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ORIGINAL ARTICLE

Force-free magnetosphere of an aligned rotator with differential rotation of open magnetic field lines

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Abstract Here we briefly report on results of self-consistent numerical modeling of a differentially rotating force-free magnetosphere of an aligned rotator. We show that differential rotation of the open field line zone is significant for adjusting of the global structure of the magnetosphere to the current density flowing through the polar cap cascades. We argue that for most pulsars stationary cascades in the polar cap can not support stationary force-free configurations of the magnetosphere.

Keywords Stars: magnetic fields · Pulsars: general · MHD

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1 Introduction

Aligned rotator with the force-free magnetosphere is being considered as a first order approximation for the real pulsar magnetosphere since introduction of the model by Goldreich and Julian (1969). Any pulsar model should be tested for this simplest case. Recently substantial progress in modeling of pulsar magnetospheres was achieved. The magnetosphere of an aligned rotator was modeled using stationary (Contopoulos et al. 1999; Goodwin et al. 2004; Gruzinov 2005; Timokhin 2006) and time-dependent (Komissarov 2006; Bucciantini et al. 2006; McKinney 2006; Spitkovsky 2006) codes. The structure of the magnetosphere has been obtained even for an inclined rotator (Spitkovsky 2006). In these works the angular velocity of plasma rotation was assumed to be constant, although the case when the open field lines rotate with a constant angular velocity different from the angular velocity of the Neutron Star (NS) was addressed in some works (see e.g. Contopoulos 2005; Beskin and Malyshkin 1998). It was also implicitly assumed that the current density in the magnetosphere could adjust to any distribution required by the global structure of the magnetosphere. However, in the pulsar magnetosphere current carriers are produced mainly by the electron-positron cascades and therefore not every current density distribution can be realized (see Timokhin 2006, hereafter Paper I). Moreover, in Paper I it was shown that the current density distribution supporting the magnetospheric configuration frequently used in theoretical pulsar models, that assume the closed field line zone extending up to the Light Cylinder (LC) and the open field lines rotating with the same angular velocity as the NS, could not be realized regardless of a particular model of the polar cap cascades. Such models require presence of an electric current flowing against the accelerating electric field in some parts of the pulsar polar cap, which cannot be naturally explained.

For the natural stationary configuration of the magnetosphere, when the last closed field line lies in the equatorial plane and magnetic field lines become radial at a large distance from the LC (the so-called Y-configuration), the system has two physical "degrees of freedom", namely (i) the size of the closed field line zone and (ii) the angular velocity of rotation of the open field lines. In Paper I the angular velocity of the magnetic field lines was fixed, but the size of the closed field line zone was varied. In any of the obtained configurations the current density is much less than the Goldreich-Julian (GJ) current density along field lines passing close to the boundaries of the polar cap (see Fig. 5 in

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Paper I). However, the current density, which could be produced by stationary polar cap cascades operating in space charge limited flow (SCLF) regime, can not be significantly less than the corresponding GJ current density (e.g. Harding and Muslimov 1998). On the other hand, the current density flowing in the polar cap of pulsar having partially screened polar gap (PSG) cannot be close to zero (Gil et al. 2003). Hence, aligned pulsar with stationary polar cap cascades operating in SCLF or PSG regime cannot have stationary force-free magnetosphere rotating with a constant velocity. As stationary SCLF and PSG are being considered as the most probable regimes for the polar cap of pulsar, it is worth to find configuration of the force-free magnetosphere compatible with them.

Here for the first time we consider self-consistently the case of a differentially rotating force-free magnetosphere of an aligned rotator in Y-configuration, i.e. we are able to explore *all* possible configurations of such magnetosphere. Our aim is to study the impact of differential rotation on the current density distribution in the force-free magnetosphere of an aligned rotator. We also try to find combinations of the angular velocity distribution and the size of the closed field line zone, which could be compatible with stationary polar cap cascade models.

2 Equations for differentially rotating force-free magnetosphere

Magnetic field lines in the closed zone are equipotential and plasma there corotates with the NS. In the open field line domain the angular velocity of plasma Ω_{F} is different from the angular velocity of the NS rotation Ω. The angular velocity of the open magnetic field lines Ω_{F} is determined by the potential difference in the polar cap of pulsar. In stationary cascade models Ω_{F} is close to Ω for young pulsars (see Sect. 2.2 in Paper I).

The force-free magnetosphere of an aligned rotator with differential rotation can be described by a solution of the so-called pulsar equation, derived by Okamoto (1974). In notations of Paper I (see (20) there) it has the form

$$(\beta^2 x^2 - 1)(\partial_{xx}\psi + \partial_{zz}\psi) + \frac{\beta^2 x^2 + 1}{x}\partial_x\psi - S\frac{dS}{d\psi} + x^2\beta\frac{d\beta}{d\psi}(\nabla\psi)^2 = 0. \tag{1}$$

Here $\beta \equiv \Omega_{\mathrm{F}}/\Omega$, S and ψ are normalized poloidal current and magnetic flux functions correspondingly. z axis is coaligned with $\vec{\Omega}$. All coordinates are normalized to the LC radius of a corotating magnetosphere $R_{\mathrm{LC}} \equiv c/\Omega$. The last term in the equation takes into account contribution of differential rotation into the force balance across magnetic field lines. The current density in the open field line zone of the magnetosphere depends on differential rotation through condition at the true LC, which is at $x(\psi) = c/\Omega_{\mathrm{F}}(\psi)$ (see Sect. 2.3 in Paper I)

$$S\frac{dS}{d\psi} = 2\beta\partial_x\psi + \frac{1}{\beta}\frac{d\beta}{d\psi}(\nabla\psi)^2. \tag{2}$$

Changes of β result in changes of the poloidal current density j being proportional to $dS/d\psi$.

So, the differential rotation contributes to the force balance perpendicular to the magnetic field lines, changes the position of the LC and affects the current density distribution, required for smooth transition of the solution trough the LC. Although, the differential rotation can modify the current density distribution, it is a quantitative question whether or not a given differential rotation $\Omega_F(\psi)$ and the corresponding current density distribution $j(\psi)$ could be supported by stationary cascades in the polar cap of pulsar. We note that $\beta(\psi)$ is a free parameter in this problem, while $j(\psi)$ is obtained from the solution.

For solution of (1) we developed an advanced version of the multigrid code described in Paper I. Now the position of the Light Cylinder is found self-consistently on each iteration step and poloidal current is calculated from (2) at the LC. The return current flowing in the current sheet between the closed and open field line zones is smeared over the interval $[\psi_{\mathrm{last}}, \psi_{\mathrm{last}} + d\psi]$,[1] and the angular velocity is continuously changing in the same interval from some value at the last open filed line $\Omega_{\mathrm{F}}(\psi_{\mathrm{last}})$ to Ω at $\psi = \psi_{\mathrm{last}} + d\psi$. Equation (1) is solved in the whole domain including the current sheet. Smearing of the return current and continuous changing of Ω_{F} in the current sheet allows us to take into account the contribution of the current sheet into the force balance across magnetic field lines at the boundary between closed and open field line zones.

3 Main results

Stationary polar cap cascades without particle inflow from the magnetosphere would provide more or less constant current density distribution over the polar cap (see e.g. Harding and Muslimov 1998; Gil et al. 2003). However, in configurations of the force-free magnetosphere with constant Ω_{F} the current density goes to zero near the polar cap boundaries. Here we try to find configurations of a differentially rotating force-free magnetosphere with current density, which is nearly constant over the polar cap at the NS surface. Current density in the magnetosphere rotating with a constant angular velocity decreases toward the polar cap boundary because of the boundary condition $\psi(x > x_0) = \psi_{\mathrm{last}}$ (x_0 is

[1] ψ_{last} is the value of the normalized magnetic flux function corresponding to the field line separating the closed and open field line domains.

the position of the point, where the last closed field line intersects the equatorial plane). In the equation for the poloidal current (see (2)) the first term on the left hand side goes to zero for ψ approaching $\psi_{\rm last}$. Hence, in order to increase the current density near the polar cap boundaries $d\beta/d\psi$ must be positive, i.e. the maximum value of β should be achieved at the polar cap boundary, for $\psi = \psi_{\rm last}$. Here we consider the case when the maximum possible value of β is 1. Such behavior of β is expected in models of the polar cap cascades.

We have performed computations for different sizes of the closed field line zone x_0. In each of these simulations the distribution of β over ψ was adjusted ad hoc in order to obtain nearly constant current density over the polar cap, which approaches zero only at field lines very close to the polar cap boundary.[2] For any size of the closed field line zone x_0 it is possible to construct a set of force-free magnetospheric configurations with the current density distribution being almost constant over the polar cap, $j(\psi) \simeq \hat{j} \equiv \text{const}$ by choosing different $\beta(\psi)$.

One of the important properties of the obtained set of solutions is that for each x_0 the constant current density $\hat{j}$ cannot be made greater than some maximum value $\bar{j}$. Distribution $j(\psi) \simeq \bar{j}$ corresponds to a distribution $\bar{\beta}(\psi)$, which achieves 1 at the polar cap boundary, $\bar{\beta}(\psi_{\rm last}) = 1$. As it was stressed above, the maximum value of β is achieved at $\psi = \psi_{\rm last}$. So, $\bar{\beta}$ differs from other distributions $\beta(\psi)$ which provide constant current density, in that it achieves the maximum possible value. Both $\bar{j}$ and $\bar{\beta}(\psi)$ depend on x_0.

The differential rotation in the magnetosphere of a young pulsar with stationary polar cap cascades is rather small (see Sect. 2.2 in Paper I). So, solutions with the minimal deviation of $\beta(\psi)$ from 1 would be of most interest for us. These turned out to be solutions with $\beta = \bar{\beta}(\psi)$, i.e. with $j \simeq \bar{j}$. One of such magnetospheric configurations with $x_0 = 0.7$ is shown in Fig. 1. Important properties of solutions with $j \simeq \bar{j}$ are:

- The value $\bar{j}$ increases with decreasing of x_0, however it does not exceeds $j_{\rm GJ}/2$, the half of the GJ current density near the NS surface (see Fig. 2).
- The angular velocity of rotation of the open magnetic field lines has smaller deviation from Ω with decreasing of x_0, see Fig. 3. As in the case of the current density distribution there exist an asymptotic distribution of $\Omega_{\rm F}(\psi)$. This asymptotic distribution deviates strongly from $\beta(\psi) \equiv 1$.
- The asymptotic form of $\bar{\beta}(\psi)$ and $\bar{j}$ for $x_0 \to 0$ matches the corresponding distributions in the split monopole configuration.

[2] We assume that the current sheet carries only the return current, so j changes the sign at $\psi_{\rm last}$.

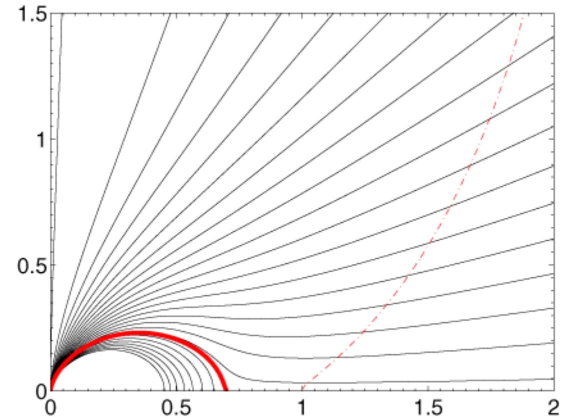

Fig. 1 Structure of the magnetosphere for $x_0 = 0.7$ and the angular velocity of the open magnetic field lines $\Omega_{\rm F}(\psi) = \Omega\bar{\beta}(\psi)$ (see text). Magnetic field lines are shown by the *thin solid lines*, the last closed field line—by the *thick solid line*, the position of the Light Cylinder is shown by the *dot-dashed line*

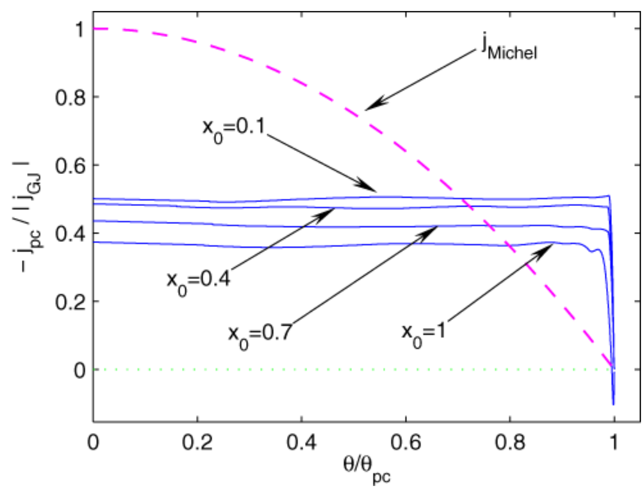

Fig. 2 Current density distributions in the polar cap of pulsar $j_{\rm pc}(\theta)$ corresponding to $j(\psi) \approx \bar{j}$ (see text) are plotted for different x_0 as functions of the colatitude θ. $j_{\rm pc}$ is normalized to the Goldreich-Julian current density $|j_{\rm GJ}|$. θ is normalized to the colatitude of the polar cap boundary $\theta_{\rm pc}$. The current density corresponding to the Michel's solution (Michel 1973) is shown by the *dashed line*. The distribution for $x_0 = 0.1$ practically coincides with the distribution for the split-monopole case

- Energy losses of the aligned rotator increases with decreasing of x_0. This increase goes a bit faster than in the case of constant $\Omega_{\rm F}$, cf. Fig. 4 here and Fig. 8 in Paper I.
- The total energy of the magnetosphere increases with decreasing of x_0.
- In none of the considered configurations the force-free condition is violated in the calculation domain, i.e. the electric field is everywhere smaller than the magnetic field.

Let us now discuss implications of these solutions for the physics of pulsars.

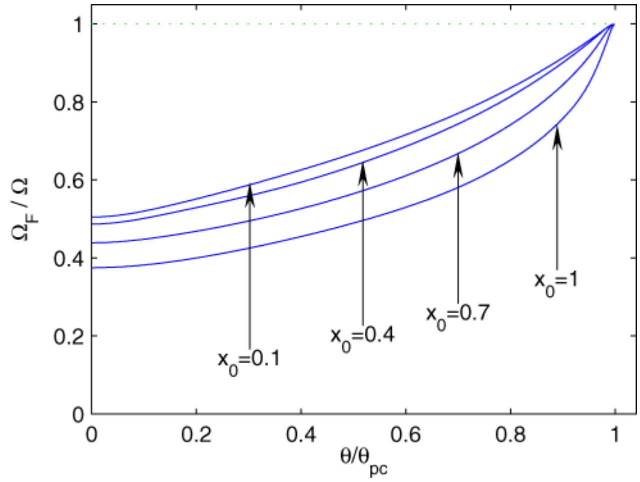

Fig. 3 Angular velocities of rotation of the open field lines $\Omega_F(\theta) = \Omega\bar{\beta}(\theta)$ (see text) for different x_0 as functions of the colatitude θ in the polar cap of pulsar. θ is normalized to the colatitude of the polar cap boundary θ_{pc}. $\Omega_F(\theta)$ is normalized to the angular velocity of the NS rotation Ω. The distribution for $x_0 = 0.1$ practically coincides with the distribution for the split-monopole case

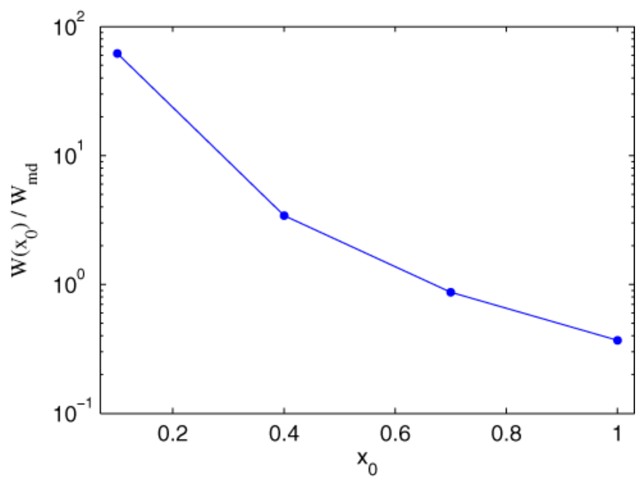

Fig. 4 Energy losses of aligned rotator normalized to the magnetodipolar energy losses as a function of x_0 for differentially rotating magnetosphere with $\Omega_F(\psi) = \Omega\bar{\beta}(\psi)$ (see text)

4 Discussion

If the polar cap cascades operate in stationary SCLF regime the current density in the polar cap of pulsar near the NS surface is very close to the local GJ current density. However, due to inertial frame dragging (i.e. due to changing of the effective Ω) the local GJ charge density decreases slower with the distance than the charge density carried by the charge-separated flow from the NS surface (see Muslimov and Tsygan 1992). This discrepancy achieves ∼15% at a distance of several radii of the NS. So, the current density at that distance will be by ∼15% less then the corresponding local GJ current density. Here we solve the problem in flat space-time. The boundary conditions on the NS surface and, hence, the physical quantities we used for normalization should be taken at some distance from the NS, where GR effects are negligible. The GR corrections will result in ∼15% higher ratio of j to the *local* value of j_{GJ} at the NS surface than the one obtained in our solution. However as it follows from our results, there is no configuration of the magnetosphere with almost constant current density larger than $j_{GJ}/2$. Hence, stationary force-free magnetosphere of an aligned rotator can not be supported by stationary polar cap cascades operating in SCLF regime.

Polar cap cascades operating in Ruderman and Sutherland (1975) regime with almost vacuum electric field in the gap allow larger deviations of the current density from j_{GJ} as well as larger deviation of β from 1. For older pulsar, when β could be rather small, even configuration with x_0 approaching 1 could be supported by the polar cap cascades. In this case the characteristic current density flowing through the gap must be also much less than the GJ current density.

For the partially screened polar gap (PSG) (Gil et al. 2003) the presence of a strong multipolar component of the magnetic field in the polar cap of pulsar is essential. In this model the current density at the NS surface for young pulsars should be close to the local value of j_{GJ}. However, the local GJ charge density is determined by the non-dipolar magnetic field. For this field the angle between $\vec{B}$ and $\vec{\Omega}$ can be rather large and the GJ charge density will be accordingly less. At some distance, where magnetic field becomes dipolar and the angle between $\vec{B}$ and $\vec{\Omega}$ is small, the current density of the flow would become smaller than the local GJ current density. This effect can be much stronger than the difference between j and j_{GJ} due to inertial frame dragging. Hence, for some specific configurations of the local magnetic field at the NS surface, stationary polar cap cascades could support a stationary force-free configuration of the magnetosphere.

So, aligned pulsars with polar cap cascades operating in SCLF regime, young pulsars with "Ruderman-Sutherland" cascades as well as pulsars with PSG and arbitrary surface magnetic field configuration can not have both polar cap cascades operating in stationary regime and a stationary force-free magnetosphere. From energetic point of view, configurations with non-stationary polar cap cascades would seem to be preferable. The energy of the magnetosphere increases with decreasing of x_0. The magnetosphere would try to achieve the configuration with the smallest possible energy, i.e. with the largest possible x_0, compatible with the physical conditions set by the polar cap cascades. If there are several possible self-consistent configuration of the magnetosphere and polar cap acceleration zone, the configuration with the largest $x_0$will be realized. With aging of the pulsar the conditions in the cascades change. This would result in changing of the magnetosphere configuration and, hence, in changes of energy losses relative to the corresponding magnetodipolar losses. This yields the breaking index different from 3. However, more detailed study of polar cap cascade properties is necessary in order to construct self-consistent magnetospheric configurations and to obtain the value of the breaking indexes. The current adjustment is necessary also for inclined rotator and in this case similar effects would be present too.

More detailed description of the results is given in Timokhin (2006).

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References

Beskin, V.S., Malyshkin, L.M.: Mon. Not. Roy. Astron. Soc. **298**, 847 (1998)
Bucciantini, N., Thompson, T.A., Arons, J., et al.: Mon. Not. Roy. Astron. Soc. **368**, 1717 (2006)
Contopoulos, I. Astron. Astrophys. **442**, 579 (2005)
Contopoulos, I., Kazanas, D., Fendt, C.: Astrophys. J. **511**, 351 (1999)
Gil, J., Melikidze, G.I., Geppert, U.: Astron. Astrophys. **407**, 315 (2003)
Goldreich, P., Julian, W.H.: Astrophys. J. **157**, 869 (1969)
Goodwin, S.P., Mestel, J., Mestel, L., Wright, G.A.E.: Mon. Not. Roy. Astron. Soc. **349**, 213 (2004)
Gruzinov, A.: Phys. Rev. Lett. **94**, 021,101 (2005)
Harding, A.K., Muslimov, A.G.: Astrophys. J. **508**, 328 (1998)
Komissarov, S.S.: Mon. Not. Roy. Astron. Soc. **367**, 19 (2006)
McKinney, J.C.: Mon. Not. Roy. Astron. Soc. **368**, L30 (2006)
Michel, F.C.: Astrophys. J. **180**, 207 (1973)
Muslimov, A.G., Tsygan, A.I.: Mon. Not. Roy. Astron. Soc. **255**, 61 (1992)
Okamoto, I.: Mon. Not. Roy. Astron. Soc. **167**, 457 (1974)
Ruderman, M.A., Sutherland, P.G.: Astrophys. J. **196**, 51 (1975)
Spitkovsky, A.: Astrophys. J. Lett. **648**, L51 (2006)
Timokhin, A.N.: in preparation (2006)
Timokhin, A.N.: Mon. Not. Roy. Astron. Soc. **368**, 1055–1072 (2006)