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OLBERS'S PARADOX AND THE SPECTRAL INTENSITY OF THE EXTRAGALACTIC BACKGROUND LIGHT

PAUL S. WESSON
Space Sciences Laboratory, University of California, Berkeley, and Department of Physics, University of Waterloo
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ABSTRACT

There is still confusion in some quarters as to why the intensity of the intergalactic radiation field is low and why Olbers's paradox is resolved. To remedy this in a fashion which is simple but different from previous bolometric work, the intensity as a function of wavelength is calculated, first in expanding uniform cosmological models and then in equivalent static ones. This separates the influence of the expansion of the universe from that of the age of the galaxies. In realistic expanding models, results are given for the spectral intensity of the extragalactic background light at 5100 Å for a range of cosmological parameters and for galaxies which form at different redshifts and either have constant luminosities or a reasonable degree of evolution. The theoretical intensities in expanding models are mostly within observational limits. In hypothetical static models the intensities are higher, but only by modest factors. For most combinations of cosmological model, galaxy formation redshift and galaxy evolution, the expansion only reduces the intensity by a factor of about 3–4. (To reduce it by an order of magnitude from what it would be in a static model would require that the universe be close to de Sitter dynamically.) This confirms the conclusion drawn from earlier bolometric calculations of the extragalactic background light by Wesson, Valle, and Stabell, and shows Harrison is right about Olbers's paradox. Contrary to what is implied in some books, the latter is not resolved mainly by the cosmological redshift. The darkness of intergalactic space is a result primarily of the finite age of the galaxies, in conjunction with other factors including the finite speed of light, and only secondarily of the expansion of the universe.

Subject headings: cosmic background radiation — cosmology — galaxies: photometry — relativity

1. INTRODUCTION

The historical subject of Olbers's paradox is close to being tied down by modern observations of remote galaxies and the extragalactic background light (EBL). The intensity of the intergalactic radiation field due to stars in galaxies is low mainly because of two factors. These are the finite age of the galaxies, which limits the amount of light they have produced, and the expansion of the universe, which increases the volume of space and redshifts the light. Some workers, especially in cosmology, understand how these factors figure in setting the level of the EBL and resolving Olbers's paradox (see, e.g., Harrison 1965, 1977, 1981; Tyson 1986, 1988a; Waldtrop 1986; Wesson, Valle, & Stabell 1987). But others, especially in more traditional areas of astronomy, continue to misunderstand the two noted effects. Specifically, there is a history of neglecting the age factor at the expense of the expansion factor. An attempt to remove confusion about these things was made a few years ago by Wesson et al. (1987). They calculated the bolometric intensity of the EBL in expanding cosmological models and equivalent static ones, thereby isolating the effect of the expansion of the universe and showing it was minor compared to the age effect. That analysis has led to some improvement in understanding. But as will be shown below, confusion persists in some quarters. So it was felt necessary to return to this topic, and approach it in a somewhat different way, namely in terms of the intensity at some particular wavelength. Also, attempts to detect the real EBL are made at particular wavelengths, and to many people Olbers's paradox involves the darkness of the night sky only at visible wavelengths. For these reasons, the present account is a pedagogical one that deals with the intensity of intergalactic radiation as a function of wavelength. The aim is to give precise numerical estimates of the effects of age and expansion on the spectral intensity of the EBL, so that recalcitrant accounts of Olbers's paradox can be improved, to the point where every text which mentions this problem can make a quantitative statement about its solution.

This is necessary. Wesson et al. (1987) reviewed textbooks that dealt with Olbers's paradox, and noted that some had the wrong explanation for it. Without going into detail, it should be noted that the situation is better now, but still not satisfactory. For example, one of the books that had it wrong has simply dropped the subject in its new edition (Zeilik 1988). Another book which is otherwise excellent still has a somewhat confused discussion in its revised edition (Silk 1989). And other books have been noticed which while of older vintage are still widely used but contain misleading accounts (e.g., Lang 1974). As regards books available now, it can be stated without going to the trouble of listing them that most are still less than satisfactory, in that they do not contain a quantitative estimate of the relative importance of the age and expansion factors in explaining the low intensity of the radiation from galaxies. A notable exception is the new edition of Pasachoff (1989), which is conceptually and numerically correct.

In the hope of improving this situation, new spectral calculations will be presented which augment the earlier bolometric ones of Wesson et al. (1987). The latter authors showed that the intensity of the EBL integrated over all wavelengths is determined to order of magnitude by the age of the galaxies and reduced only by about a factor of 2 by the expansion of the universe. However, it was noted that the intensity at a particular wavelength is given by a different kind of function (see below), and several workers have asked why this was not evaluated. This will be done in what follows. Specifically, theoretical calculations of the intensity of the EBL will be made for a range of models at 5100 Å, which may be compared to the observational upper limit at this wavelength set by Dube, Wickes, & Wilkinson (1977, 1979). Also, it will be shown that for realistic source spectra, Olbers's paradox is still resolved primarily by the finite age of the galaxies (in conjunction with other parameters), and that for most cosmological models there is only a modest effect from the expansion of the universe.

2. THE SPECTRAL INTENSITY OF THE EBL IN VARIOUS MODELS

The spectral intensity of the radiation field due to galaxies has been considered from the theoretical side principally by McVittie & Wyatt (1959), Whitrow & Yallop (1964, 1965), Valle (1983), and Wesson et al. (1987, hereafter WVS). It is the energy received per unit time per unit area per unit wavelength interval, and is given (WVS, eq. [15]) by

$$I(\lambda_0) = cn_0 \int_{t_f}^{t_0} F(\lambda, t)\left(\frac{R}{R_0}\right)^2 dt \, . \tag{1}$$

Here c is the speed of light, and n_0 is the number density at the present epoch t_0 of galaxies which formed at t_f (measured in general from the big bang; a subscript zero will henceforth denote the present epoch). The spectrum of a galaxy as a function of wavelength and time is described by $F(\lambda, t) = F(\lambda_0 R/R_0, t)$ in equation (1), where the wavelengths emitted and received are related by $\lambda/\lambda_0 = R/R_0$. This expresses the redshift caused by the expansion of the universe in terms of the scale factor $R(t)$ of a Friedmann-Robertson-Walker (FRW) cosmological model. The expression (1) is the intensity at wavelength λ_0 of the combined radiation from all the galaxies lying within an imaginary spherical surface corresponding to the time of their formation. (There is no contribution to the EBL from distances beyond this surface or times prior to the epoch of galaxy formation, since the EBL is usually taken to be the background formed from photons emitted by stars in galaxies, as opposed to photons from the bing bang fireball which later formed the microwave background.) Thus equation (1) is an all-sky intensity. But it can of course be easily converted to one involving a prescribed solid angle by multiplying by $d\omega/4\pi$. The units of equation (1) may be taken to be ergs $\mathrm{s^{-1}\ cm^{-2}\ Å^{-1}}$ if the intention is to make a direct comparison with observational upper and lower bounds on the EBL (Dube et al. 1977, 1979; Toller 1983). The units may equivalently be taken to be ergs $\mathrm{s^{-1}\ cm^{-3}}$, however, and these are more convenient for theoretical work because then I turns out to be a number of order unity for most choices of parameters (see below). But before numerical values of I can be given, it is necessary to inquire how to determine the functions $F(\lambda, t)$ and $R(t)$ that appear in equation (1) above.

The rate of energy emission per galaxy in the waveband λ to $\lambda + d\lambda$ is $F(\lambda, t)d\lambda$ ergs $\mathrm{s^{-1}}$. Ideally, F could be determined directly from observations of galaxies made at different wavelengths over a range of time or redshift. Observations at modest redshifts do indeed appear to indicate that galaxies were bluer when younger, as expected from models of their formation and evolution (see Butcher & Oemler 1978, 1984; Tyson & Jarvis 1979; Peterson et al. 1979; Kron 1980; Windhorst et al. 1985; Tyson 1988b, and other papers in Part 3 of the volume edited by Kron & Renzini 1988). But there are inadequate data for higher redshifts and negligible data for galaxies seen as they were near formation when they might have been highly luminous (see the models of Partridge & Peebles 1967a, b; and the reports of recent observations by Cowie et al. 1988, 1990; Cowie & Lilly 1989; Lilly, Cowie, & Gardner 1990; Tyson 1988a; Tyson & Seitzer 1988; Elston, Rieke, & Rieke 1988, 1989; Djorgovski 1988). Another option is that F could be determined indirectly from galaxy-evolution models. Something like this was done in an early study of the spectral intensity of the EBL by Tinsley (1973; see also 1977). The analogous bolometric case was treated by Stabell & Wesson (1980), who used the models of Tinsley (1977, 1978) and Larson (1974; see also Tinsley 1976; Larson 1976, 1977; and Spinrad 1977). More sophisticated galaxy-evolution models were developed later, notably by Bruzual & Kron (1980) and Bruzual (1981a; see also Bruzual 1983a, b). These models were used in two short accounts of the spectral intensity of the EBL, namely those by Bruzual (1981b; see also Jakobsen 1980, and Paresce & Jacobsen 1980) and by Code & Welch (1982). Since then other models for the evolution of galaxies have appeared, notably those of Arimoto & Yoshii (1986, 1987). These have also been used in a short account of the spectral EBL by Yoshii & Takahara (1988; see also 1989). More work could be done wherein models for the formation and evolution of galaxies are used to calculate the intensity of the EBL as a function of wavelength. However, such models are complicated while at the same time containing uncertainties. This makes their use somewhat incompatible with the aim of the present article, which is to make a simple, first-order estimate of the contending effects of galaxy age and expansion on the spectral intensity. Now the simplest nontrivial way to model a galaxy spectrum is to assume it is blackbody, and this was done by McVittie & Wyatt (1959), Whitrow & Yallop (1964, 1965), and Valle (1983). And while we expect to use more refined models in the future, for now we elect to follow these other authors and adopt this basic spectrum.

Then $F(\lambda, t)$ can be written as the product of a wavelength-independent parameter C and a Planck function:

$$F(\lambda, t) \equiv C \frac{2hc^2}{\lambda^5 \left[\exp\left(hc/kT\lambda\right) - 1\right]} \, . \tag{2}$$

Here h is Planck's constant and k is Boltzmann's constant as usual, and T is the effective temperature of the source. The function F is normally regarded as decreasing with time (see the references above), and in equation (2) this may in principle be accommodated by allowing C or T to decrease with time. The former choice corresponds to a situation where the luminosity of a galaxy decreases while its spectrum remains unchanged, as might happen if stars were simply to die. The second choice corresponds to a situation in which the luminosity of a galaxy decreases as its spectrum becomes redder, as may happen as its stellar populations age. The second choice is more realistic, and will be adopted here (a combination is also possible in principle). The bolometric luminosity is then given (WVS, eq. [16]) by

$$L(t) = \int_0^\infty F(\lambda, t) d\lambda = \frac{C\sigma}{\pi} T^4(t), \qquad \sigma \equiv \frac{2\pi^5 k^4}{15c^2 h^3} \, . \tag{3}$$

Here σ is the Stefan-Boltzmann constant, and it is seen that Stefan's law holds at each time. The parameter C is now a constant independent of time and is identified via equation (3) as $C = \pi L_0/\sigma T_0^4$. Putting this back into equation (2) and the result into equation (1) gives

$$I(\lambda_0) = \left(\frac{15c^5h^4}{\pi^4k^4}\right)\left(\frac{n_0 L_0}{\lambda_0^5 T_0^4}\right)\int_{t_f}^{t_0}\left(\frac{R_0}{R}\right)^3 \frac{dt}{[\exp(hcR_0/kT\lambda_0 R) - 1]} . \quad (4)$$

This is the intensity of the radiation due to many galaxies, whose light is emitted at various wavelengths and redshifted by various amounts, but which is all in a waveband centered on λ_0 when it arrives at us.

The spectral intensity I as given by equation (4) clearly depends on the age of the galaxies $(t_0 - t_f)$ via the limits of integration, and on the expansion of the universe via the scale factor $R(t)$. As with the bolometric case, it is interesting to know the relative sizes of these two factors. And as in WVS, they can be separated by asking what the intensity would be in an equivalent static universe. This can be defined most logically by setting $R = R_0$ in equation (4). *For the case of no evolution* with $T = T_0$ under the integral, the static intensity is

$$I_s(\lambda_0) = \left(\frac{15c^5h^4}{\pi^4k^4}\right)\left(\frac{n_0 L_0}{\lambda_0^5 T_0^4}\right)\frac{(t_0 - t_f)}{[\exp(hc/kT_0\lambda_0) - 1]} . \quad (5)$$

A comparison of equations (4) and (5) shows that the intensity is determined by the age of the galaxies modified by a factor that depends on the history of the expansion and a typical galaxy's spectrum. The latter is important. From the general expression (1), it is apparent that if the spectral information contained in $F(\lambda, t)$ is temporarily ignored then the expansion of the universe leads via the factor $(R/R_0)^2 < 1$ to a weakening of the radiation. This is like the bolometric case. [The analog of expression (1) above is WVS eq. (10), and they are formally the same if $F(\lambda, t)(R/R_0)^2$ is replaced by $L(t)(R/R_0)$, where the extra power of R/R_0 comes in because observations of the spectral intensity are made in a wavelength interval from λ_0 to $\lambda_0 + d\lambda_0$ as opposed to λ to $\lambda + d\lambda$.] But from equation (4) it is seen that if anything like a realistic spectrum is specified in $F(\lambda, t)$, then it is not obvious how the expansion and the redshift affect the observed intensity. It is in particlar not obvious what the relative sizes are of the intensity I in an expanding model and the intensity I_s in an equivalent static model (though the exponential in the denominator of equation (4) with $R_0 > R$ is expected to dominate and make $I < I_s$). To give specific data on the effect of expansion, values of I/I_s will be tabulated below for a range of cosmological models.

To do this, equation (4) must be put into a form suitable for numerical integration, particularly with regard to the scale factor $R(t)$. This is given by a solution of the Friedmann equations. It is not necessary to discuss these, except to state that they are taken with zero pressure in accordance with the derivation of expression (1) but with finite cosmological constant for generality. It is useful to parameterize the Friedmann equations in terms of the present values of the Hubble parameter $H_0 \equiv \dot{R}_0/R_0$ and the deceleration and density parameters

$$q_0 \equiv \frac{-\ddot{R}_0 R_0}{\dot{R}_0^2} = -\frac{\ddot{R}_0}{R_0 H_0^2} , \quad (6a)$$

$$\sigma_0 \equiv \frac{4\pi G\rho_0 R_0^2}{3\dot{R}_0^2} = \frac{4\pi G\rho_0}{3H_0^2} . \quad (6b)$$

Here a dot denotes the derivative with respect to time, G is the gravitational constant, and ρ_0 is the present density of matter. [An often-used alternative to equation (6b) is $\Omega_0 \equiv 2\sigma_0$, but the latter will be used here for convenience.] Then equation (4) can be converted to a relation in $y \equiv R/R_0$ using an expression for $dt = dt(y, H_0, q_0, \sigma_0)$ that follows from the Friedmann equations with zero pressure. (See WVS eq. [19]; the integral of this gives the age of the galaxies as WVS eq. [20], and this will be used to obtain eq. [8] below. Some other authors, like Weinberg 1972, have similar relations, but restricted to the case of zero cosmological constant.) The result is

$$I = \alpha\beta \int_{y_f}^{1} \frac{y^{-5/2}\, dy}{(e^{\gamma/y} - 1)[2\sigma_0 + (1 + q_0 - 3\sigma_0)y + (\sigma_0 - q_0)y^3]^{1/2}} ,$$

$$\alpha \equiv \frac{15c^5h^4}{\pi^4k^4} , \qquad \beta \equiv \frac{n_0 L_0}{\lambda_0^5 T_0^4 H_0} , \qquad \gamma \equiv \frac{hc}{kT_0\lambda_0} . \quad (7)$$

A similar procedure can be applied to the static expression (5), giving

$$I_s = \frac{\alpha\beta}{(e^{\gamma} - 1)} \int_{y_f}^{1} \frac{y^{1/2}\, dy}{[2\sigma_0 + (1 + q_0 - 3\sigma_0)y + (\sigma_0 - q_0)y^3]^{1/2}} . \quad (8)$$

The expressions (7) and (8) can if so desired by evaluated as they stand. (An expression analogous to expression [7] was used by Stabell & Wesson 1980 in the bolometric case; and some results based on expressions like [7] and [8] were given by Valle (1983) in the spectral case.) But while y is a convenient parameter in purely theoretical studies, it is better to replace it by the redshift z in studies aimed at making some contact with observation. The two are related via $y = (1 + z)^{-1}$, and the appropriate limit is now z_f, the redshift of galaxy formation. Equations (7) and (8) can now be converted to the new variable (note that the latter will now involve an integral over redshift, which however is merely an algebraic way of expressing the age of the galaxies and does not imply a physical effect). The results are

$$I = \alpha\beta \int_0^{z_f} \frac{(1+z)^2\,dz}{[e^{\gamma(1+z)} - 1][2\sigma_0(1+z)^3 + (1 + q_0 - 3\sigma_0)(1+z)^2 + \sigma_0 - q_0]^{1/2}} , \tag{9}$$

$$I_s = \frac{\alpha\beta}{(e^{\gamma} - 1)} \int_0^{z_f} \frac{(1+z)^{-1}\,dz}{[2\sigma_0(1+z)^3 + (1 + q_0 - 3\sigma_0)(1+z)^2 + \sigma_0 - q_0]^{1/2}} . \tag{10}$$

These are the final expressions for the intensities of light emitted by galaxies with constant luminosities, and received by us at a wavelength λ_0 which is now present in the parameters β and γ defined above.

The parameters α, β, and γ defined in equation (7) need to be evaluated before equations (9) and (10) can be used to derive results. The first depends only on atomic constants and numerically is $\alpha = 2.0 \times 10^{10}$ cm^5 s^{-1} K^4. The second requires knowledge of the luminosity density ϵ_0 ($\equiv n_0 L_0$) of galaxies and choices for the effective blackbody temperature T_0 of their stellar populations and the wavelength λ_0 of observation. For the luminosity density, $\epsilon_0 = 2.5 \times 10^8 h\, L_\odot$ Mpc^{-3} can be taken following Shapiro (1971). Here $H_0 \equiv 100\, h$ km s^{-1} Mpc^{-1} as usual, and it should be noted that since β depends on the ratio ϵ_0/H_0 the uncertainty in h does not affect the value of this parameter (see WVS). There are of course more recent determinations of ϵ_0, including that of Gott & Turner (1976). But it is convenient to use the value of Shapiro because it is close to the midrange of determinations of this quantity as reviewed by Peebles (1980), and because it has been used in deriving bolometric results which one may wish to compare with the spectral results to be presented below. For the temperature, $T_0 = 6000$ K can be taken following Whitrow & Yallop (1964, 1965) and Valle (1983). For the wavelength of observation, $\lambda_0 = 5100$ Å can be taken following Dube et al. (1977, 1979), who have set the most useful upper limit on the EBL in the optical at this wavelength. Combining the data just noted gives the second parameter as $\beta = 2.2 \times 10^{-8}$ ergs cm^{-8} K^{-4}. The third parameter using the same values of T_0 and λ_0 is $\gamma = 4.70$. This is dimensionless, whereas the product $\alpha\beta = 440$ ergs s^{-1} cm^{-3} and defines convenient absolute units for the spectral intensity as noted previously.

Results in these units are given in Table 1. There, values of I and I_s from equations (9) and (10) are given in the form of ratios for various q_0, σ_0, and z_f. The latter are chosen to allow a direct comparison with the results in the bolometric case (WVS, Table 1). Intensities have also been derived for larger values of z_f, but have not been tabulated because the quantity of most interest I shows hardly any change as z_f increases (even between $z_f = 3$ and 6, the change in I appears only in the fourth decimal place). The physical reason for this will be discussed below. For now, it can be noted that values of I/I_s lie typically around 1/3. Some smaller values do occur toward the bottom left-hand corners of the two parts of the table; but these are not typical, and the extreme case of the de Sitter model with $q_0 = -1$, $\sigma_0 = 0$ is in any case unrealistic (see below). Thus for realistic models, the expansion reduces the spectral intensity of the EBL in the optical by a modest factor of 1/3 or so.

This conclusion is similar to that reached by WVS for the bolometric case. However, in the latter the ratio of intensities was typically 1/2, so the expansion of the universe is relatively more important for the spectral case. Bolometrically, WVS argued that because the age of the galaxies determines the order of magnitude of the intensity whereas the expansion reduces it only by a factor of order unity, the former is the dominant factor in explaining the low intensity of the EBL and resolving Olbers's paradox. Spectrally, a similar argument can be made. In both cases, however, it should be recalled that other quantities also enter the problem, notably the number density and luminosities of galaxies (or their luminosity density equivalently), and the speed of light.

TABLE 1

VALUES OF I/I_s

q_0	σ_0 = 0	0.01	0.1	0.5	1
			$z_f = 3$		
+1	0.917/2.454	0.916/2.447	0.911/2.390	0.893/2.206	0.873/2.058
+0.5	0.969/2.688	0.968/2.678	0.962/2.596	0.939/2.356	0.914/2.174
0	1.037/3.029	1.036/3.012	1.027/2.884	0.995/2.547	0.964/2.316
−0.5	1.132/3.621	1.130/3.583	1.116/3.331	1.068/2.804	1.026/2.496
−1	1.296/5.599	1.291/5.271	1.257/4.231	1.171/3.185	1.107/2.736
			$z_f = 6$		
+1	0.917/2.763	0.916/2.751	0.911/2.653	0.893/2.389	0.873/2.200
+0.5	0.969/3.044	0.968/3.025	0.962/2.887	0.939/2.547	0.914/2.320
0	1.037/3.462	1.036/3.430	1.027/3.212	0.995/2.748	0.964/2.467
−0.5	1.132/4.221	1.130/4.145	1.116/3.718	1.068/3.017	1.026/2.651
−1	1.296/7.859	1.291/6.552	1.257/4.726	1.171/3.411	1.107/2.897

NOTE.—Values derived from eqs. (9) and (10), which give the spectral intensity of the EBL in an expanding FRW model (I) and an equivalent static one (I_s) in terms of the deceleration parameter q_0, the density parameter σ_0, and the redshift of galaxy formation z_f. The unit of intensity is ergs s^{-1} cm^{-3}, or 10^{-8} ergs s^{-1} cm^{-2} Å^{-1} in units more attuned to observations. A galaxy is modeled as a blackbody source with an effective temperature of 6000 K which is constant, and observations are made at 5100 Å. The ratio I/I_s is typically about 1/3, showing that expansion has a relatively minor effect on the intensity of the EBL in the optical. The values of I may be used separately in connection with attempts to detect the EBL observationally.

And in the spectral case, the form of the spectrum is of major importance. This was commented on in a general sense above, and it is instructive to interject here some remarks on how the spectrum affects the intensity as a function of redshift.

Plots of $I(z_f)$ for three simple FRW models are given in Figure 1, together with the corresponding plots of $I_s(z_f)$. It is seen that $I(z_f)$ is essentially flat for $z_f \gtrsim 1$. The reason for this is that if there is little radiation emitted in the ultraviolet there is little energy to be redshifted into the optical and observed there, so I increases little as z_f is increased. This effect was commented on by Peebles (1971) and Valle (1983) and is quite pronounced for the blackbody spectrum considered here. A similar effect must exist generally, however, because the spectra of normal galaxies fall off in the ultraviolet, and while primeval galaxies may emit relatively more energy at short wavelengths their spectra are believed to be essentially cut off at the Lyman limit (Partridge & Peebles 1967a, b). As regards $I_s(z_f)$, it is seen that it goes flat for the Einstein-de Sitter and Milne models but not for the de Sitter one. The integrals in equation (10) for these three models can be done algebraically and give

$$I_s = 2.7\left[1 - \frac{1}{(1+z_f)^{3/2}}\right], \quad \text{Einstein–de Sitter } (q_0 = \sigma_0 = 1/2)\ ; \tag{11a}$$

$$I_s = 4.0\left[1 - \frac{1}{(1+z_f)}\right], \quad \text{Milne } (q_0 = \sigma_0 = 0)\ ; \tag{11b}$$

$$I_s = 4.0 \log_e (1 + z_f)\ , \quad \text{de Sitter } (q_0 = -1, \sigma_0 = 0)\ . \tag{11c}$$

By considering these it is clear that while I/I_s will tend to a finite limit in the first two models and others like them, $I/I_s \to 0$ for $z_f \to \infty$ for the last one because I_s is unbounded. However, the de Sitter model is unrealistic in having an infinite past history and no big bang, and in more realistic models where the galaxies have a finite age it is expected that I/I_s will be finite.

The results given above disregard the possibility that the spectra and luminosities of galaxies may undergo evolution, and since this may be a significant effect it is relevant to ask how I/I_s is affected by it. Some unpublished work on this was done by Valle (1983), who used a method similar to the bolometric one of Stabell & Wesson (1980). This involves integrating an expression derived from the Friedmann equations to obtain t_0, using this to form the look-back time $(t_0 - t)$ which is the parameter used in the evolution function, and using the latter in another expression which when integrated gives the intensity. Such a method, while necessary if one has a model for evolution in which time is the parameter, is cumbersome. Therefore, in the present work evolution will be described directly in terms of redshift. Specifically, one can consider a simple power-law form for the luminosity and use Stefan's law to convert this to a form for the temperature, thus

$$L = L_0(1 - z)^n, \qquad T = T_0(1 + z)^{n/4}\ . \tag{12}$$

These are plotted in Figure 2 for three values of n. We realize these forms are probably too simple to be realistic, but believe they are adequate to illustrate the effects of expansion. They have the advantages of allowing the expressions for I and I_s to be modified easily to take account of evolution and are compatible in a general sense with ideas on galaxy formation (for example, the above with $n = 2$ is compatible with the model of Patridge & Peebles 1967a, b). Results for evolution with $n = 1$ and $n = 2$ are given in Tables 2 and 3, respectively. There is a big spread of values of I/I_s in both of these tables, but if one focuses on the center entries ($q_0 = 0$, $\sigma_0 = 0.1$), then this parameter lies approximately in the range 0.2–0.3. Thus in rough terms the expansion reduces the intensity from what it would be in an equivalent static model by a factor of 1/4 or so.

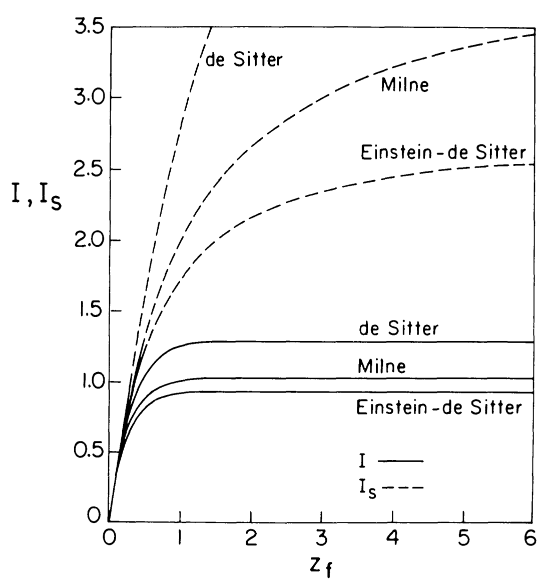

FIG. 1

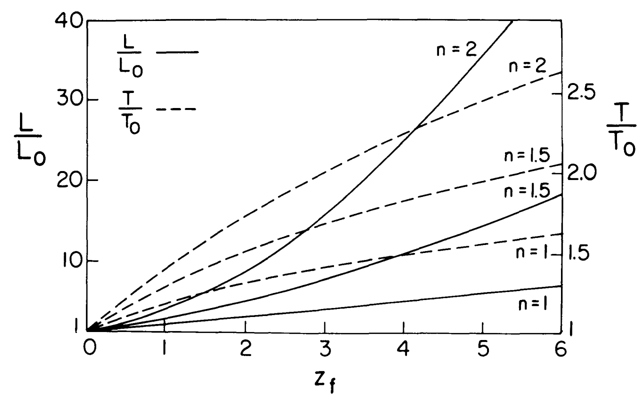

FIG. 2

FIG. 1.—Plots of the intensities as given by eqs. (9) and (10) for the Einstein-de Sitter ($q_0 = \sigma_0 = 1/2$), Milne ($q_0 = \sigma_0 = 0$), and de Sitter ($q_0 = -1, \sigma_0 = 0$) models

FIG. 2.—Plots of the luminosity and temperature as given by eq. (12) for three choices of n

TABLE 2

VALUES OF I/I_s

q_0	σ_0 = 0	0.01	0.1	0.5	1
			$z_f = 3$		
+1	1.366/4.593	1.365/4.573	1.352/4.417	1.304/3.942	1.258/3.580
+0.5	1.464/5.100	1.462/5.072	1.444/4.849	1.382/4.226	1.325/3.787
0	1.597/5.865	1.593/5.817	1.567/5.461	1.480/4.590	1.406/4.039
−0.5	1.798/7.264	1.792/7.155	1.745/6.442	1.611/5.082	1.509/4.357
−1	2.235/12.698	2.207/11.622	2.061/8.509	1.801/5.812	1.646/4.781
			$z_f = 6$		
+1	1.367/6.159	1.366/6.110	1.353/5.746	1.305/4.859	1.258/4.294
+0.5	1.465/6.903	1.463/6.830	1.445/6.314	1.383/5.185	1.325/4.520
0	1.598/8.059	1.595/7.931	1.568/7.115	1.481/5.597	1.406/4.793
−0.5	1.800/10.310	1.794/9.997	1.746/8.386	1.612/6.146	1.509/5.135
−1	2.241/24.401	2.211/18.096	2.062/10.985	1.801/6.944	1.647/5.584

NOTE.—As in Table 1, except that evolution of a galaxy is included according to eq. (12) with $n = 1$.

This conclusion, however, is only a middle-of-the-park one. A considerably smaller value of I/I_s may occur if there is strong evolution in a universe that is dynamically close to the de Sitter one and has a strong redshift effect. Clearly the cosmological redshift and galaxy evolution are intertwined as they affect the intensity. By and large, if galaxies are more luminous at large redshifts and relatively more of the EBL originates there, then the effect of expansion in reducing the intensity is expected to be more marked than without evolution. But while this trend is straightforward in the bolometric case it is complicated in the spectral case, because if the effective temperature is higher at higher redshifts then relatively more radiation is produced in the ultraviolet that may be redshifted into the optical band and contribute to the intensity there. Another complicating factor, which may be relevant in practice but has not been included in the theoretical calculations, is absorption. This could be by dust associated either with galaxies that block the line of sight or with an intergalactic medium, and may be significant enough to obscure objects with redshifts greater than 3–4 (Ostriker & Heisler 1984). It is worth noting that if absorption at high redshifts is significant, especially in the ultraviolet, then it will have the effect of reducing the absolute intensity of the EBL as observed in the optical. That is, evolution and absorption may be offsetting effects as regards the observed intensity of the EBL in the optical.

Direct measurements of the EBL in the optical, as opposed to estimates of it by smearing out the light from resolved galaxies, are hard to make. A review of observations has been given by Mattila (1990), who noted that none has to date provided a generally accepted detection. However, direct upper limits at various optical wavelengths have been set by several workers, most notably Dube et al. (1977, 1979). They made observations at 5100 Å, and concluded that the intensity of the EBL was $1 \pm 1.2\ S_{10}$ at this wavelength, with an upper limit at the 90% confidence level of $3.4\ S_{10}$ (where $1\ S_{10}$ is the intensity equivalent to one tenth magnitude star per square degree). An indirect lower limit in the optical of about $0.5\ S_{10}$ was inferred from the work of various people by Toller (1983). A figure of about the same size was arrived at by Tyson (1988a), who however used a considerably more sophisticated technique. He imaged faint galaxies, integrated the flux corresponding to differential number-magnitude counts, and arrived at an intensity of $0.57\ S_{10}$ at 4500 Å. He noted, though, that his technique effectively smears out the light from resolved objects less than

TABLE 3

VALUES OF I/I_s

q_0	σ_0 = 0	0.01	0.1	0.5	1
			$z_f = 3$		
+1	2.593/8.580	2.587/8.535	2.532/8.173	2.353/7.104	2.203/6.321
+0.5	2.837/9.615	2.827/9.548	2.750/9.030	2.515/7.631	2.330/6.685
0	3.189/11.203	3.173/11.090	3.053/10.260	2.722/8.305	2.486/7.127
−0.5	3.789/14.200	3.756/13.939	3.522/12.262	3.002/9.217	2.684/7.682
−1	5.647/26.848	5.382/24.115	4.456/16.581	3.419/10.565	2.950/8.417
			$z_f = 6$		
+1	2.704/13.502	2.696/13.363	2.628/12.343	2.420/9.975	2.256/8.553
+0.5	2.964/15.280	2.952/15.071	2.856/13.626	2.585/10.631	2.384/8.977
0	3.344/18.099	3.323/17.732	3.172/15.445	2.796/11.455	2.542/9.484
−0.5	4.004/23.776	3.958/22.867	3.663/18.348	3.081/12.543	2.742/10.111
−1	6.411/63.980	5.844/44.440	4.640/24.316	3.503/14.101	3.010/10.924

NOTE.—As in Table 1, except that evolution of a galaxy is included according to eq. (12) with $n = 2$.

30″ in size and is insensitive to a uniform optical component of the EBL, so the noted figure can only strictly be regarded as a lower limit (Tyson 1988a, p. 18). In addition to the aforementioned data on the EBL in the optical, it should be noted that useful work has been done in the infrared by Boughn & Kuhn (1986) and in the ultraviolet by Martin & Bowyer (1989). A first unequivocal detection of the EBL could come in either waveband, perhaps by the orbiting *Cosmic Background Explorer* satellite or the planned *Far Ultraviolet Space Telescope*, respectively. However, in the present account calculations have been made in the optical, mainly because it is desired to make contact with the traditional problem of Olbers's paradox, which it is the aim to clarify. The observational limits on the EBL in the optical mentioned above correspond approximately to $6 \gtrsim I \gtrsim 0.7$ ergs s^{-1} cm^{-3}. Of these limits, the former is the 90% one of Dube et al. (1977, 1979). But this may be unreasonably high, and a probable level for the EBL in the optical is nearer their detection level, which corresponds to about 2 ergs s^{-1} cm^{-3}. These numbers can be compared to the theoretical values given in Tables 1–3. It is seen that for nearly all expanding models the theoretical values are within the observational limits, though the theoretical intensities are uncomfortably high for models close to the de Sitter one with significant galaxy evolution. Better limits on the EBL, or a measurement of it perhaps from space, would allow useful constraints to be set on cosmology and the properties of galaxies.

3. CONCLUSION

Following earlier bolometric calculations by WVS, the intensity of the EBL as a function of wavelength has been studied for various choices of the cosmological parameters, redshift of galaxy formation, and galaxy evolution parameters. This has been done for expanding and equivalent static models, in order to separate the influences of the expansion of the universe and the age of the galaxies. The intensity is directly proportional to the age of the galaxies in a static model by equation (5), and the redshift effect present in the expression for an expanding model equation (4) only causes a reduction of 3–4 typically, as seen in Tables 1–3. However, the intensity also depends on other things, notably the speed of light, the luminosity density of galaxies, and the form of a typical galaxy spectrum. Theoretical values for the intensity are in agreement with presently available limits on the EBL from observations. Further theoretical work could be done in the direction of developing more realistic models, as outlined previously, with the idea of comparison with better observational data which should be available in the future.

A main motivation for doing the calculations described above is to better understand why the intensity of intergalactic light is low even in a universe that may be infinite in extent and uniformly populated by luminous galaxies (Olbers's paradox). Harrison and a few others have long argued that age and not expansion is the dominant factor. (See, for example, Harrison 1965, 1977, 1981; Whitrow & Yallop 1964; this does not imply anything directly about the intensity as a function of time in the real universe, which may decrease if the expansion is fast enough according to these authors.) Harrison's view is endorsed implicitly by many people who are engaged in work on the EBL, because for plausible choices of the parameters involved it is a consequence of the equations that define the intensity, both in the bolometric case and the spectral case. Unfortunately, this topic has a long history of confusion which still persists. What has been shown here using a toy model is that the intensity of intergalactic radiation would still be low even if the universe were static, and that the expansion reduces it only by a relatively modest amount.

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P. S. WESSON: Space Sciences Laboratory, University of California, Berkeley, CA 94720