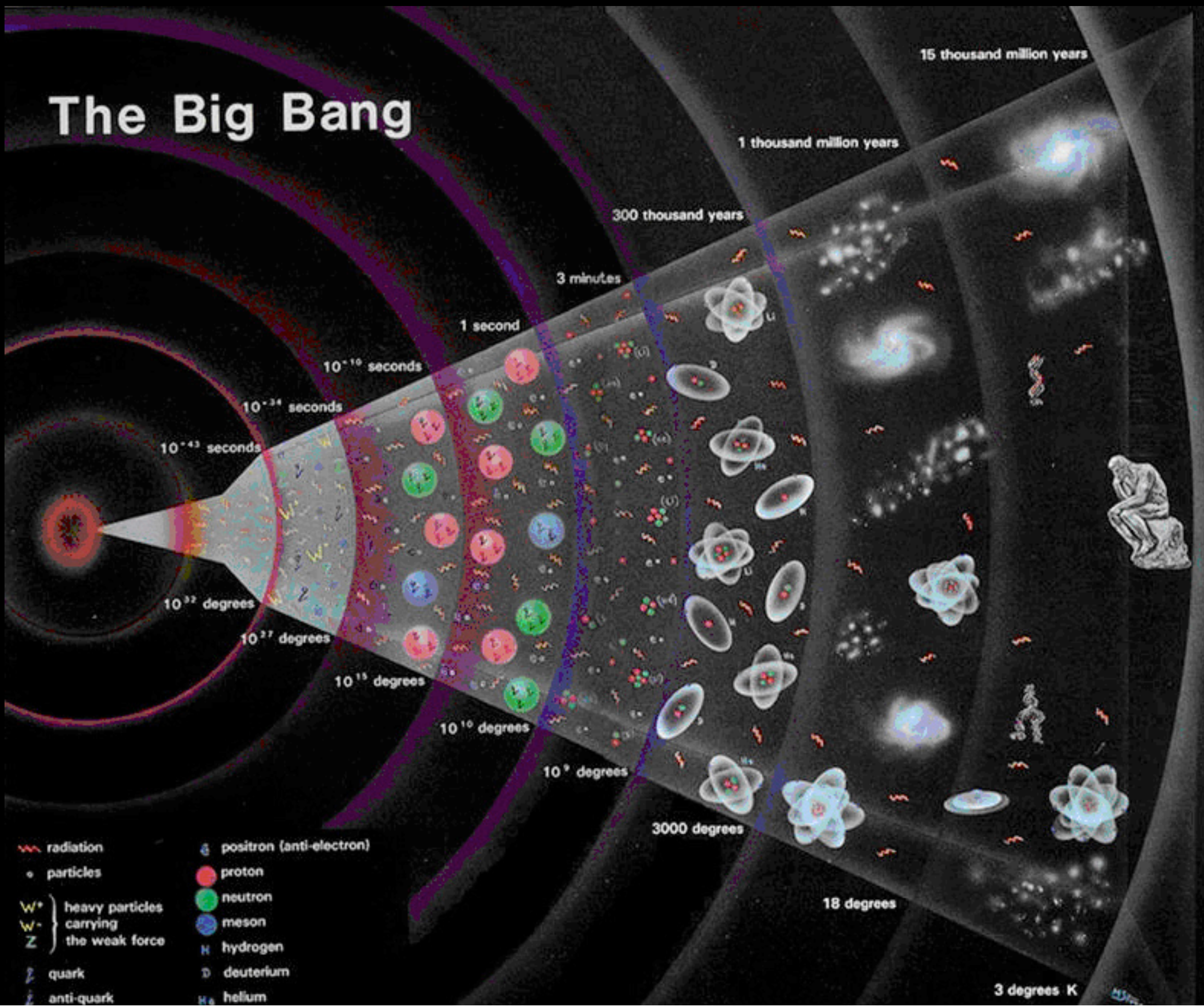
Scale factor a(t) in a lambda-only universe



Matter, empty, radiati



Scale factor a(t) in a flat, single component universe



The Big Bang
15 thousand million years
1 thousand million years
300 thousand years
3 minutes
1 second
10⁻¹⁰ seconds
10⁻³⁴ seconds
10⁻⁴³ seconds
10³² degrees
10²⁷ degrees
10¹⁵ degrees
10¹⁰ degrees
10⁹ degrees
3000 degrees
18 degrees
3 degrees K
radiation
particles
W+ W- Z heavy particles carrying the weak force
quark
anti-quark
positron (anti-electron)
proton
neutron
meson
hydrogen
deuterium
helium

Universe with matter and curvature only

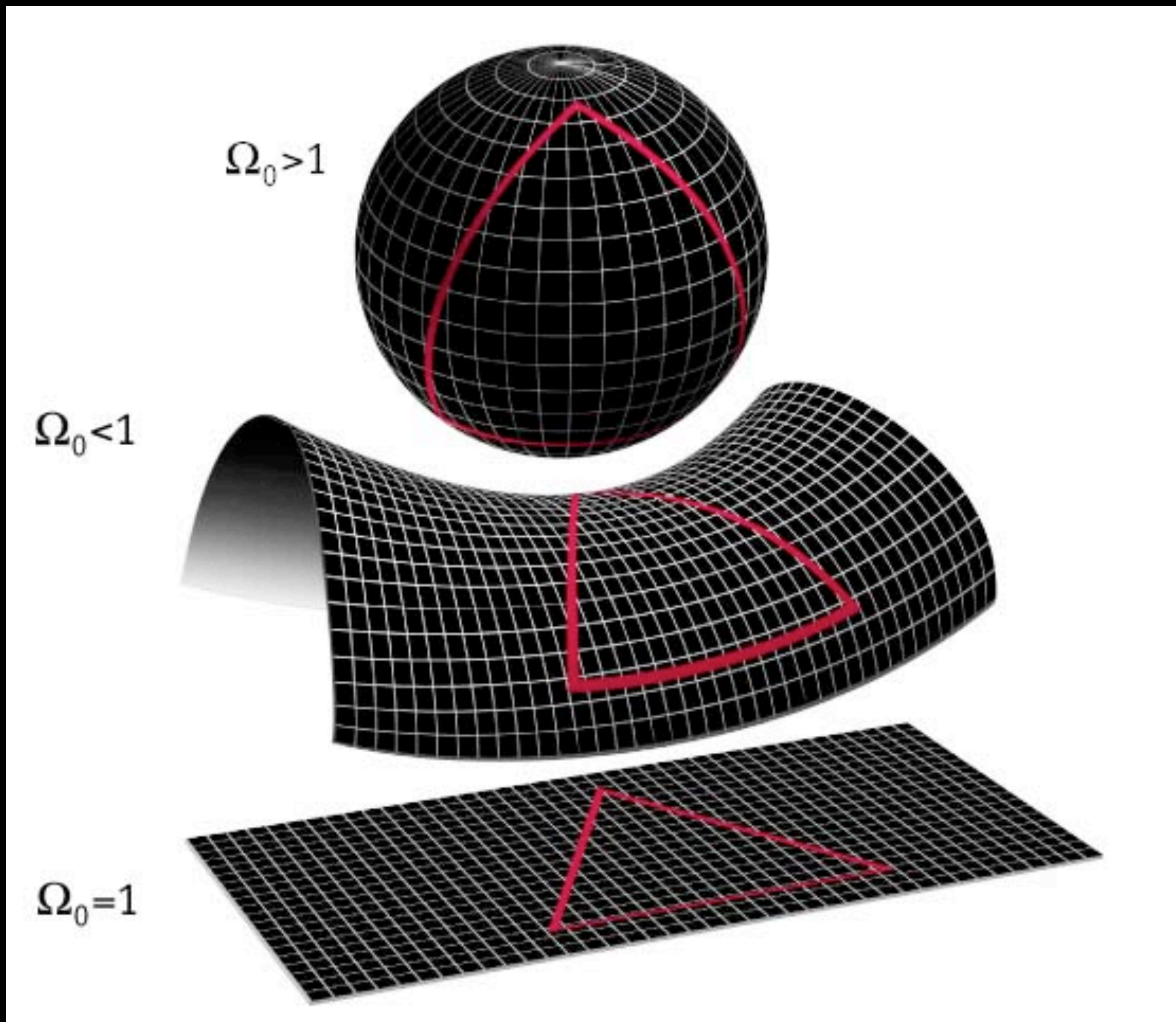
$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$$P = \omega\,\epsilon$$

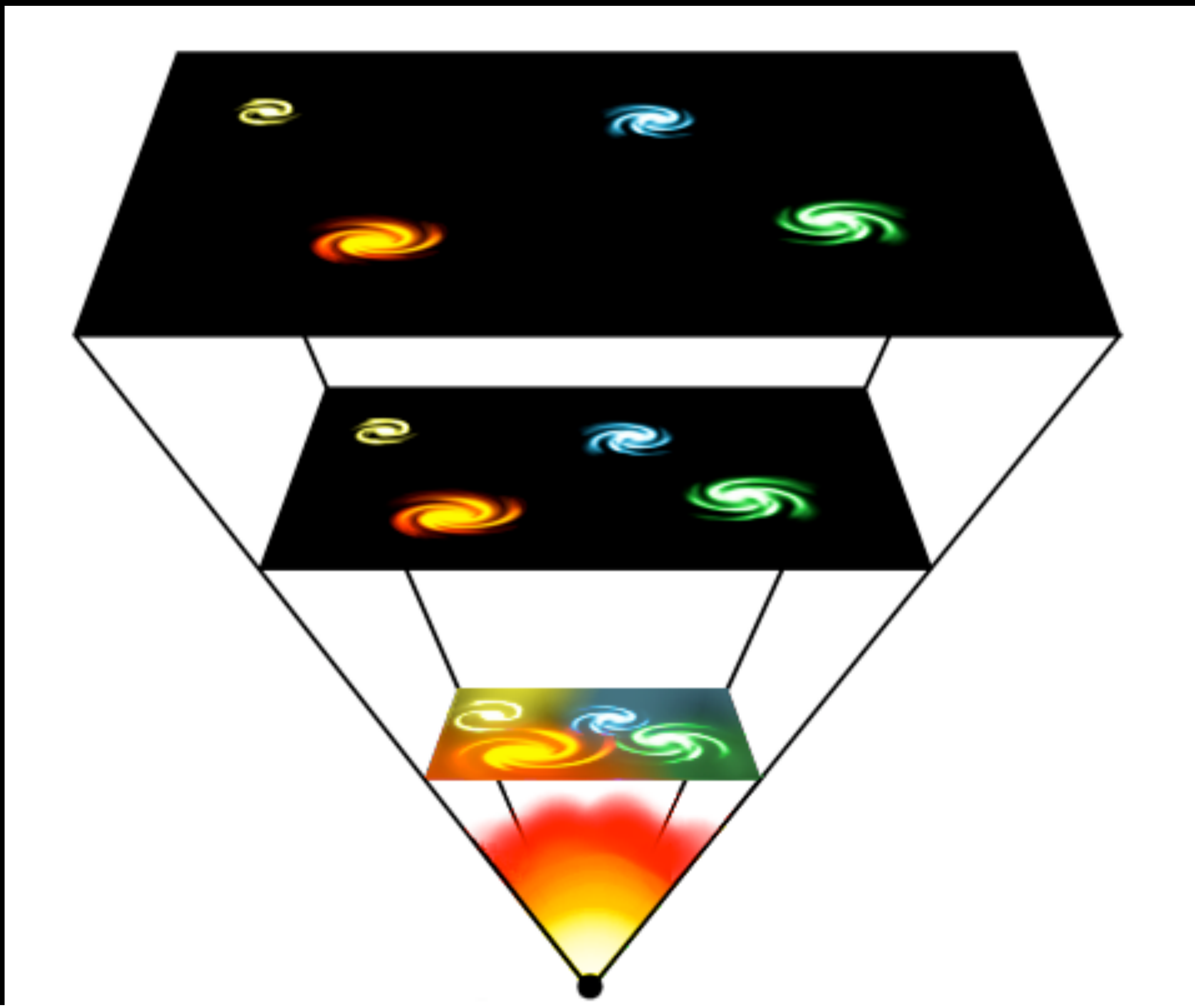
Density parameter:

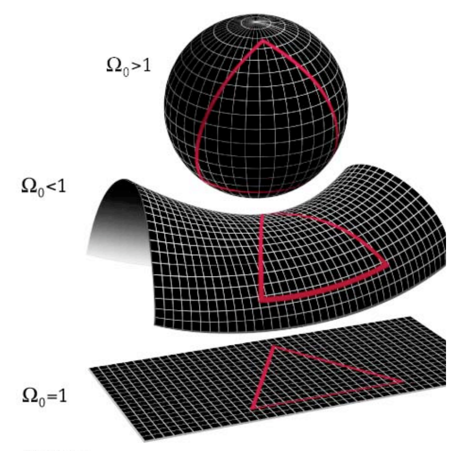
$$\Omega(t) \equiv \frac{\epsilon(t)}{\epsilon_c(t)}$$

Universe with matter and curvature only



Universe with matter and curvature only



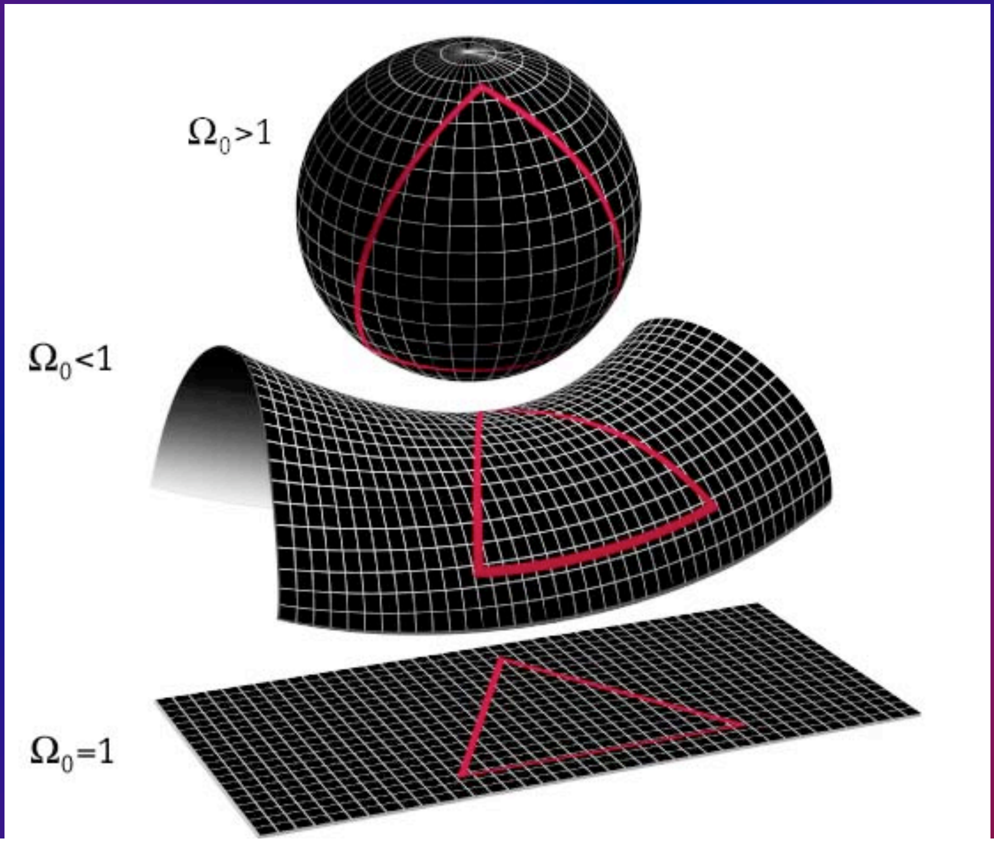


Curved, matter dominated Universe

$\Omega_0 < 1$ $\kappa = -1$ Big Chill ($a \propto t$)

$\Omega_0 = 1$ $\kappa = 0$ Big Chill ($a \propto t^{2/3}$)

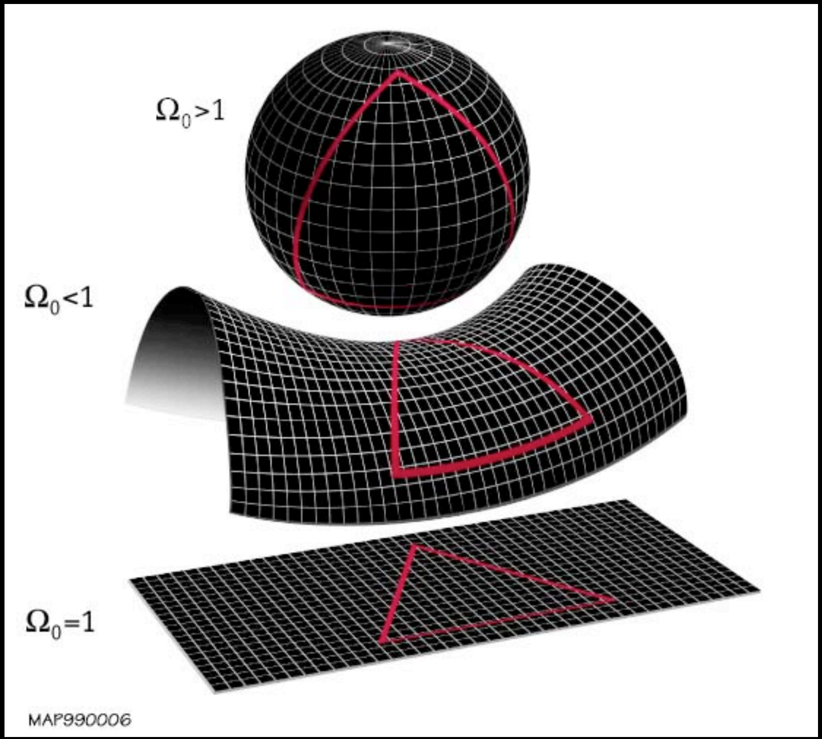
$\Omega_0 > 1$ $\kappa = -1$ Big Crunch



Universe with matter and curvature only

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

What is a(t) ?

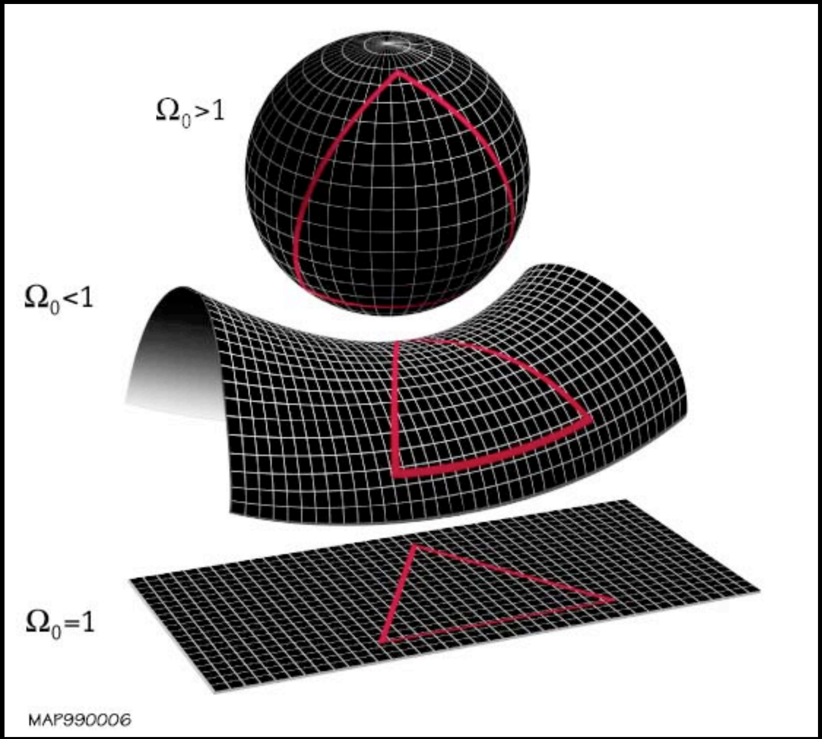


Universe with matter and Λ

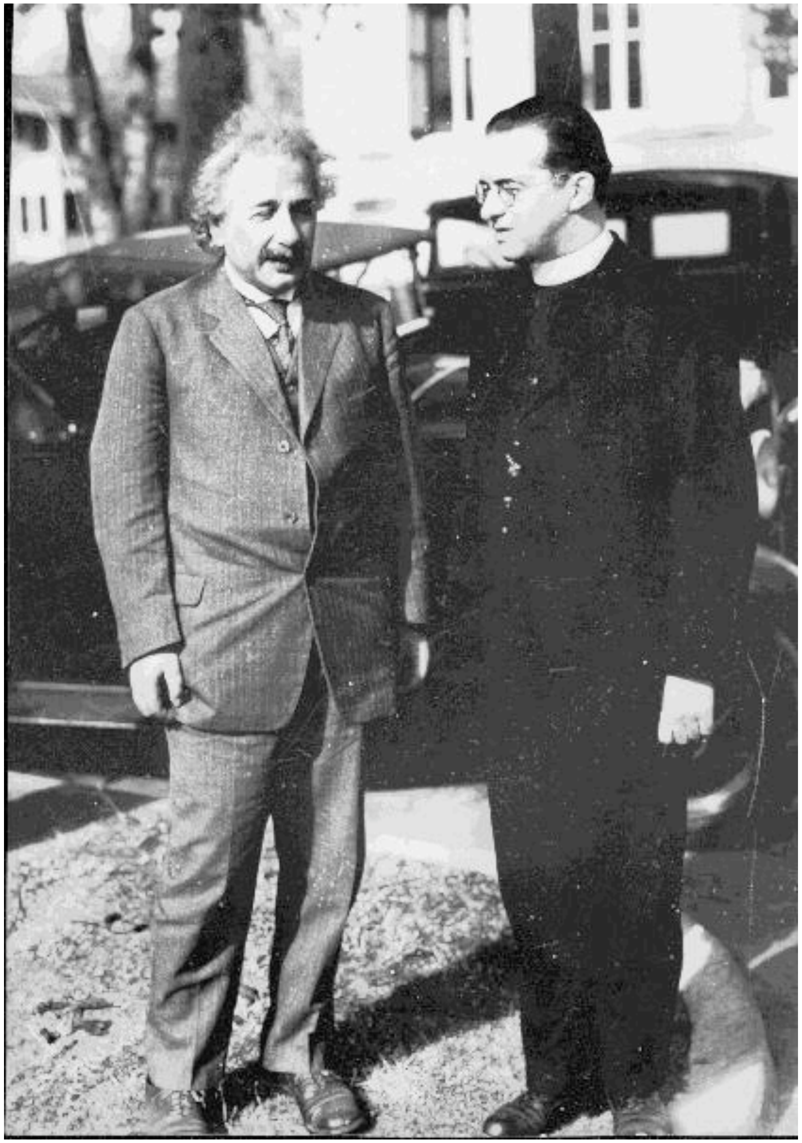
(matter + cosmological constant, no curvature)

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

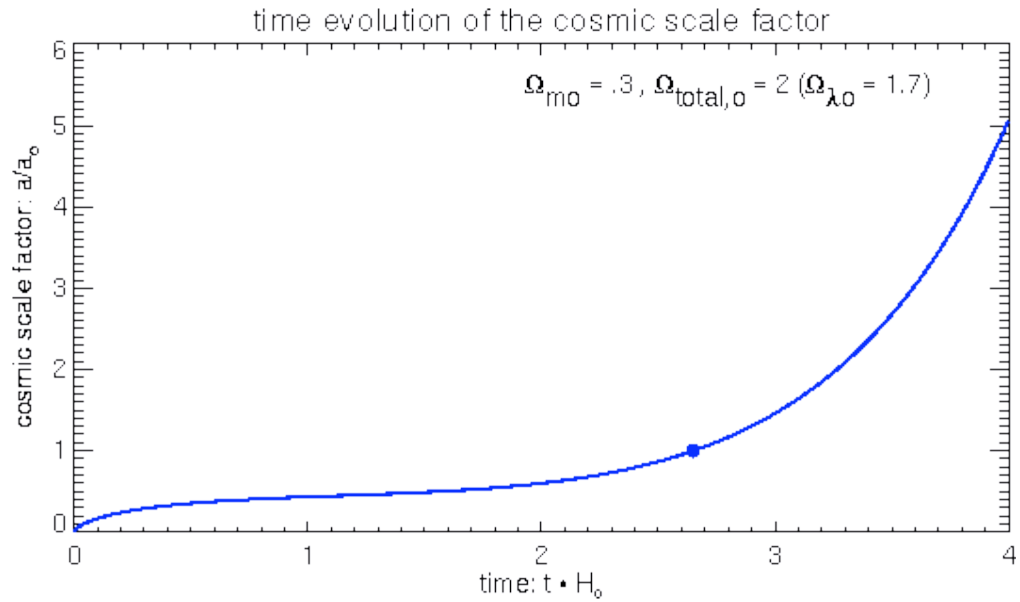
What values for Ω in order to get a flat Universe?

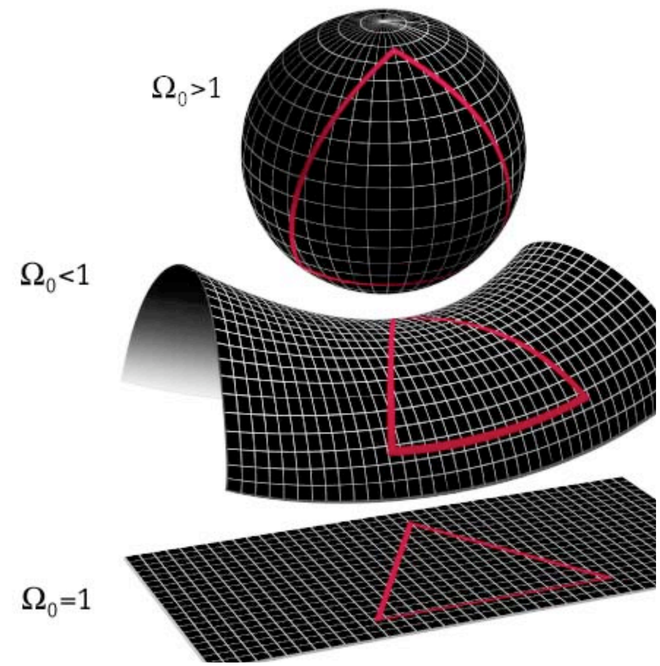


Possible universes containing matter and dark energy



Abbe George Lemaître (1894-1966)
and Albert Einstein in Pasadena 1933





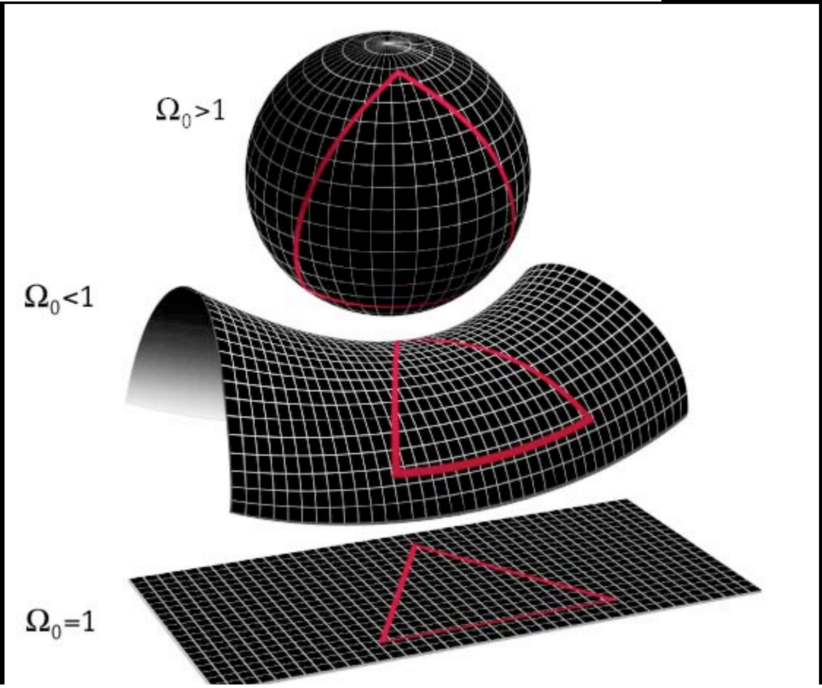
Universe with matter, Λ, and curvature

(matter + cosmological constant + curvature)

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{1-\Omega_{m,0}-\Omega_{\Lambda,0}}{a^2} + \Omega_{\Lambda,0}$$

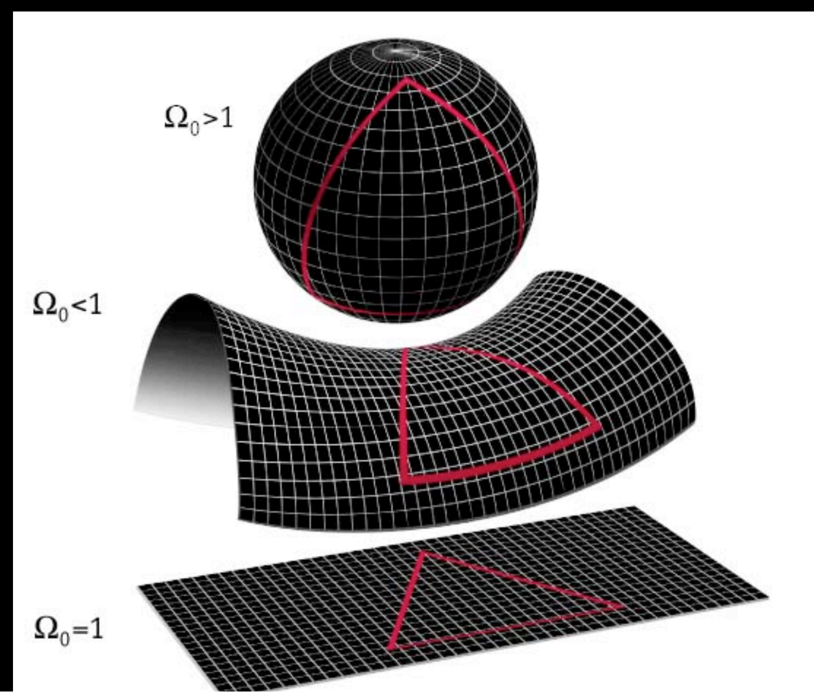
$$\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0}$$



Flat Universe with matter, radiation
(e.g. at $a \sim a_{rm}$)

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

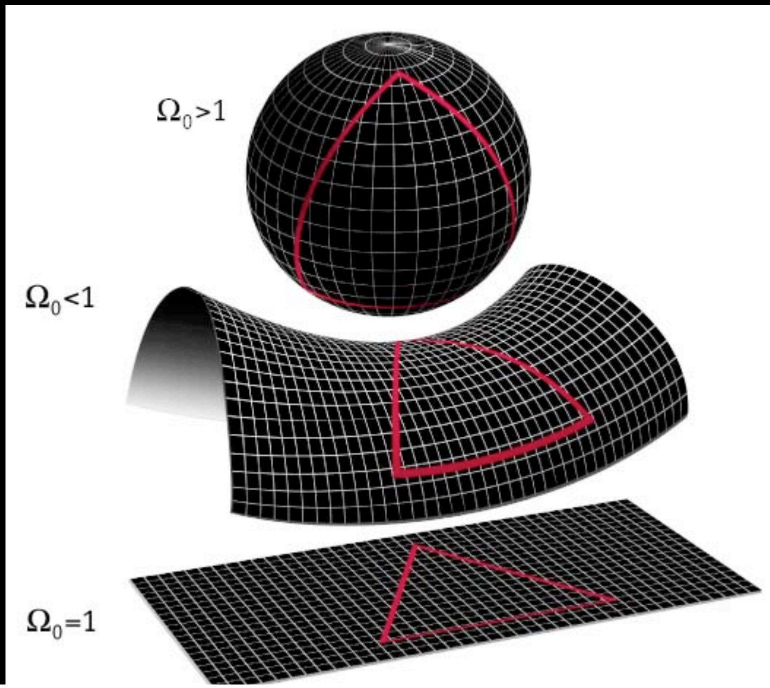
$$\Omega_0 = \Omega_{m,0} + \Omega_{r,0}$$



Describing the real Universe
- the "benchmark" model

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$\Omega_0 = \Omega_L + \Omega_{m,0} + \Omega_{r,0}$
$= 1.02 \pm 0.02$
(Spergel et al. 2003, ApJS, 148, 175)

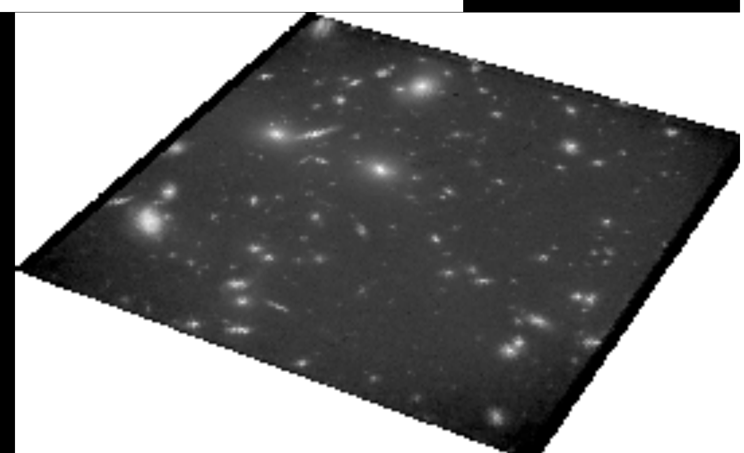


The “benchmark” model

photons	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic	$\Omega_{bary,0} = 0.04$
dark matter	$\Omega_{dm,0} = 0.26$
total matter	$\Omega_{m,0} = 0.30$
dark energy	$\Omega_{\Lambda,0} \simeq 0.70$

$$\Omega_0 = \Omega_L + \Omega_{m,0} + \Omega_{r,0}$$
$$= 1.02 \pm 0.02$$

(Spergel et al. 2003, ApJS, 148, 175)

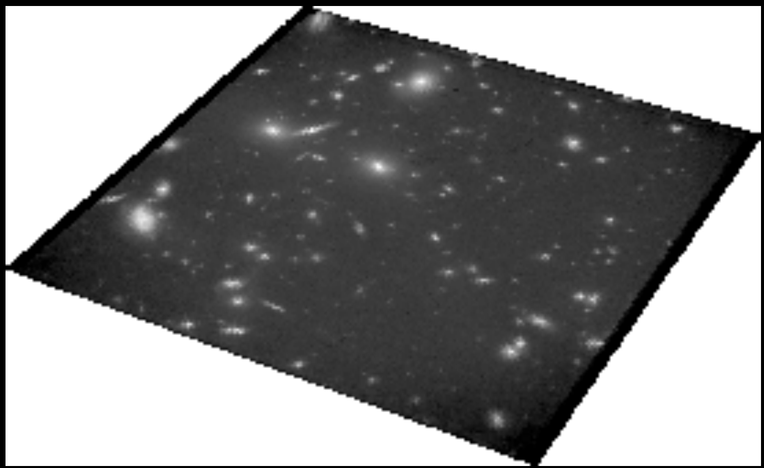


“The Universe is flat and full of stuff we cannot see”

The "benchmark" model

Important Epochs in our Universe:

Epoch	scale factor	time
radiation-matter	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 47,000\,\mathrm{yr}$
matter-lambda	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8\,\mathrm{Gyr}$
Now	$a_0 = 1$	$t_0 = 13.5\,\mathrm{Gyr}$



"The Universe is flat and full of stuff we cannot see -
and we are even dominated by dark energy right now"

The "benchmark" model

Some key questions:

- Why, out of all possible combinations, we have $\Omega_0 = \Omega_\Lambda + \Omega_{m,0} + \Omega_{r,0} = 1.0$?
- Why is $\Omega_\Lambda \sim 1$?
- What is the dark matter?
- What is the dark energy?
- What is the evidence from observations for the benchmark model?

"The Universe is flat and full of stuff we cannot see"

How do we verify our models with observations?

$$H_0 \cdot t = \int_0^a \frac{da}{\sqrt{\Omega_{r,0}\, a^{-2} + \Omega_{m,0}\, a^{-1} + \Omega_{\Lambda,0}\, a^2 + (1 - \Omega_0)}}$$

Hubble Space Telescope Credit: NASA/STS-82

Taylor Series

A one-dimensional Taylor series is an expansion of a real function f(x) about a point x==a is given by

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots.$$

Brook Taylor
(1685 - 1731)

Scale factor as Taylor Series

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots.$$

$$a(t) \simeq 1 + H_0(t-t_0) - \tfrac{1}{2} q_0 H_0^2 (t-t_0)^2$$

qo = deceleration parameter

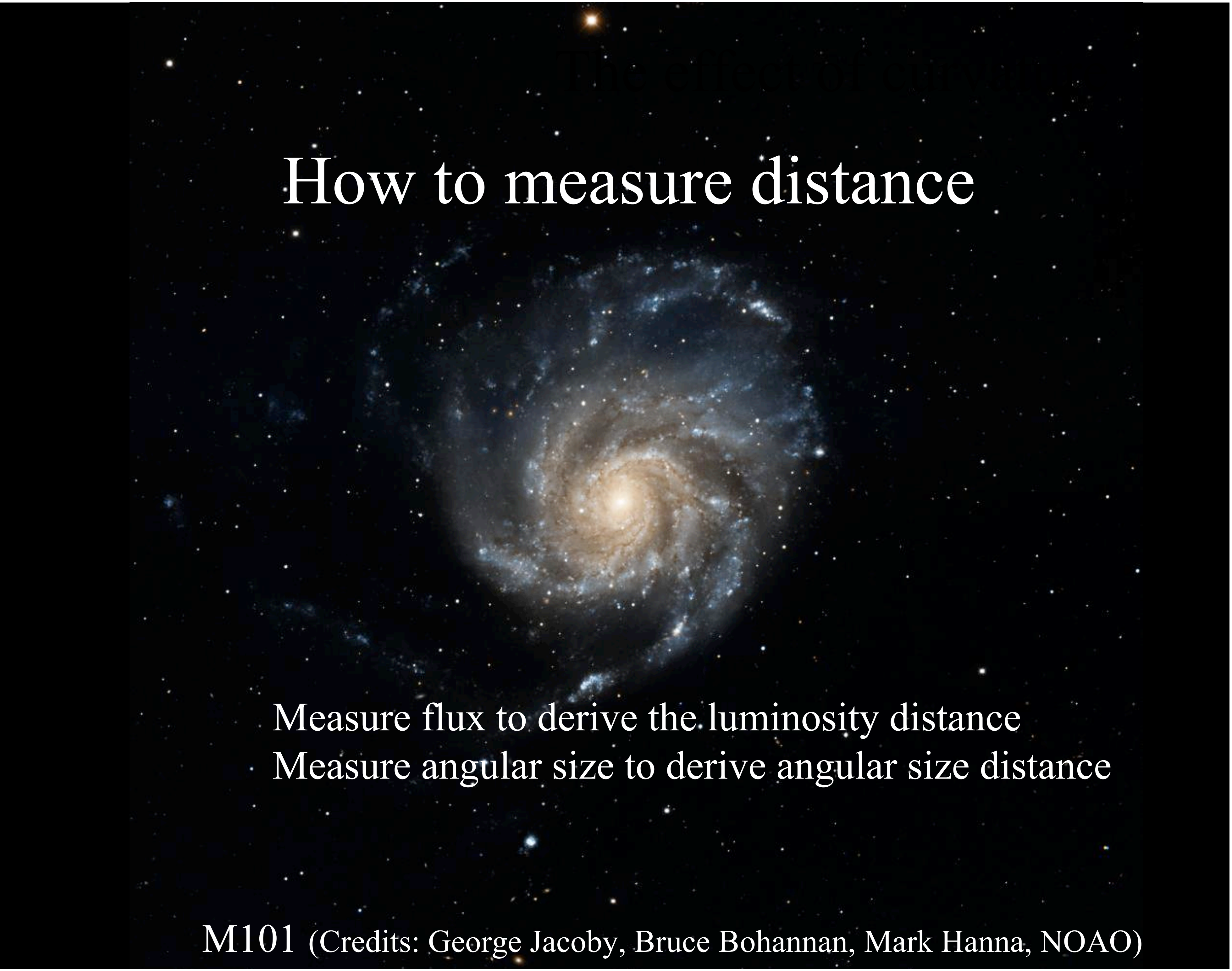
Deceleration parameter q_0

$$a(t) \simeq 1 + H_0(t - t_0) - \tfrac{1}{2} q_0 H_0^2 (t - t_0)^2$$

Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)$$

$$-\frac{\ddot{a}_0}{a_0 H_0^2} = q_0 = \frac{1}{2} \sum_{\omega} \Omega_\omega (1 + 3\omega)$$



M101 (Credits: George Jacoby, Bruce Bohannan, Mark Hanna, NOAO)

Luminosity Distance

In a nearly flat universe:

$$d_L \simeq \frac{c}{H_0} z \left[1 + \frac{1-q_0}{2} z\right]$$

How to determine a(t) :

- determine the flux of objects with known luminosity to get luminosity distance

- for nearly flat: $d_L = d_p(t_0)\ (1+z)$

- measure the redshift

- determine Ho in the local Universe

$\rightarrow$ qo

Angular Diameter Distance

$$d_A = \text{length} / \delta\Theta = d_L / (1+z)^2$$

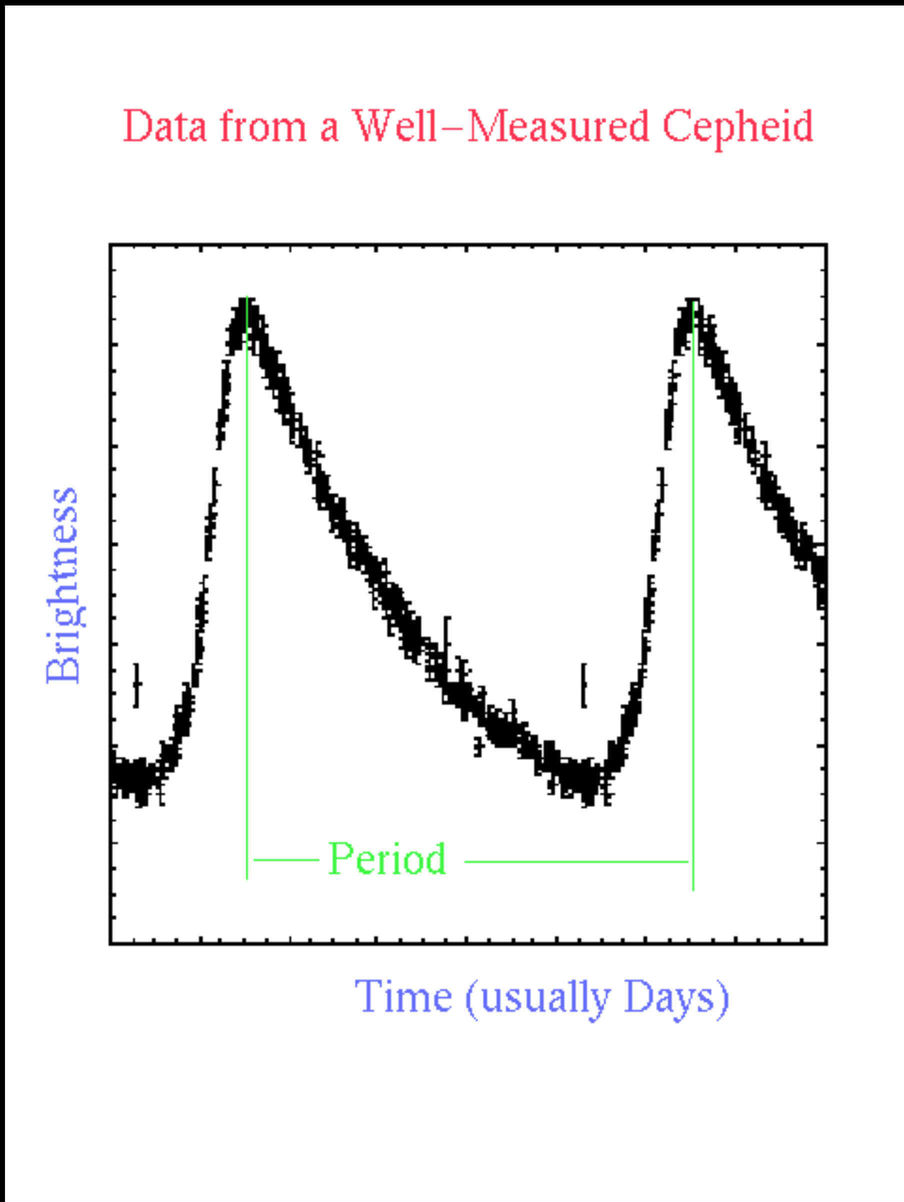
For nearly flat universe:
$d_A = dp(t_0) / (1+z)$

Angular Diameter Distance $d_A = l / \delta\Theta$

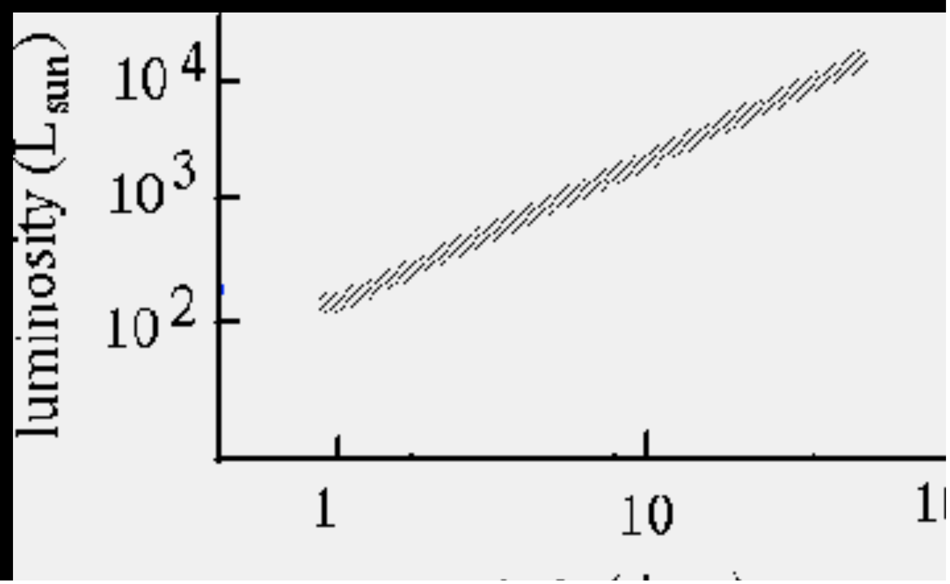
Credits: Adam Blo[illegible]

Measuring Distances - Standard Candles

Cepheids as standard candles

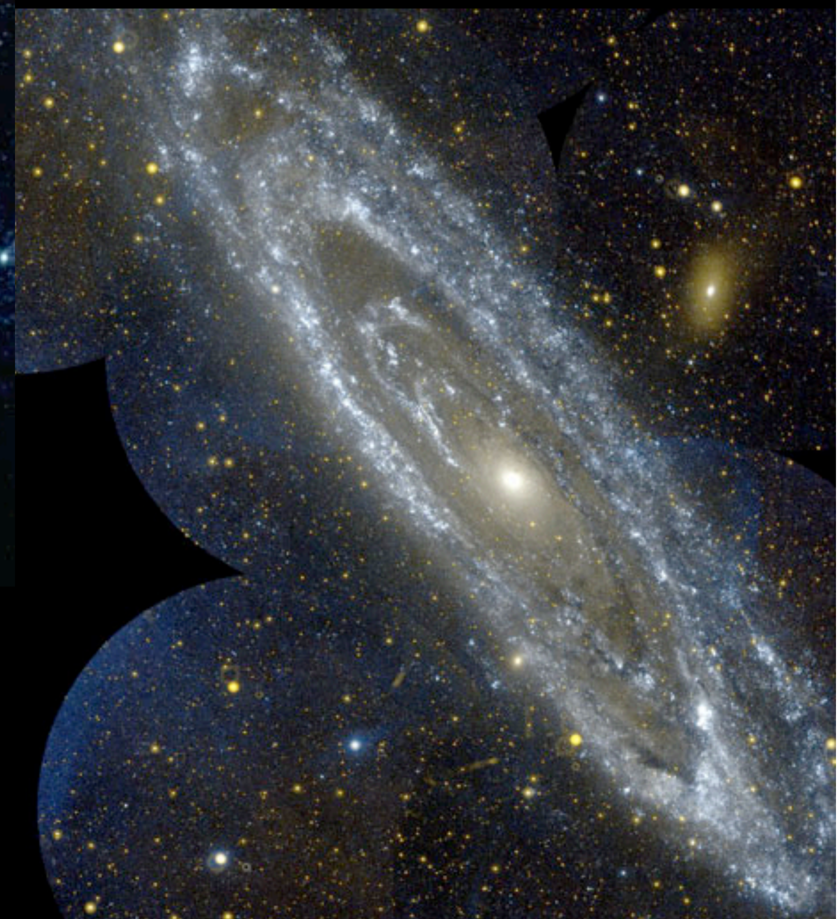


Henrietta Lea



Cepheids as standard candles

Large Magellanic Cloud (Credit: NOAO)



M31 (Andromeda galaxy) Credit:

Cepheids as standard candles

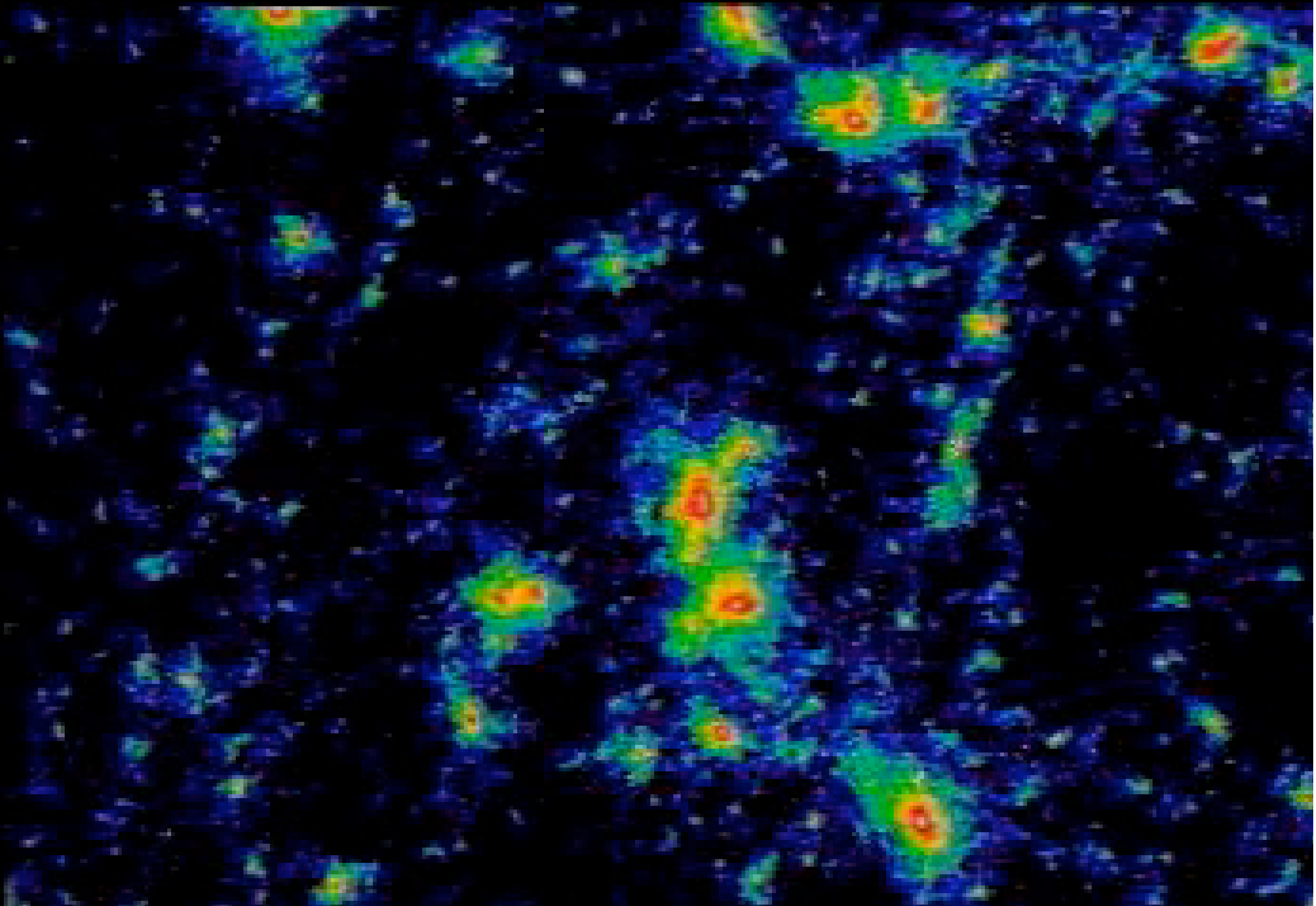
Hipparcos astrometric satellite (Credit: ESA)

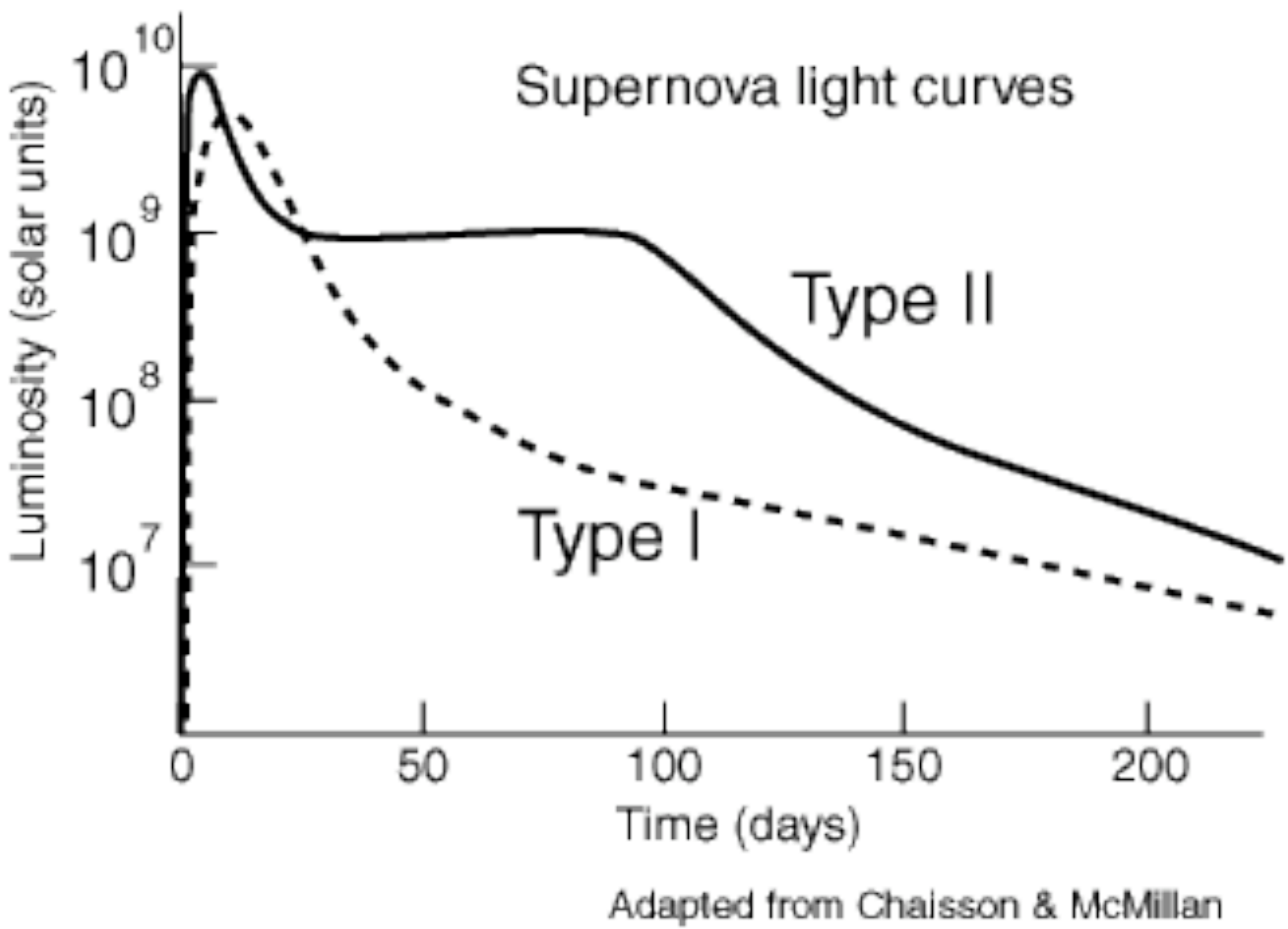
Hubble Space Telescope Credit: NASA/STS-82

For nearly flat Universe:

$$d_L \simeq \frac{c}{H_0} z \left[1 + \frac{1-q_0}{2} z\right]$$

Simulation of galaxy merging

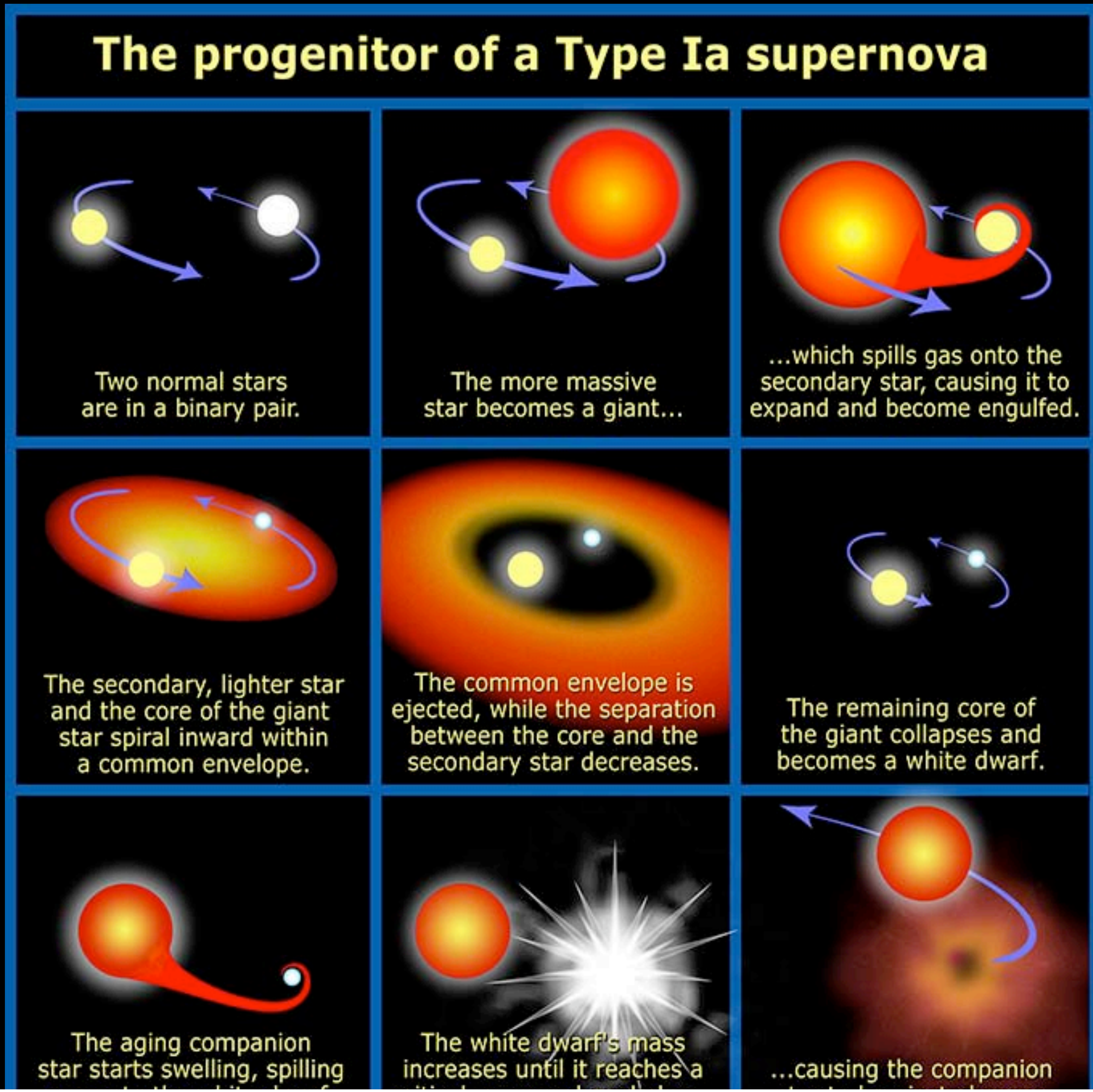




Super Nova types - distinguish by spectra and/or lightcurves

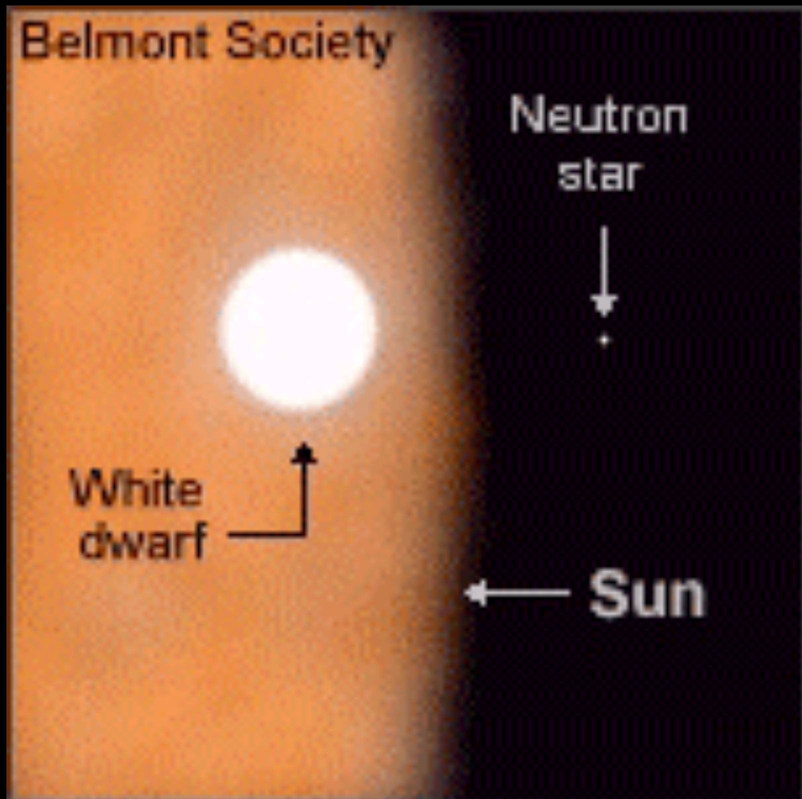
Super Nova Type II - “core collapse” Super Nova

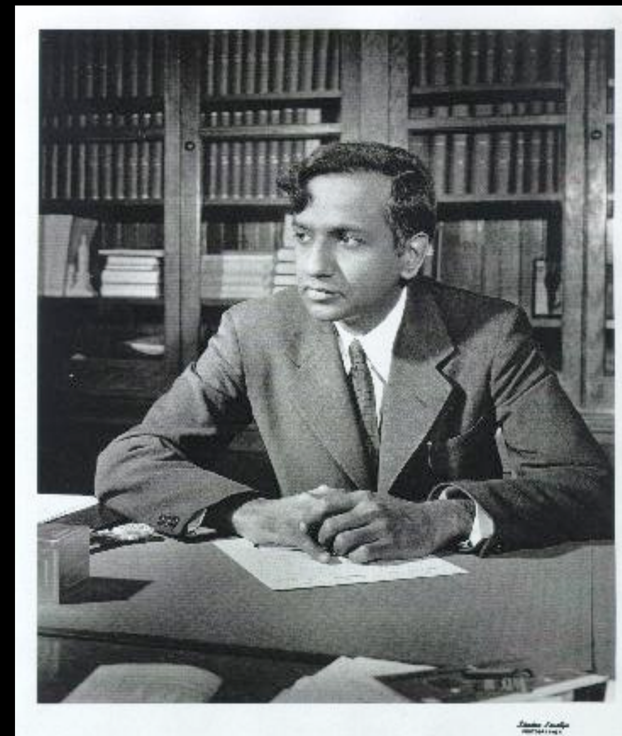
A white dwarf in NGC 2440 (Credits: Hubble Space Telescope)



The progenitor of a Type Ia supernova
Two normal stars are in a binary pair.
The more massive star becomes a giant...
...which spills gas onto the secondary star, causing it to expand and become engulfed.
The secondary, lighter star and the core of the giant star spiral inward within a common envelope.
The common envelope is ejected, while the separation between the core and the secondary star decreases.
The remaining core of the giant collapses and becomes a white dwarf.
The aging companion star starts swelling, spilling
The white dwarf's mass increases until it reaches a
...causing the companion

Chandrasekhar limit





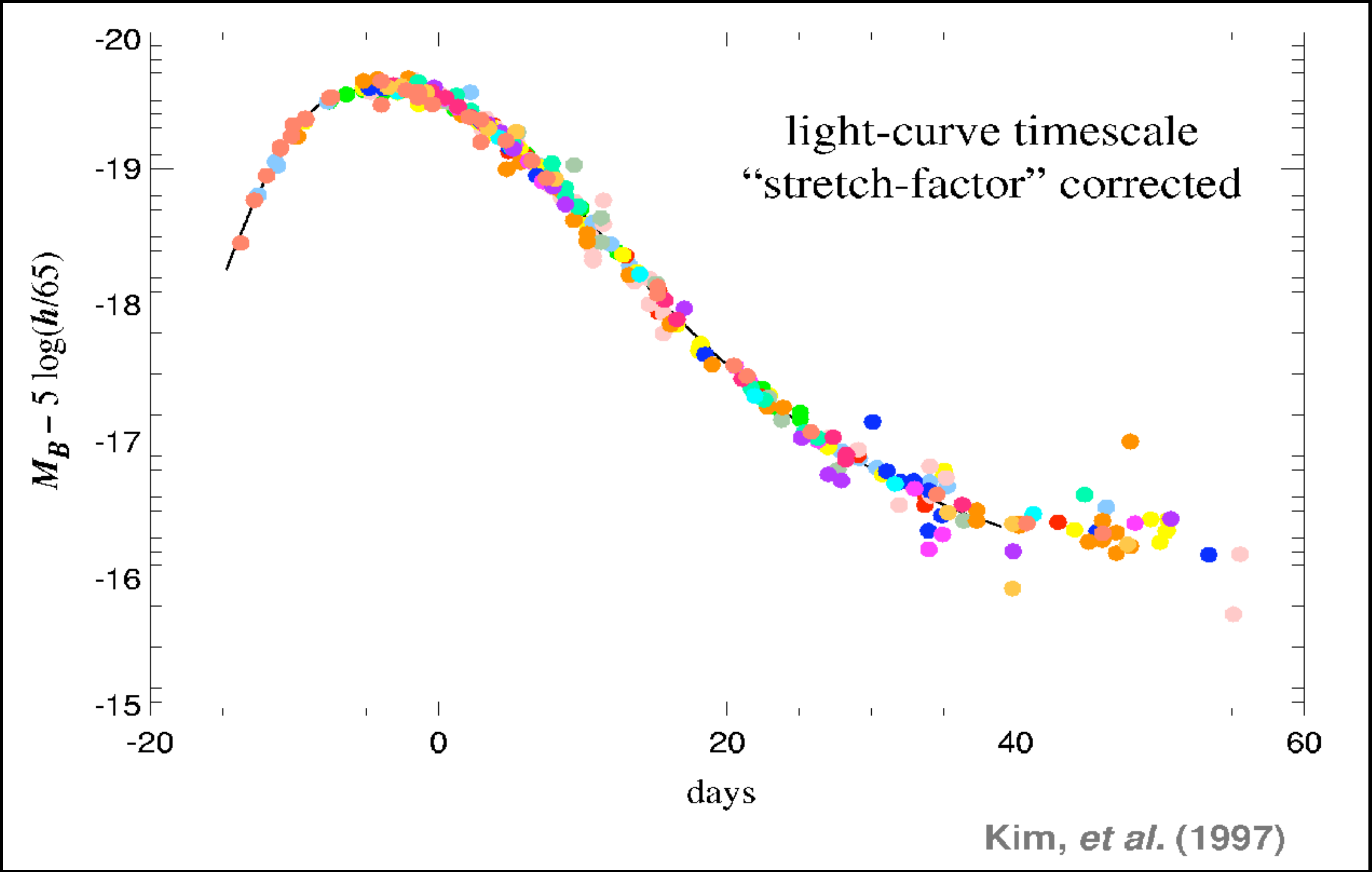
Subrahmanyan Chandrasekhar (1910-1995)

Nobel prize winner

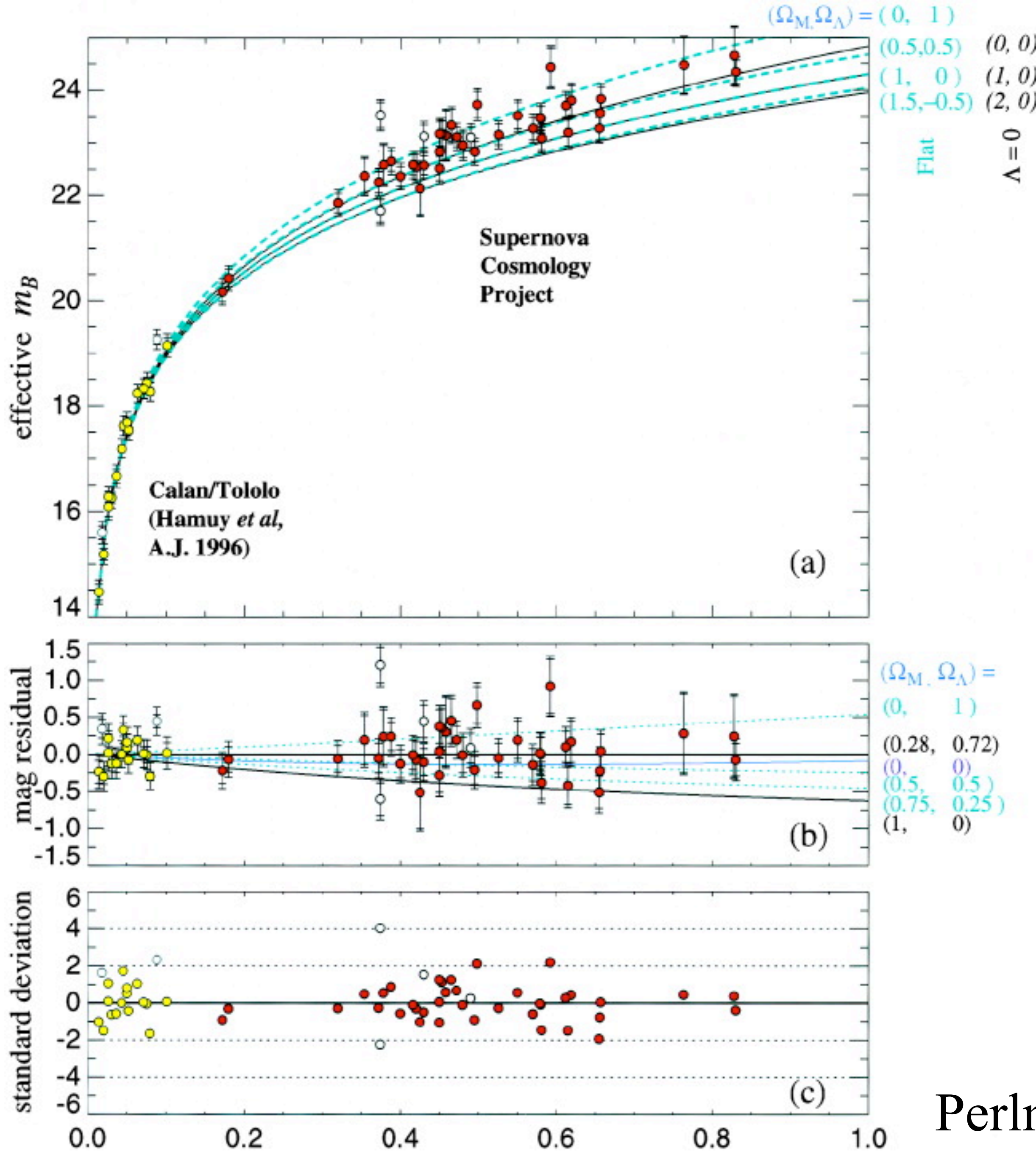
Electron degeneracy pressure can support an electron star (White Dwarf) up to a size of ~1.4 solar masses

$$M_{Ch} \simeq \frac{3\sqrt{2}\pi}{8}\left(\frac{hc}{2\pi G}\right)^{3/2}\left[\frac{Z}{A}\frac{1}{m_H}\right]^2$$

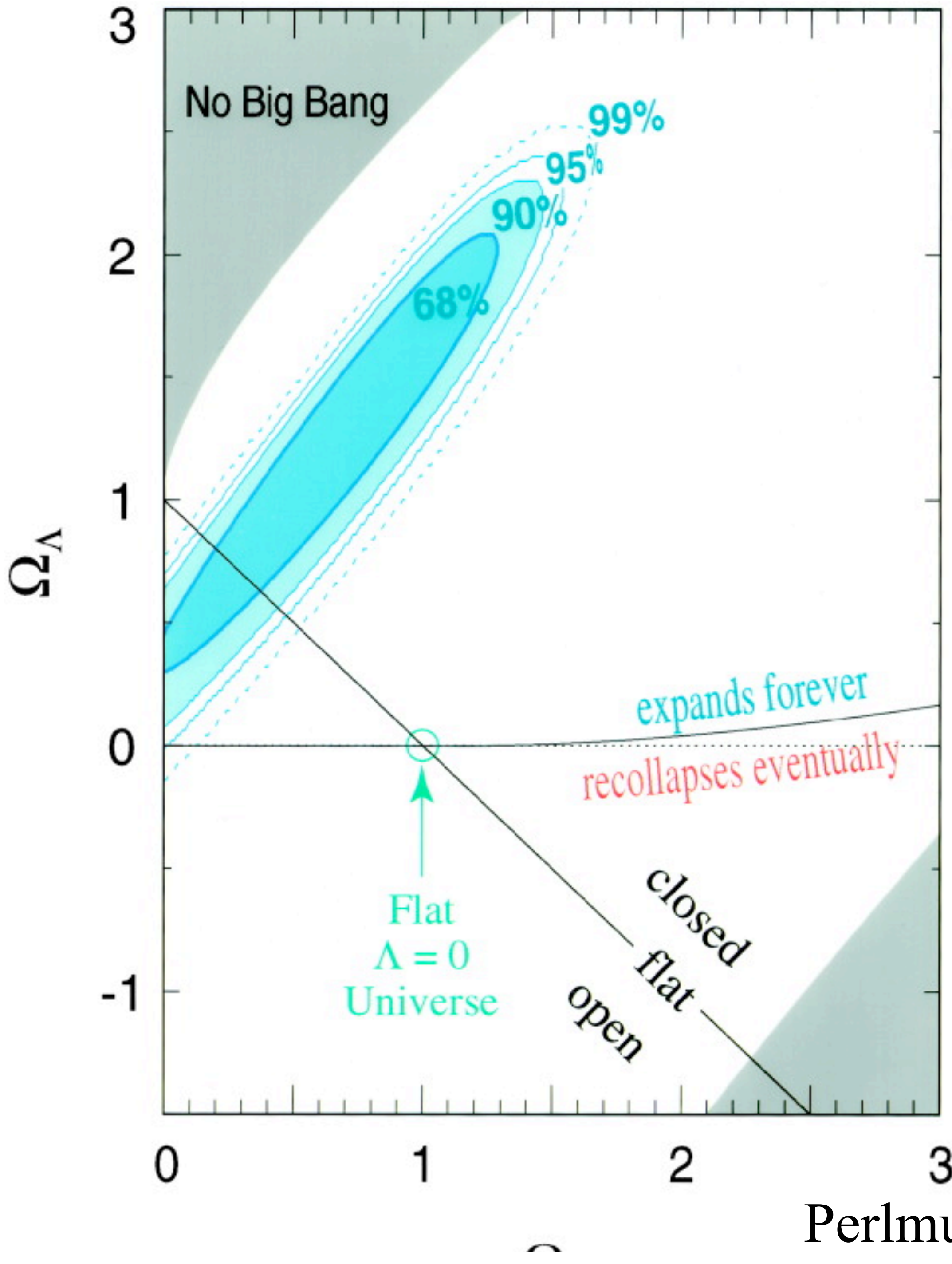
Super Nova Type Ia lightcurves



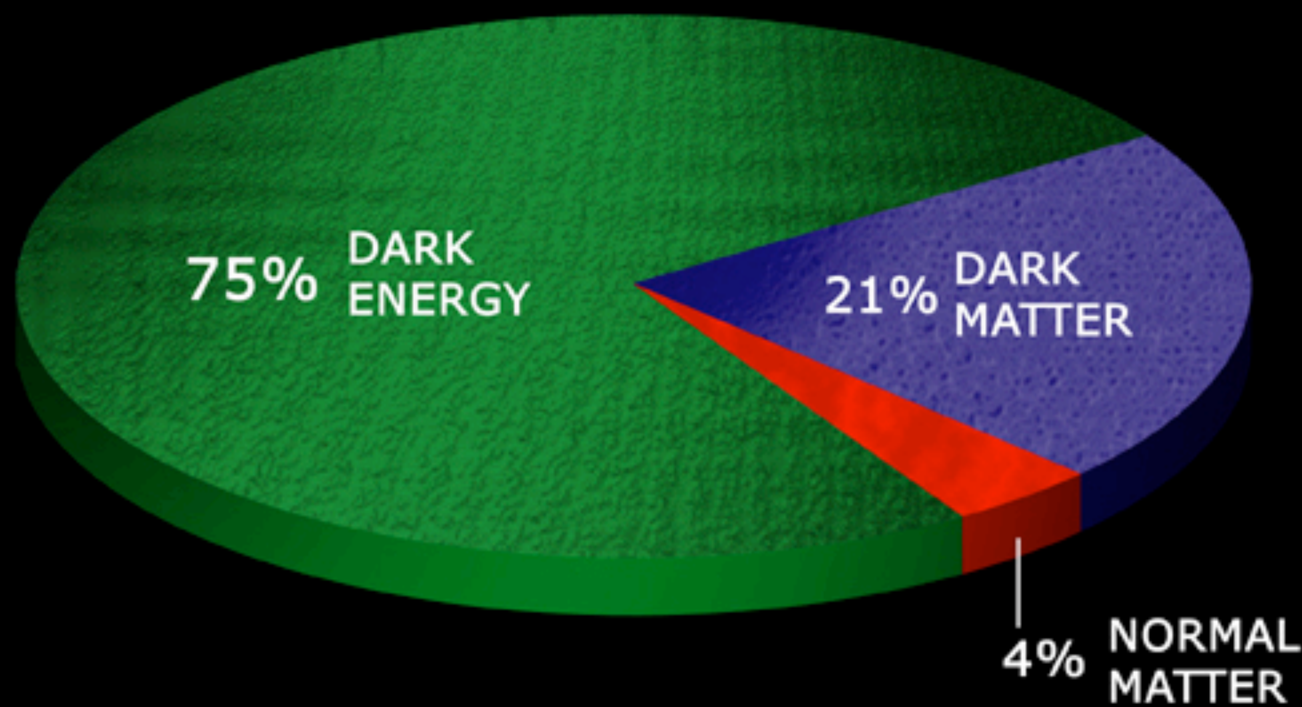
Corrected lightcurves



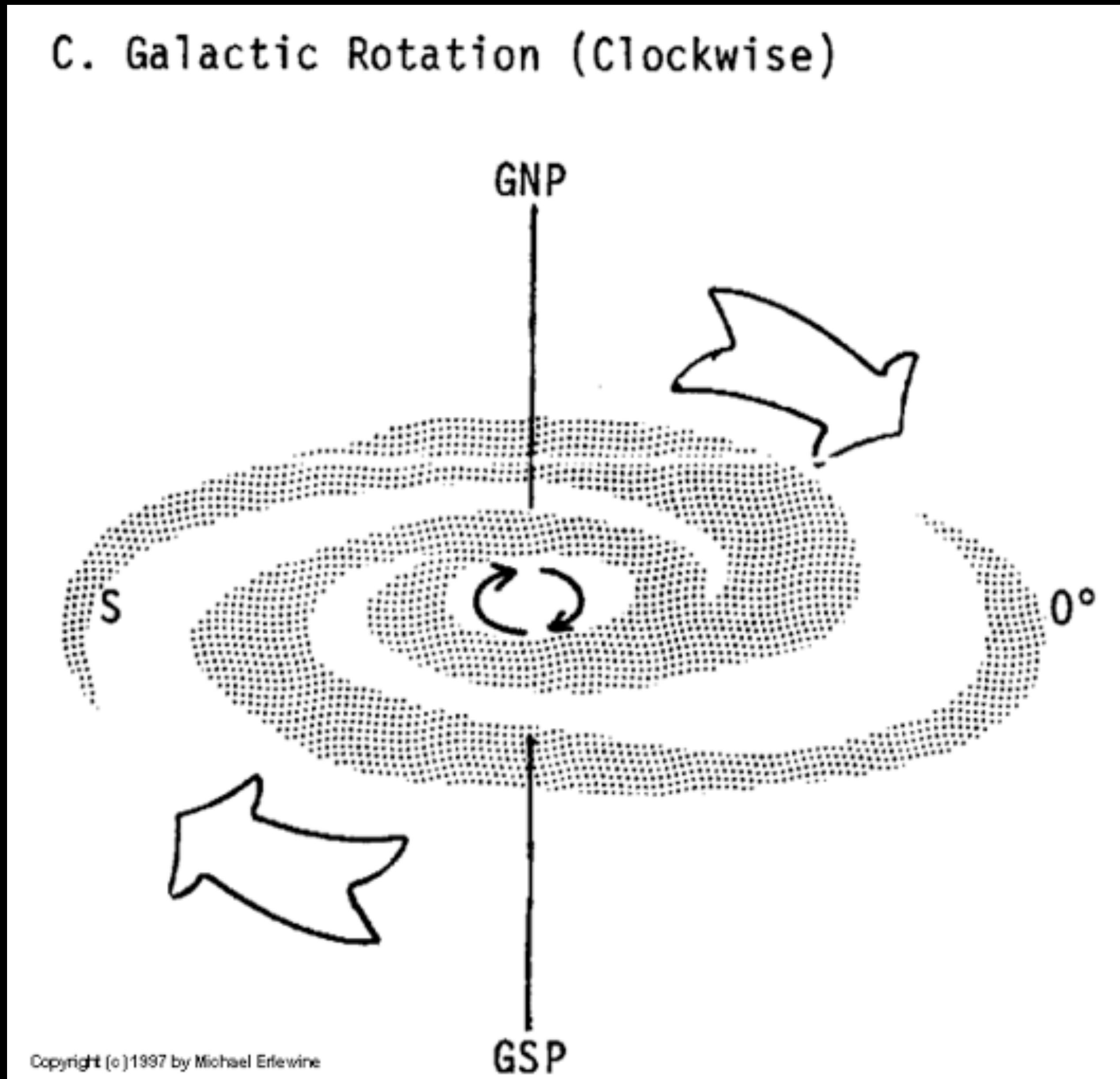
Perlmutter et al. 1999



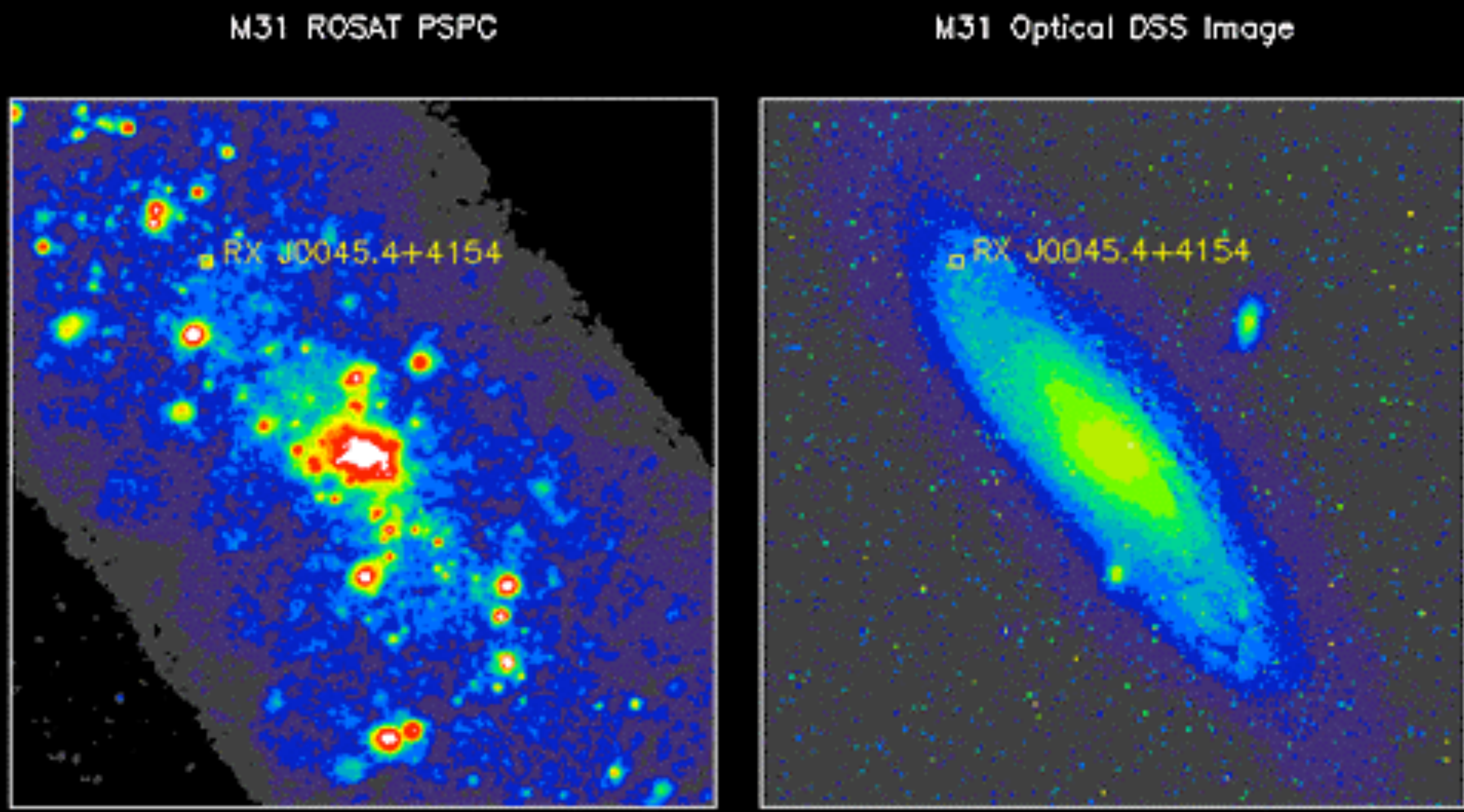
Perlmutter et al. 1999



Primordial Nucleosynthesis: $\Omega_{bary,0} = 0.04 \pm 0.01$



Circular orbit: acceleration $g = v^2/R = G\,M(R)/R^2$



Andromeda galaxy M31 in X-rays and optical

Second midterm exam

- Tuesday, 8:30 a.m. - 9:45 a.m. (here)
- Prof. Ian George
- show up 5 minutes early
- bring calculator, pen, paper
- no books or other material
- formulas provided on page 3 of midterm exam
- start with ‘easy’ questions (50%)

Second midterm exam preparation

- Ryden Chapter 5 - 8 (incl.)
- look at homework (solutions)
- look at midterm #1
- check out the resources on the course web page (e.g. paper about benchmark model)
- careful with other web resources: different notation!